Author:
Yu, Yin

Title:
Analysis of structural vulnerability.

ANALYSIS OF STRUCTURAL VULNERABILITY

by

Yin Yu

A thesis submitted to the University of Bristol in accordance with the requirements for the degree of Doctor of Philosophy in the Faculty of Engineering, Department of Civil Engineering, February, 1997

DEDICATION

to *My Parents*

PAGE NUMBERING AS ORIGINAL

ABSTRACT

Safety is a very important quality of civil engineering structures. Two safety and reliability procedures used by designers are limit state design and reliability theory. Problems and limitations rise from the difficulty in reliability theory of dealing with system failure and the complex dependencies between random variables describing the system. The intention of this research is to look at the whole problem from a different perspective. Instead of trying to find what is the most likely limit state failure condition and probability of failure, the aim of the theory to be presented is to identify the weakest links in a structure.

The vulnerability of a structural system is its susceptibility to disproportionate consequences in the event of damage or failure. Internal vulnerability lies within the system and stems from its internal configuration and form.

Structural vulnerability theory was first introduced by Wu (1991). The purpose of the theory is to identify the most vulnerable part or parts of a structural system so that they may be suitably protected and monitored.

In this thesis, the original structural vulnerability theory is re-examined and improved. The major improvements include: (a) a redefined concept of structural rings to improve and clarify the original theory; (b) two generalised structural ring types with which most cases of structure can be represented; (c) a new clustering criteria consisting of an ordered set of measures to improve the single measure criteria in the original theory; (d) a redefined measure of the failure consequences to improve a technical problem of the original theory, and (e) a vulnerability index as a measure of vulnerability which allows comparisons to be made between various failure scenarios.

Also in this thesis, an algorithm is developed and implemented into a computer program SAVE. The aim of the program is to demonstrate structural vulnerability theory as a workable theory. A selection of various types of structure are used to demonstrate structural vulnerability analysis, such analysis is concerned with the identification of various vulnerable failure scenarios in a structural system.

Finally, the potential application of structural vulnerability theory in planning a structural monitoring scheme is discussed.

ACKNOWLEDGEMENTS

I wish to express my gratitude to **Professor D. I. Blockley** and **Dr. N. J. Woodman**, my supervisors, for their invaluable advice and guidance throughout this research and in producing this thesis.

Thanks are due to all the former and present members of the Civil Engineering Systems Group for their support and many interesting discussions at every stage of the research.

Thanks are also due to the **Engineering and Physical Science Research Council** and **Building Research Establishment** for funding the research.

Finally, special thanks go to my husband, Martin, and my parents and friends for their precious support.

DECLARATION

This thesis entitled: Analysis of Structural Vulnerability. is submitted for the Degree of Doctor of Philosophy, in the Faculty of Engineering, at the University of Bristol.

The research, on which this thesis is based was carried out between October 1992 and February 1997 under the joint supervision of Professor D. I. Blockley and Dr. N. J. Woodman. It is due entirely to the author except where otherwise acknowledged in the text and has not formed the basis of a submission for any other degree.

Signed ....

Date 16/5/97

CONTENTS

Title page I

Dedication II

Abstract III

Acknowledgements IV

Declaration V

List of Figures XII

List of Tables XIV

Notation XVI

Part I *Introduction*

1. Introduction 1

1.1 Objectives of the thesis 1

1.2 Background of this research 2

1.3 Layout of the thesis 3

1.4 Key concepts 5

2. Vulnerability of structural systems 10

2.1 Objectives 10

2.2 Introduction 10

2.3 General aspects 10

2.3.1 Structural systems 11

2.3.2 Safety and reliability 11

2.3.3 Limitations 12

2.4 Concept of vulnerability 13

2.4.1 Internal vulnerability 14

2.4.2 Specific action-related vulnerability 15

2.4.3 Overall vulnerability 16

2.4.4 An example 17
2.5 Modelling system internal vulnerability 18
2.5.1 Vulnerability in the form 18
2.5.2 Searching for possible failure scenarios 19
2.5.3 General search strategies 19
2.5.4 Adopted methodology 19
2.5.4.1 Graph model 20
2.5.4.2 Cluster and hierarchy 20
2.5.4.3 Searching for the most vulnerable failure scenarios 20
2.6 Conclusions 21

Part II Theory

3. Structural Rings & Well-formedness 22
3.1 Objectives 22
3.2 Introduction 22
3.3 Graph theory 23
3.3.1 Concept of graphs 23
3.3.2 Incidence and adjacency matrices 25
3.3.3 Degree of vertex 27
3.3.4 Sub-graphs 27
3.3.5 Paths and cycles in a graph 28
3.3.6 Connectivity of graphs 28
3.3.7 Weighted graphs 29
3.4 The graph representation of a structural system 29
3.4.1 Structural systems 30
3.4.2 Representing a structural system with a graph 31
3.4.3 The association and fixity matrices 32
3.4.4 Structural paths and loops 33
3.4.5 Degree of joints 34
3.4.6 Comparison of terms 34

3.5 Structural rings 35
3.5.1 Concept of structural rings 35
3.5.2 Types of structural rings 36
3.5.3 String patterns of structural rings 39
3.5.4 Redundancy, deterioration and failure of a structural ring 40
3.5.5 Deterioration hierarchy of structural rings (DHSR) 41
3.6 Well-formedness of a structure 43
3.6.1 Concept of well-formedness 43
3.6.2 Stiffness matrix and principal stiffness coefficients 44
3.6.3 Well-formedness of a structural joint 46
3.6.4 Well-formedness of a structure 50
3.7 Conclusions 52

4. Structural clusters 54

4.1 Objectives 54
4.2 Introduction 54
4.3 Cluster Analysis 55
4.3.1 Techniques of cluster analysis 56
4.3.2 Clusters 57
4.3.3 Clustering criteria 58
4.4 Structural Clusters 59
4.4.1 Method of clustering structural systems 59
4.4.2 Definition of a structural cluster 60
4.4.3 Type of structural clusters 61
4.4.4 Structural rings and structural clusters 62
4.4.5 Measures of a structural cluster 63
4.4.5.1 Well-formedness of structural clusters 63
4.4.5.2 Minimum Damage demand of structural clusters 64
4.4.5.3 Nodal connectivity of structural clusters 66
4.4.5.4 Distance from the reference 67
4.5 Cluster formation 68
4.5.1 Principles of cluster formation 69

4.5.2 Criteria 70
4.5.3 Initial, secondary and reference clustering 71
4.6 Example 73
4.7 Conclusions 83

5. Hierarchical representation of a structural system 85
5.1 Objectives 85
5.2 Introduction 85
5.3 Systems approach 86
5.4 Hierarchy 87
5.4.1 The structure of a complex system 87
5.4.2 Holons 88
5.4.3 Levels 89
5.5 The hierarchical representation of a structural system 91
5.5.1 Internal and external connectivity of structural clusters 91
5.5.2 Complex joints 92
5.5.3 Forming a structural ring at a higher level of description 93
5.5.4 The structural system as a hierarchy 95
5.6 Hierarchy Formation 96
5.7 Conclusions 108

6. Failure scenarios and vulnerability analysis 110
6.1 Objectives 110
6.2 Introduction 110
6.3 Failure scenarios 111
6.3.1 Structural rings and a structural system 111
6.3.2 Damage process and failure state 114
6.3.3 Failure scenarios 115
6.3.4 The damage demand of a failure scenario 118
6.4 Vulnerability analysis 119
6.4.1 Failure scenarios in a structural system 119
6.4.2 Failure consequences 120

6.4.3 Damage scale & separateness 122
6.4.4 The vulnerability index of a failure scenario 123
6.4.5 Vulnerable scenarios 126
6.4.5.1 Minimal failure scenarios 127
6.4.5.2 Maximal failure scenarios 127
6.4.5.3 Interesting failure scenarios 128
6.4.6 Identification of vulnerable scenarios 128
6.4.7 Applications of the vulnerability analysis 130
6.5 Conclusions 131

Part III Implementation

7. Algorithm 133
7.1 Objectives 133
7.2 Introduction 133
7.3 General outline of SAVE 134
7.4 Data structure 134
7.5 Data input 139
7.6 Data preparation and preliminary calculation 142
7.7 Hierarchy formation 142
7.8 Identification of failure scenarios 148
7.8.1 Minimal failure scenarios 149
7.8.2 Maximal failure scenarios 150
7.9 Conclusions 161

8. Structural Analysis for Vulnerability Estimation 163
8.1 Objectives 163
8.2 Introduction 163
8.3 Truss structures 164
8.4 Frame structures 183

8.5 Combined structures 193

8.6 Conclusions 205

Part IV *System vulnerability and structural monitoring*

9. Structural Monitoring with Vulnerability Analysis 207

9.1 Objectives 207

9.2 Introduction 207

9.3 Current practice of structural monitoring 208

9.3.1 General purposes 208

9.3.2 Methods and instrumentation 209

9.3.3 Problems 211

9.4 Vulnerability analysis and structural monitoring 211

9.5 Conclusions 214

Part V *Conclusion*

10. Conclusions and recommendations for future work 215

10.1 Conclusions 215

10.2 Recommendations for future work 218

References 220

Appendix-1 228

Appendix-2 231

Appendix-3 233

LIST OF FIGURES

No.	Title	Page
2.1	Vulnerability research in relation to other safety procedures	13
2.2	St Louis bridge, October, 1969	14
2.3	Rayland bridge, April, 1968	15
2.4	Relationship between internal, specific action-related and overall vulnerability	17
3.1	Graphs	24
3.2	Graph definitions	25
3.3	Sub-graphs	27
3.4	Paths and cycles in a graph	28
3.5	Disconnected and connected graphs	28
3.6	Illustration of a 3-link-ring	36
3.7	Illustration of a 2-link-ring	37
3.8	Different forms of a 2-link-ring	38
3.9	Deterioration of a 2-link-ring	41
3.10	A partial deterioration hierarchy of structural rings	42
3.11	Three different structures with the same structural ring pattern	43
4.1	Example of a set of two-dimensional data	57
4.2	Clusters in a graph	58
5.1	A structural system represented at two different levels of description	92
5.2	Complex joints	93
6.1	A structure	111
6.2	The hierarchical representation	112

6.3	A damage process of a portal frame	114
6.4	A failure scenario	116
6.5	A branch cluster containing other branch clusters	118
6.6	Failure consequences of various clusters in a structure	119
6.7	Failure consequence depending on different Reference	120
7.1	The data structure for a structural cluster	133
7.2	Hierarchical representation of a structure	142
7.3	Decision-making process of unzipping	150
7.4	Detail of the unzipping process	151
7.5	Detail-1 in Figure 7.4	152
7.6	Detail-2 in Figure 7.4	154
8.1	Information in the hierarchical representation	161
8.2	The structure --- Truss-1	161
8.3	Hierarchical representation of Truss-1	166
8.4	The structure --- Truss-2	167
8.5	Hierarchical representation of Truss-2	175
8.6	The structure --- Frame-1	180
8.7	Hierarchical representation of Frame-1	188
8.8	The structure --- Combined-1	190
8.9	Hierarchical representation of Combined-1	192
9.1	Principle of active control of automatic monitoring system	206

LIST OF TABLES

No.	Title	Page
3.1	Type of joint in graph model	31
3.2	Joints in graph model and in real-life	32
3.3	Comparison of terms in graph model and mathematics	34
3.4	String pattern for objects in a structural ring	39
3.5	Well-formedness for structures (a)	51
3.6	Well-formedness for structures (b)	52
4.1	Step-by-step cluster formation of a structure	74
5.1	Step-by-step hierarchy formation of a structure	97
6.1	Details of structural rings in the hierarchy	112
6.2	Joint co-ordinate detail of the structure in Figure 6.6(a)	122
6.3	Member detail of the structure in Figure 6.6(a)	123
6.4	Constraint condition of the structure in Figure 6.6(a)	123
6.5	Calculation of the vulnerability index	123
8.1	Joint co-ordinate table of Truss-1	162
8.2	Member properties table of Truss-1	162
8.3	Constraint condition of Truss-1	162
8.4	Step-by-step cluster formation --- Truss-1	163
8.5	Minimum failure scenarios for Truss-1	167
8.6	Maximal failure scenarios for Truss-1	167
8.7	Joint co-ordinate table of Truss-2	168
8.8	Member properties table of Truss-2	169
8.9	Constraint condition of Truss-2	168

8.10	Step-by-step cluster formation --- Truss-2	170
8.11	Minimum failure scenarios for Truss-2	176
8.12	Maximal failure scenarios for Truss-2	176
8.13	Some interesting failure scenarios for Truss-2	179
8.14	Joint co-ordinate table of Frame-1	180
8.15	Member properties table of Frame-1	181
8.16	Constraint condition of Frame-1	180
8.17	Step-by-step cluster formation --- Frame-1	181
8.18	Minimum failure scenarios for Frame-1	188
8.19	Maximal failure scenarios for Frame-1	188
8.20	Some interesting failure scenarios for Frame-1	189
8.21	Joint co-ordinate table of Combined-1	190
8.22	Member properties table of Combined-1	191
8.23	Constraint condition of Combined-1	192
8.24	Minimum failure scenarios for Combined-1	193
8.25	Maximal failure scenarios for Combined-1	193
8.26	Some interesting failure scenarios for Combined-1	201

NOTATIONS

G	Graph
V	Vertex set in G
E	Edge set in G
v_i	ith vertex
e_i	ith edge
M(G)	Incidence matrix of G
A(G)	Adjacency matrix of G
D(G)	Degree of vertices matrix of G
d(v_i)	Degree of the ith vertex
P	Path in G
S	Structural system
M	Member set in S
J	Joint set in S
m_i	ith member
j_i	ith joint
C(S)	Association matrix of S
F(S)	Fixity matrix of S
D(j)	Degree of joint matrix of S
d(j_i)	Degree of ith joint
R	Structural ring
DOF	Degree of freedom
D_M	String pattern for a member object
D_J	String pattern for a joint object
Red[R]	Redundant DOF in R
K	Structure stiffness matrix
K_{ii}	Submatrix of K associated with ith joint
det(K_{ii})	Determinant of K_{ii}
λ	Eigenvalue
q_i	Quality of the well-formedness of ith joint
l	Level of description
C^l	Structural cluster at a level of description
Q(C^l)	Well-formedness of C^l
δ	Validity factor for Q(C^l)
$d^l_{i,j,k}$	A DOF in a joint object
$s^l_{i,j,k}$	A DOF in a cluster
$f^l_{i,j,k}$	The kth deteriorating event causing the loss of a DOF in a joint
$g^l_{i,j,k}$	The kth deteriorating event causing the loss of a DOF in a cluster
$e(f^l_{i,j,k})$	Damage demand for a deteriorating event $f^l_{i,j,k}$
$e(g^l_{i,j,k})$	Damage demand for a deteriorating event $g^l_{i,j,k}$

$e_{\min}(C^l)$	Minimum damage demand of a cluster
$\eta(C^l)$	Nodal connectivity of a cluster
$\Delta(C^l)$	Distance from reference of a cluster
$F_h(R^l)$	A failure scenario
$E[F_h(R^l)]$	Damage demand of a failure scenario
$E_r[F_h(R^l)]$	Relative damage demand of a failure scenario
$\gamma[F_h]$	Separateness of a failure scenario
$\xi[F_h]$	Vulnerability index of a failure scenario

Part I Introduction

Chapter 1

Introduction

1.1 Objectives of this thesis

The objectives of this thesis are to:

1. present an refined version of the theory of structural vulnerability first formulated by Wu (1991). Vulnerability theory is a theory of form, the purpose of which is to identify the most vulnerable part or parts of a structural system;

2. define the concept of vulnerability in the context of engineering safety;

3. review the theoretical background of graph theory and redefine the concept of structural ring;

4. define two types of structural rings with which all cases of structural configuration can be represented;

5. review the concept of well-formedness as a measure of the form of a structural system, and refine the calculation of the measure;

6. review the concept of structural clusters and define four types of structural clusters;

7. refine the clustering criteria, which is the vital part of the cluster formation algorithm, i.e. add three further measures to form an ordered set of measures as the clustering criteria instead of the original single measure criteria;

8. represent a structural system in the form of hierarchy;

9. redefine Wu's definition of vulnerable failure scenarios (the minimal and maximal failure scenarios) to include: the minimum demand failure scenario, the least well-formed cluster scenarios, the total failure scenario and the maximum failure scenario;

10. redefine separateness as a measure of the failure consequences of a failure scenario;

11. define a vulnerability index as a measure of the potential for disproportionate consequence of a failure scenario;

12. develop an algorithm to implement structural vulnerability analysis;

13. develop a computer program which will demonstrate structural vulnerability analysis;

14. illustrate structural vulnerability analysis with various examples;

15. discuss the potential application of structural vulnerability theory in structural monitoring.

1.2 Background of this research

The previous work by Wu (1991) and Hashimoto (1994) has laid down the theoretical foundation for this research. Many important concepts were first introduced by Wu. There are several issues in the original work which need to be improved. These issues, some conceptual, some technical, are identified in order to implement the theory. In this research, the major objective is to further develop structural vulnerability theory and to produce a computer program to demonstrate it as a workable theory.

In this thesis, the theoretical foundation of the original theory is re-examined and improved. The major improvements required of the theory are to:

1. clarify the important concept of structural rings;
2. generalise structural rings into two types, within which most cases of structure can be represented;
3. improve the clustering criteria which is the central issue for the hierarchical representation of the structural system;
4. redefine separateness as a measure of the failure consequence to improve some technical problems in the original theory;
5. define a vulnerability index as a measure of vulnerability which allows comparisons to be made between various failure scenarios.

1.3 Layout of the thesis

The material in this thesis is organised into five parts: introduction, theory, implementation, application and conclusion.

In the introduction, the objectives of the research are stated in Chapter 1. The concept of vulnerability is discussed in depth and the framework of the research is set in term of engineering safety in Chapter 2.

The theory is set out in Chapter 3, 4, 5, and 6.

In Chapter 3, the theoretical background of graph theory is reviewed and the graph model which represents a structural system is introduced. The concept of structural ring is redefined and the concept of the well-formedness as a measure of the form of a structure is reviewed.

In Chapter 4, the use of cluster analysis and the concept of structural clusters is reviewed. Developing from Chapter 3, the concept of structural rings and the measure of well-formedness are important in establishing structural clusters. Structural clusters are categorised into four types. The clustering criteria, which is the vital part of the cluster formation process, is refined. The principle and process of cluster formation is demonstrated with an example in Chapter 4.

In Chapter 5, systems concepts and methodology for modelling complex systems are introduced and represented as a structural system in the form of a hierarchy of structural clusters. The hierarchical representation allows a structural system being modelled at different levels of detail and each structural cluster is a holon at a level of description.

In Chapter 6, the cluster analysis and hierarchical representation are brought together to form a basis for vulnerability analysis. Various failure scenarios are defined and identified using the hierarchical model of the structural system. Chapter 6 defines a measure of vulnerability (vulnerability index) to evaluate the potential of disproportionate consequences of failure scenarios.

The implementation is set out in Chapter 7 and 8.

In Chapter 7, the algorithm for the computer program SAVE (Structural Analysis for Vulnerability Estimation) is introduced and each of the five modules of the program are discussed in detail. They are: data input, data preparation and preliminary calculation, hierarchy formation, search for minimal failure scenarios and search for maximal failure scenarios.

In Chapter 8, a variety of examples are given to demonstrate the vulnerability analysis.

The application is set out in Chapter 9, in which the potential application of structural vulnerability theory in structural monitoring as an integral part of the whole design, construction and maintenance process for a systems approach to safety management is discussed.

Finally, in the concluding Chapter 10, the conclusions of the thesis and some recommendations for future research are presented.

1.4 Key concepts

Some key concepts which are used in or related to this thesis are listed as follows:

Safety:
The correspondence between a required state of the world and the actual state of the world (Blockley, 1992b).

Reliability:
A measure of the safety margin of the artefact, i.e. the "distance" between the required state of the world and the actual state of the world (Blockley, 1992b).

Structural system:
The subsystem in an engineering artefact that sustains its form. (Chapter 2)

Vulnerability:
The susceptibility to disproportionate consequences in the event of damage or failure. Internal vulnerability is in the form of a system which stems from its internal configuration. Specific action-related vulnerability is concerned with not only the form of the system, but also the nature of a specific action. Overall vulnerability is concerned with a certain period of time and the possible actions to which the system may be subjected. (Chapter 2)

Graph model of a structural system:

A graph model which represents a structural system and consists of a set of joint objects and a set of member objects. Each member object is defined by two joint objects.

Structural ring:

A pattern in the graph model of structure which has sufficient degrees of freedom to maintain equilibrium. The pattern defines the structural characteristics of a set of objects in the graph model. (Chapter 3)

Degree of freedom:

The capacity of a structural object (a joint or a member object) to permit the transmission of force in a principal co-ordinate direction. (Chapter 3)

Structural cluster:

A subset in the graph model of a structure, the objects of which must be (1) able to form a structural ring or a set of overlapping structural rings, and (2) more tightly connected to each other than to those not inside the cluster. (Chapter 4)

Leaf/primitive cluster:

A structural cluster which contains a single member object. (Chapter 4)

Branch/intermediate cluster:

A structural cluster which contains more than one member object. (Chapter 4)

Root/complete cluster:

A structural cluster which contains the entire set of objects in the structural system. (Chapter 4)

Reference cluster:

A specific structural cluster which must not be damaged or is undamagable. For building structures, it is normally the ground. (Chapter 4)

Well-formedness:

A measure of the form of a structure, which is independent of the co-ordinate system, but closely related to the principal stiffness coefficients of the joints, the type of joint, the stiffness of the members and the configuration of the members in the structure. (Chapter 3)

Holon:

A concept which is both a part and a whole. It is a part of a wider system and is itself a whole with respect to its subsystems. (Koestler, 1967)

Hierarchy:

A multi-levelled structure of an ordered set of entities, each of which is a holon. Each entity is a sub-system of the whole system. The entities at lower levels are a more detailed description of the system than those at higher levels. There may be emergent properties that are associated with a specific level, i.e. properties only meaningful at a particular level and do not obtain at any other levels. (Chapter 5)

Deteriorating event:

The loss of one degree of freedom resulting from actions. (Chapter 3)

Failure state:

The state where a structure lacks sufficient capacity to maintain static equilibrium, i.e. becomes a mechanism.

Deterioration hierarchy of structural rings:

A hierarchy which include all possible patterns in which a structural ring deteriorates into a mechanism. (Chapter 3)

Scenario:

An ordered set of possibilities (Blockley, 1992b).

Failure scenario:

A scenario in which the final state is a failure. More specifically, it is a path in the DHSR such that the final element is a mechanism. (Chapter 6)

Damage demand:

A measure of the effort which is required to cause a deteriorating event. (Chapter 6)

Nodal connectivity:

The total number of member objects connecting to the joint objects in a structural cluster. It is an indication of potential capacity of the cluster to form further structural rings with other clusters. (Chapter 4)

Separateness:

A measure of the failure consequence. It is calculated as the ratio of the loss in structural well-formedness of the separated structures, which is caused by the failure scenario, to the well-formedness of the intact structure. (Chapter 6)

Vulnerability index:

A measure of vulnerability in a structural system. For a failure scenario, it is calculated as the ratio of the separateness to the relative damage demand. (Chapter 6)

Minimum demand scenario:

The failure scenario which requires the least damage demand to cause the failure of a structural ring at a level of description. It is therefore the easiest possible way to cause damage to the structural system. (Chapter 6)

The least well-formed cluster scenario:

The scenario where the connection between clusters in a structural system is loose, hence poorly formed. It is the weak link in the form of the structure. (Chapter 6)

Total failure scenario:

The failure scenarios which requires the least effort to cause total failure, i.e. complete disconnection of the structure from the reference cluster.

Maximum failure scenario:

The failure scenario which has the highest value of vulnerability index.

Vulnerability analysis:

Vulnerability analysis is concerned with the identification of various vulnerable scenarios, such like:

- the minimum demand failure scenario;
- the least well-formed cluster scenario;
- the total failure scenario;
- the maximum failure scenario, and
- any interesting failure scenarios.

Chapter 2

Vulnerability of Structural Systems

2.1 Objectives

The objectives of this chapter are to:

- set a framework for the research;
- review briefly some general aspects of engineering safety;
- introduce the concept of vulnerability;
- introduce the methodology of modelling system vulnerability.

2.2 Introduction

In this chapter, some general aspects of engineering safety will be reviewed briefly, to give the background and motivation for this research. The concept of vulnerability will be discussed under three headings: internal vulnerability, specific action-related vulnerability and overall vulnerability. The differences between them will be illustrated with an example. And finally, a methodology of modelling internal vulnerability of a structural system will be outlined.

2.3 General Aspects

Structural engineering is about designing and building structures. The quality of structures is essentially fitness for purpose. The quality or purpose requirements include function, safety and reliability, economy and environmental impact, etc.
Our prime concern in this research is with safety.

2.3.1 Structural systems

The term "structural system" will be used throughout this thesis. In a structural system, components are configured in a certain way in order to achieve a set of structural functions. The interrelation between components is important as well as the properties and characteristics of the components.

A structural system is part of a system of engineering artefact. Take a watch as an example:

A watch, as a whole, has the function of telling time. The "structure" of the watch, in general sense, means all its components and the way they are connected. There is a clear distinction between the "structure" and the "structural system" of a watch. The watch may consist of several systems, such as energy system, timing system, presentation system and structural system, etc. The energy system includes the parts handling energy input, storage and output. The timing system includes the parts that giving correct time. The presentation system includes the choice of material, colour, shape and form of the watch. The *structural system* includes the metal frame and some other components which *hold the watch together*. The structural system of a watch may be concerned with some components but not all of them.

Slightly different from the case of a watch, in civil engineering practice, the term structure may be used to mean a building, a bridge, or other engineering artefacts. By structural system, we mean simply the sub-system in an engineering artefact that sustains its form.

2.3.2 Safety and reliability

The safety of an engineering artefact has been defined as the correspondence between a required state of the world and the actual state of the world (Blockley, 1992b). The central issue of general techniques and procedures for dealing with engineering safety is modelling uncertainty. Blockley classified uncertainty as FIR: *F*uzziness (imprecision of definition), *I*ncompleteness (open world model) and *R*andomness (lack of specific pattern in information) (Blockley, 1992a, 1994).

In civil engineering practice, the traditional approach is the use of safety factor. Current procedures used by designers include limit state design and reliability theory. Reliability theory is concerned with calculation and prediction of probability of limit state violation, i.e. failure. Much work has been done in this field in the past 50 years. The result is a rich collection of techniques and considerable depth of understanding.

The limit state design and reliability theory address only the technical aspect of the problem. Blockley argued that safety is not only a technical problem of reliability, but also a social/technical problem of responsibility and attention should be paid to hazard engineering which is concerned with the identification, by management, of incubating accidents and disasters (Blockley, 1993). Safety procedures which deals with the social aspect of the problem includes good management and safety culture in the whole process.

2.3.3 Limitations

Despite the development and success in application under appropriate situations, reliability theory has some difficult problems. Several fundamental and practical problems in the reliability theory has been addressed critically by Elms and Turkstra (Blockley, 1992a). The fundamental problem stems from basic modelling issues. Practical problems arise because:

- the system failure problem is difficult to deal with,
- complex dependencies between random variables describing the components of the system are difficult to deal with,
- complexity in some parameters and distributions, such as loading, are difficult to model,
- limit states are hard to define,
- judgement variables are difficult to incorporate,
- human factors are difficult to deal with.

Where system failure is concerned, failure sometimes occurs due to highly unlikely circumstances coming together. The failure could occur even though the probability of an event is very low.

An alternative, which has not yet been paid enough attention, is to tackle the problem from a different angle and concentrate on the level of defence against the consequences. Accidents like famous Ronan Point happened because the structure had an unexpected vulnerability.

The original motivation for vulnerability theory, which will be proposed in this thesis, was to recognise the difficulty of calculating probability of failure in the light of complex dependencies and to look at the whole problem from a different perspective.

Instead of asking what is the most likely limit state failure condition and the probability of failure of a component, we are asking, for any load, where is the weakest link.
If we consider approaches and procedures for ensuring safety as a whole set, vulnerability study is to contribute to the set from a different perspective of reliability theory and general procedures (Figure 2.1).

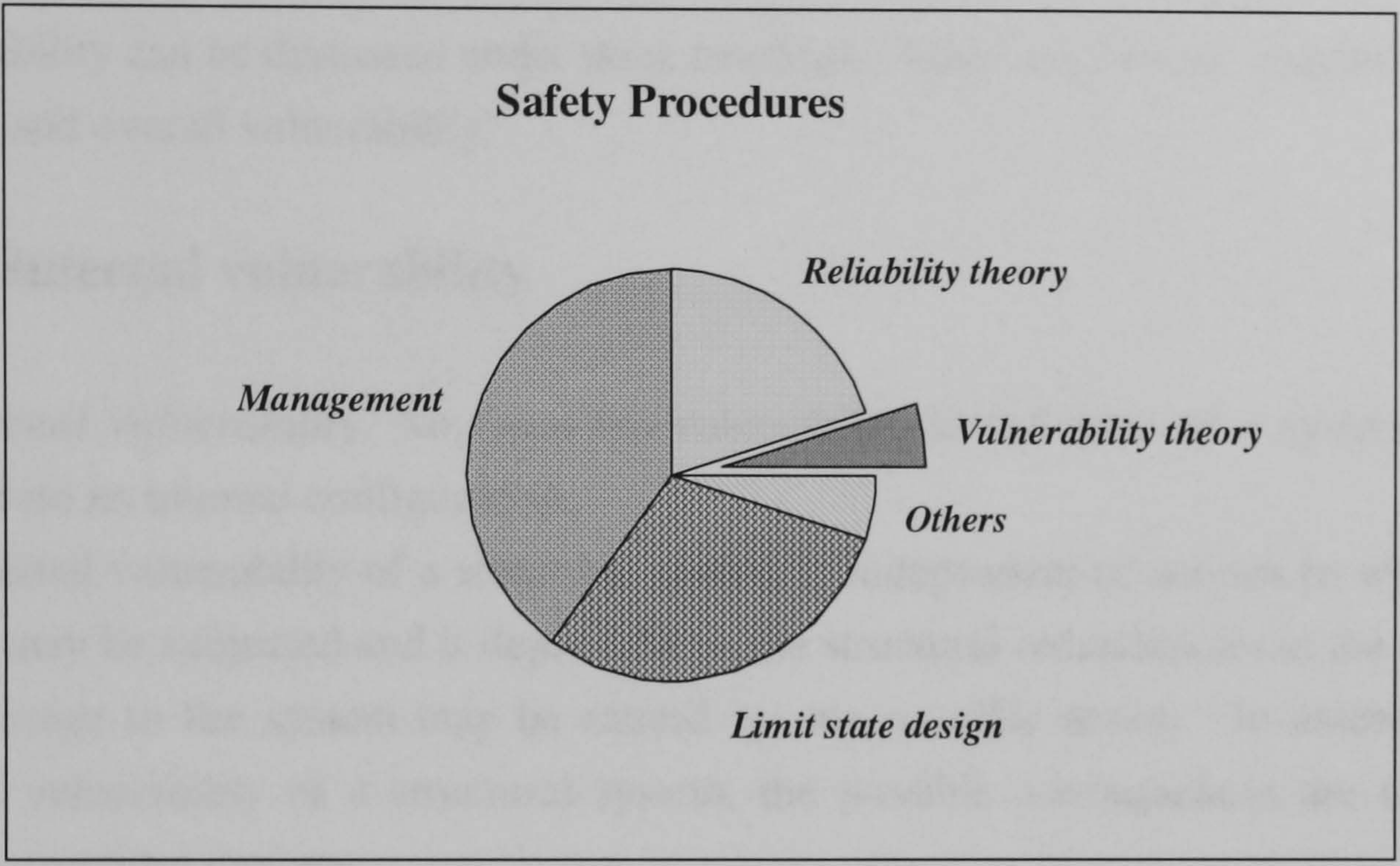


Figure 2.1 Vulnerability research in relation to other safety procedures.

2.4 Concept of Vulnerability

The word "vulnerability" is defined in the Oxford English dictionary as:

- susceptibility to injury or hurt,
- openness to temptation,
- liability to attack.

The general term "vulnerability" is used in many different ways in everyday language. In social science, it is commonly understood to mean the characteristics of a person or group in terms of their capacity to anticipate, cope with, resist and recover from the impact of a natural hazard (Blaikie, *et al*, 1994). In earthquake engineering, the term is used as the degree of loss to a given element at risk, or set of such elements, resulting from an earthquake of a given magnitude or intensity, which is usually expressed on a scale from 0 (no damage) to 1 (total loss) (EERI, 1984).
In whatever the context, the central issue of vulnerability is about susceptibility to damage and failure (Wu, 1991). It is a combination of (1) the ease with which the system may be damaged and (2) the potential consequence of such damage.

In a general system, vulnerability is a quality of the system. It is concerned with:

- the damage (or failure events) to which the system is subjected,
- the capability of the system to remain intact, and
- the consequence of the damage.

Vulnerability can be discussed under three headings. They are internal, specific action-related, and overall vulnerability.

2.4.1 **Internal vulnerability**

By *internal vulnerability*, we mean the vulnerability in the form of a system which stems from its internal configuration.
The internal vulnerability of a structural system is independent of actions to which the system may be subjected and is dependant on the structural redundancies in the system. The damage to the system may be caused by *any possible* action. In assessing the internal vulnerability of a structural system, the possible consequences are confined here to structural damage.
The internal vulnerability of different structural systems can be compared. The systems which have significant internal vulnerability are those ones that either have a "weak link" in the system or the consequences of local damage are significantly larger than the damage itself.
The following two figures are used to illustrate the internal vulnerability in the bridge shown in Figure 2.2 and Figure 2.3 (ASCE 1972).

Figure 2.2 St Louis bridge, October, 1969.

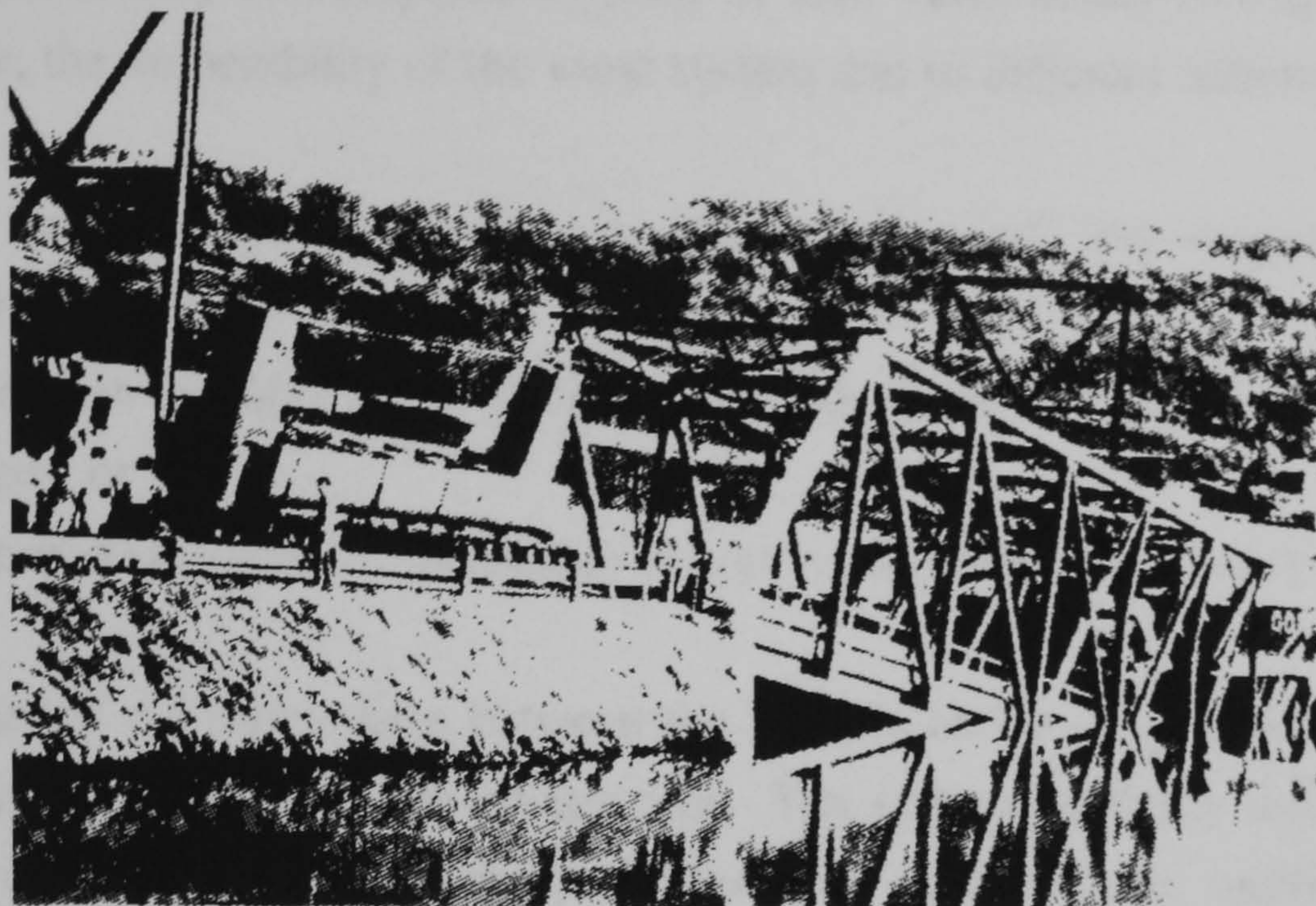


Figure 2.3 Rayland bridge, April, 1968.

In figure 2.2, a section of the lower chord of one of the arches was knocked out by a tug. The damage was confined to the section. The capability of the bridge to resist disproportionate damage is largely due to the design philosophy as its designer James B. Eads addressed at the inauguration of the bridge:

> "... the peculiar construction of the superstructure is such that any piece
> in it can be easily taken out and examined, and replaced or renewed,
> without interrupting the traffic of the bridge."

Figure 2.3 shows a case where a truck knocked out the bracing for the upper chord of a bridge and caused collapse of the whole bridge.
It is apparent that the saving in life and limb and avoidance in economic disruption due to the consideration of the internal vulnerability of the system is sometimes significant, thus very important.

2.4.2 Specific action-related vulnerability

The internal vulnerability focuses on the form of the system. In the final analysis, the vulnerability of a system is also dependent on the nature of an action. This is termed the *specific action-related vulnerability* of the system.
The nature of an action may include its magnitude, location/distribution, and frequency of occurrence, etc.

Different systems may be compared in terms of their vulnerability to a specific action. Alternatively, the vulnerability of the same system due to different actions may also be compared.
The remark "suspension bridges are vulnerable to high wind" may suggest one of two cases or both:

- *suspension bridges* are vulnerable to high wind compared to cable-stayed bridges, or
- suspension bridges are vulnerable to *high wind* compared to heavy rainfall.

The fundamental difference here between the specific action-related vulnerability and system reliability theory is in the perspective. The system reliability theory is mainly concerned with methods to assess a structural system with multiple, perhaps correlated, limit states and to deal with interdependent multiple variables (for details, see Melchers, 1987). However, the specific action-related vulnerability is an extension from the internal vulnerability and is concerned with where the structural system is most vulnerable.

2.4.3 **Overall vulnerability**

The *overall vulnerability* of a system is concerned with a certain period of time and the possible actions to which the system may be subjected. The assessment of overall vulnerability will need to take into account the probability of occurrence of actions, as well as the change in the system (damage, deterioration, etc.) with time. It is concerned with the damage and consequences of a system as a process with time.
At this top level of definition, the vulnerability assessment may also be concerned with the system as embedded in a broader social system. Therefore, the system as well as the consequence of any failure events are more complex to model.

Figure 2.4 illustrate these three types of vulnerability of a given system.

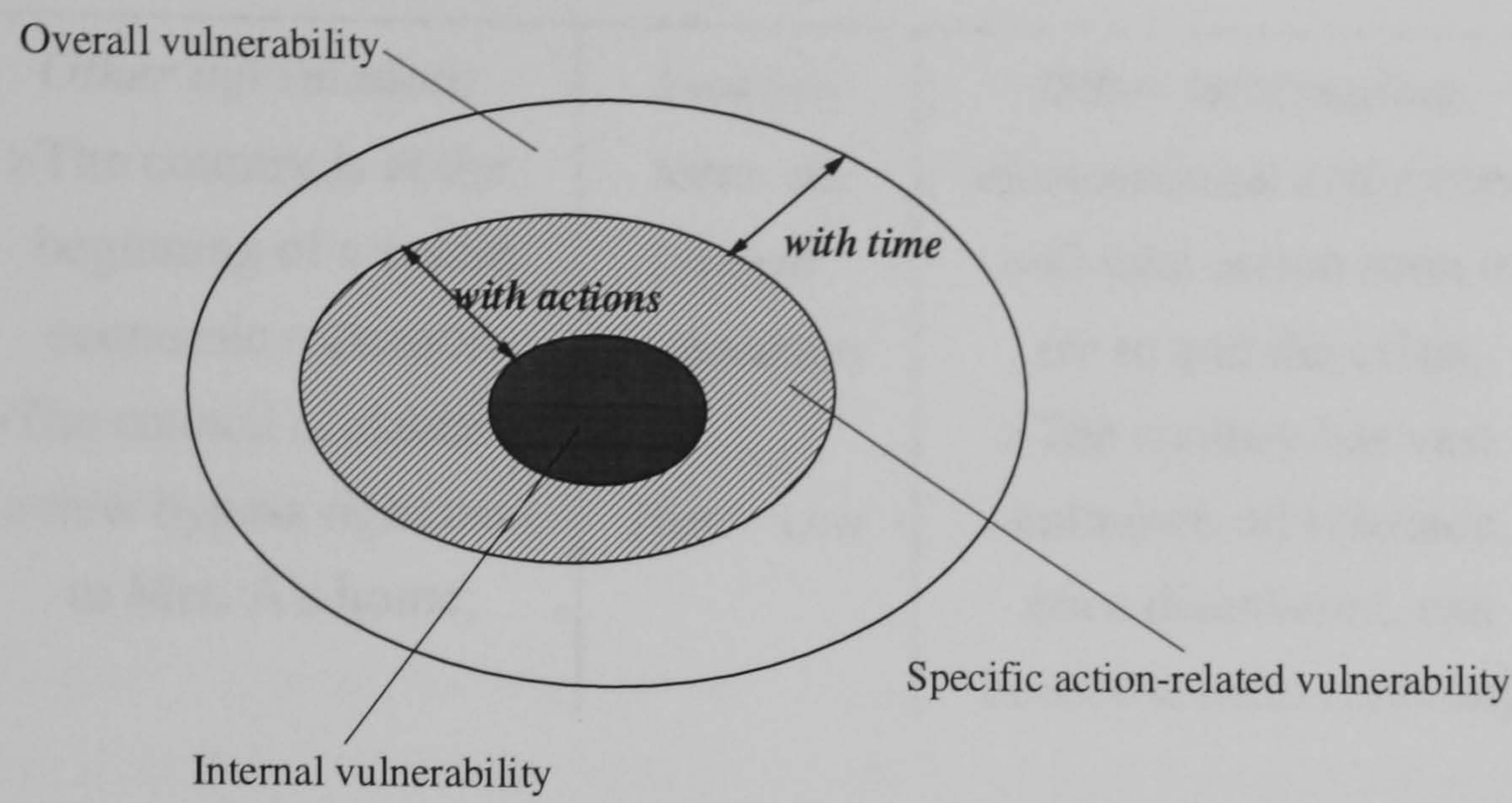


Figure 2.4 Relationship between internal, specific action-related and overall vulnerability

2.4.4 An example

Let's take a brief break from engineering systems and use two people as an example.

Mrs. A:		**Mr. B:**
>Age: 60, >Retired teacher, >Suffers from high blood pressure and some heart problems,	Internal vulnerability High -- Low ⇓	>Age: 28, >Unemployed, >No major health problem,
At a particular time: >Lives in a peaceful area with good health care and good general facilities, >Financially secure. Children all grown up, having their own family,	Action-related or environment-related vulnerability Low -- High ⇓	*At a particular time*: >Lives in a country which suffers recently from a combination of civil war, poverty and natural disasters, >Have family to support, but there is no job or food around,

Other information: >The country is at the beginning of a severe economic recession, >The council is planning a new bypass right next to Mrs. A's home,	Look into future, the overall vulnerability High -- Low	*Other information*: >International committee will take action soon to try to end the crisis, >The country has vast unknown oil resource, once discovered, can boost the local economy,

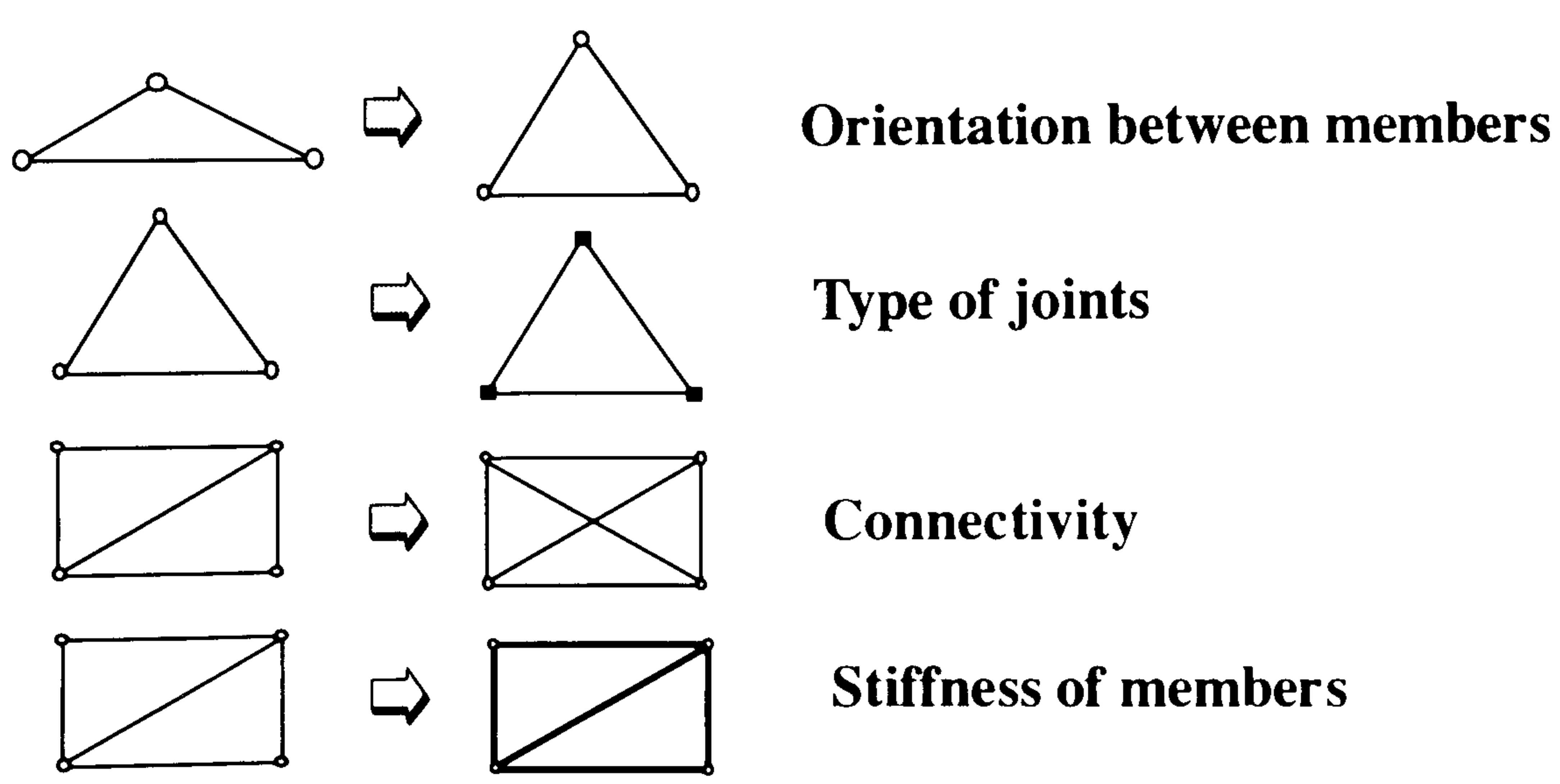
2.5 Modelling System Internal Vulnerability

Internal vulnerability is concerned with the form of a structural system. It is also concerned with the consequence of any damage due to its internal configuration. In order to assess system internal vulnerability, we need to build a model of the form of a structural system.

2.5.1 Vulnerability in the form

The form of a system determines its efficiency in terms of its functions. The internal vulnerability of a structural system is essentially the vulnerability in its form. The quality of the form is very important in assessing the vulnerability in the structural form.

In a structural system, good form is related to:



Apart from a model of the form, it is also necessary for a method to evaluate the consequences of damage and relate them to the form of a structural system, i.e. a method to identify and evaluate various failure scenarios.

2.5.2 Searching for possible failure scenarios

A failure scenario is a sequence or process through which damage propagates until a structural system fails. A failure scenario provides information on how the structural system may fail and to what extend the structural system is damaged. It records the part/parts in the structural system and how the initial damage may be triggered and then damage propagated in the whole process of failure.

In theory, a structural system can fail or be damaged in an infinite number of ways. Even with a simplified model, there may be enormous number of failure scenarios. A search strategy is required to sieve through these large number of possible failure scenarios and to get what is relevant for the purpose of the user.

2.5.3 General search strategies

Searching is generally concerned with retrieving some particular information from a large amount of previously stored information. The simplest and most general method for searching is exhaustive searching, which is to try every single one in the data set and come up with a result. This is however the most expensive method in terms of time and computational resource and may well be impossible.

There are several other efficient methods which all rely on some properties of the data set to achieve the speed and efficiency. The data is ordered in some way according to their properties prior to storing in order to be retrieved in a efficient way. Some commonly used techniques for searching are linear searching, binary searching, tree searching, and hashing, etc. (Sedgewick, 1983). Accordingly, the data structure may be an array, a table (an array with more than one dimensions), or a tree.

2.5.4 Adopted methodology

In order to achieve an efficient and computationally inexpensive method to identify and search for vulnerable failure scenarios, we will present a theory of form to deal with the following issues:

- to build a model of the form of a structural system,
- to represent the structure of the model in a hierarchy,
- to identify various failure scenarios of the structural system,
- to analyse the consequences of various failure scenarios,
- to identify vulnerable failure scenarios in the system.

2.5.4.1 **Graph**

Following previous work (Wu, 1991), graph theory will be used as a basis to develop a graph model of the form of a structural system.
The characteristics of the form of a structural system can be captured in a basic graph model, i.e. a structural ring , in the theory (see Chapter 3).
For a structural system, the graph model enables the development of a measure of the quality of its form. The measure, i.e. well-formedness, incorporates all four aspects of the quality of form as discussed in 2.5.1. The details of well-formedness will be introduced in Chapter 3.

2.5.4.2 **Clusters and hierarchy**

According to the quality of form of a structural system, its components can be organised into structural clusters using cluster analysis (see Chapter 4). The clustering technique can be performed on different levels of description of the structural system. Thus at any level, a structural cluster is a holon. It may consists of several other clusters which are at a lower level and is a constituent of other clusters which are at a upper level. Using cluster analysis, the structural system can be represented as a hierarchy with its components at the lowest level of description and the whole structural system at the top level of description (see Chapter 5). At any level in between, the structural system is represented as a set of interconnected sub-systems, i.e. structural clusters.

2.5.4.3 **Searching for the most vulnerable failure scenarios**

The graph model and the hierarchical representation of the model are both purpose-built data structures with which the vulnerable failure scenarios can be efficiently identified. With the hierarchical graph model, the structure of the form is revealed, thus any weak link in the form will be identified. Also, various failure scenarios can be found and the most vulnerable scenarios can be identified by comparing the damage demand and the ratio of consequence to damage demand between them. Detail of the identification of vulnerable failure scenarios will be given in Chapter 6.

2.6 Conclusions

Safety is the correspondence between the required state of the world and the actual state of the world. It is a very important quality of civil engineering structures.
Safety procedures used by designer are limit state design and reliability theory. Those used by the industry include management and control in the process and promotion of good safety culture.
Problems and limitations of reliability theory arise from the difficulty in dealing with system failure and the complex dependencies between random variables describing the system components. The intention of this research is to look at the whole problem from a different perspective, i.e. instead of asking what is the most likely limit state failure condition and the components probability of failure, we ask, for any possible load combinations, where is the weak link.
The basic concept of vulnerability is susceptibility to failure. For a general system, the internal vulnerability is in the form of a system due to its internal configuration. The specific action-related vulnerability is dependent on the nature of an action to which the system is subjected. The overall vulnerability is concerned with a period of time within which the state of the system and the possible actions on the system are considered as a process.
The aim of this research is to develop a theory to assess the internal vulnerability of a structural system.
The approach includes the following objectives:

- to build a graph model to model the form of a structural system,
- to use cluster analysis to reveal the structure of the form,
- to represent the structural system, in terms of its form, as a hierarchy,
- to enable the identification of various vulnerable failure scenarios related to its form.

In this thesis, a theory of structural vulnerability will be presented which is ***a theory of form***.

Part II Theory

Chapter 3

Structural Rings & Well-formedness

3.1 Objectives

The objectives of this chapter are to:

- briefly review the theoretical background of graph theory;
- represent a structural system with a graph model;
- introduce the concept of structural ring;
- define the types of structural ring and their characteristics;
- define the process of deterioration and failure of a structural ring;
- introduce the concept of Well-formedness.

3.2 Introduction

The most fundamental concepts in structural vulnerability theory are that of a structural ring and the measure of well-formedness. A structural ring is a basic model developed from graph theory which can represent a structure in general. Well-formedness is a measure of the form of a structure.

In this chapter, some preliminaries of graph theory will be introduced. A graph model of the structural system will be presented. The concept of a structural ring will be introduced as a pattern in the graph model with structural characteristics. Finally, a measure of the quality of the structural form (well-formedness) will be developed.

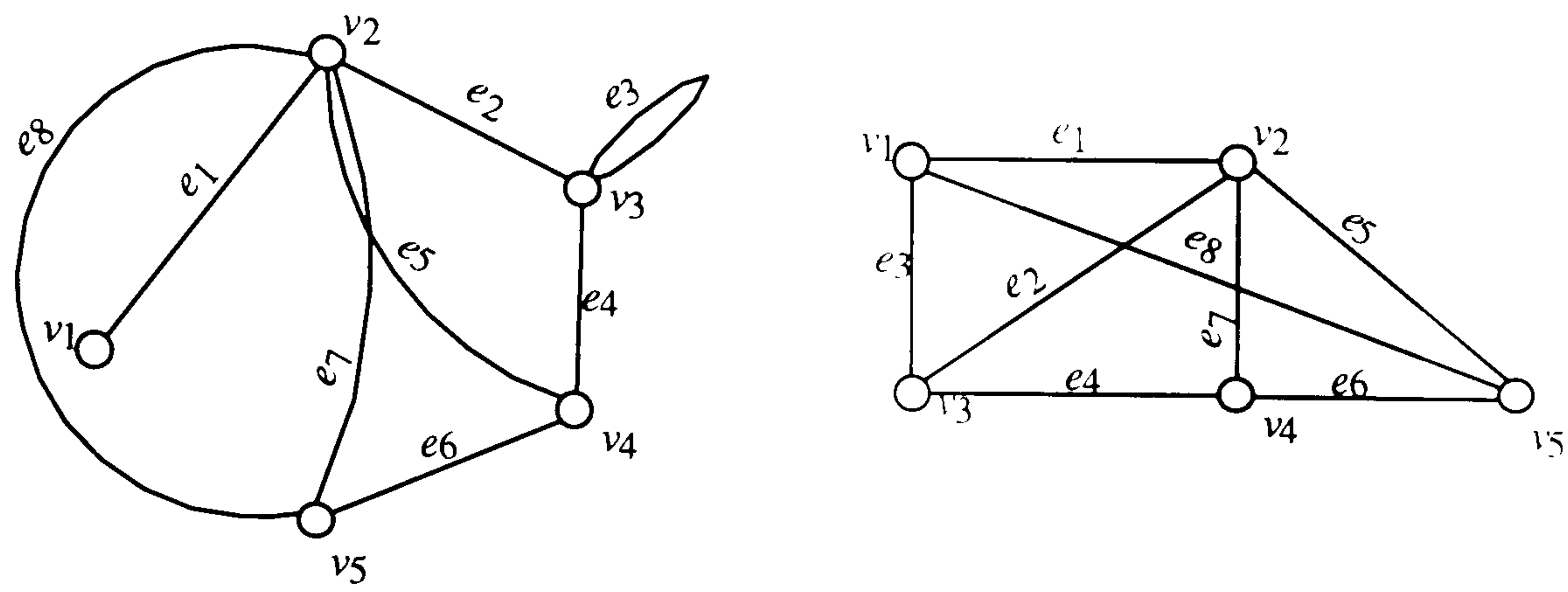
3.3 Graph Theory

There are many real-world situations that can be described abstractly by means of a diagram consisting of a group of points joined either by lines or by arrows. Graph theory is a branch of mathematics through which such diagrams are studied.
The origin of graph theory dates back to 18th century. It was originally motivated by solving problems related to the geometry of position. The generic name "graph" was first used by J. J. Sylvester in his note *Chemistry and Algebra* in the scientific magazine *Nature* at 1878 (Biggs, Lloyds & Wilson, 1976). The first full-length book on this subject, written by D. König, was published at 1936. Since then, graph theory has been systematically studied and applications have been made in solving problems in various disciplines, such as physical science, economics and management, behavioural science, organic chemistry, information technology and engineering, etc.

3.3.1 Concept of graphs

The definitions to be introduced in this section are more or less standard vocabularies in graph theory. (Bondy & Murty, 1976)
A graph is defined as a pair $\mathbf{G} = (V, E)$, where V is the set of *vertices* and E is the set of pairs of distinct vertices called *edges*. If the edges are ordered pairs of vertices, the graph is *directed*, otherwise it is *undirected*. A graph is *finite* if both the vertex set and edge set are finite. For the purposes of this research, only undirected finite graphs are discussed.

Examples:



G1 **G2**

Figure 3.1 Two example graphs

For graph G1:

$G_1 = (V_1, E_1)$

$V_1 = \{ v_1, v_2, v_3, v_4, v_5 \}$

$E_1 = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8 \}$

and

$e_1 = v_1 v_2,\quad e_2 = v_2 v_3,\quad e_3 = v_3 v_3,\quad e_4 = v_3 v_4,$

$e_5 = v_2 v_4,\quad e_6 = v_4 v_5,\quad e_7 = v_2 v_5,\quad e_8 = v_2 v_5$

For graph G2:

$G_2 = (V_2, E_2)$

$V_2 = \{ v_1, v_2, v_3, v_4, v_5 \}$

$E_2 = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8 \}$

and

$e_1 = v_1 v_2,\quad e_2 = v_2 v_3,\quad e_3 = v_1 v_3,\quad e_4 = v_3 v_4,$

$e_5 = v_2 v_5,\quad e_6 = v_4 v_5,\quad e_7 = v_2 v_4,\quad e_8 = v_1 v_5$

The example graph G1 is a general graph. Note that in G1, edge e_3 is defined by a pair of the same vertex, v_3. This type of edge is called a *loop*. All the other edges, defined by a pair of distinct vertices are *links*. Note also that edges e_7 and e_8 are defined by the same pair of vertices, v_2v_5. This type of edges are called *parallel* edges.

A graph is *simple* if it has no loop or parallel edges. The example graph G2 is such a graph. For the purposes of this research, graphs with self-loops are not to be further discussed. In later sections, a structural system will be initially represented with a simple graph.

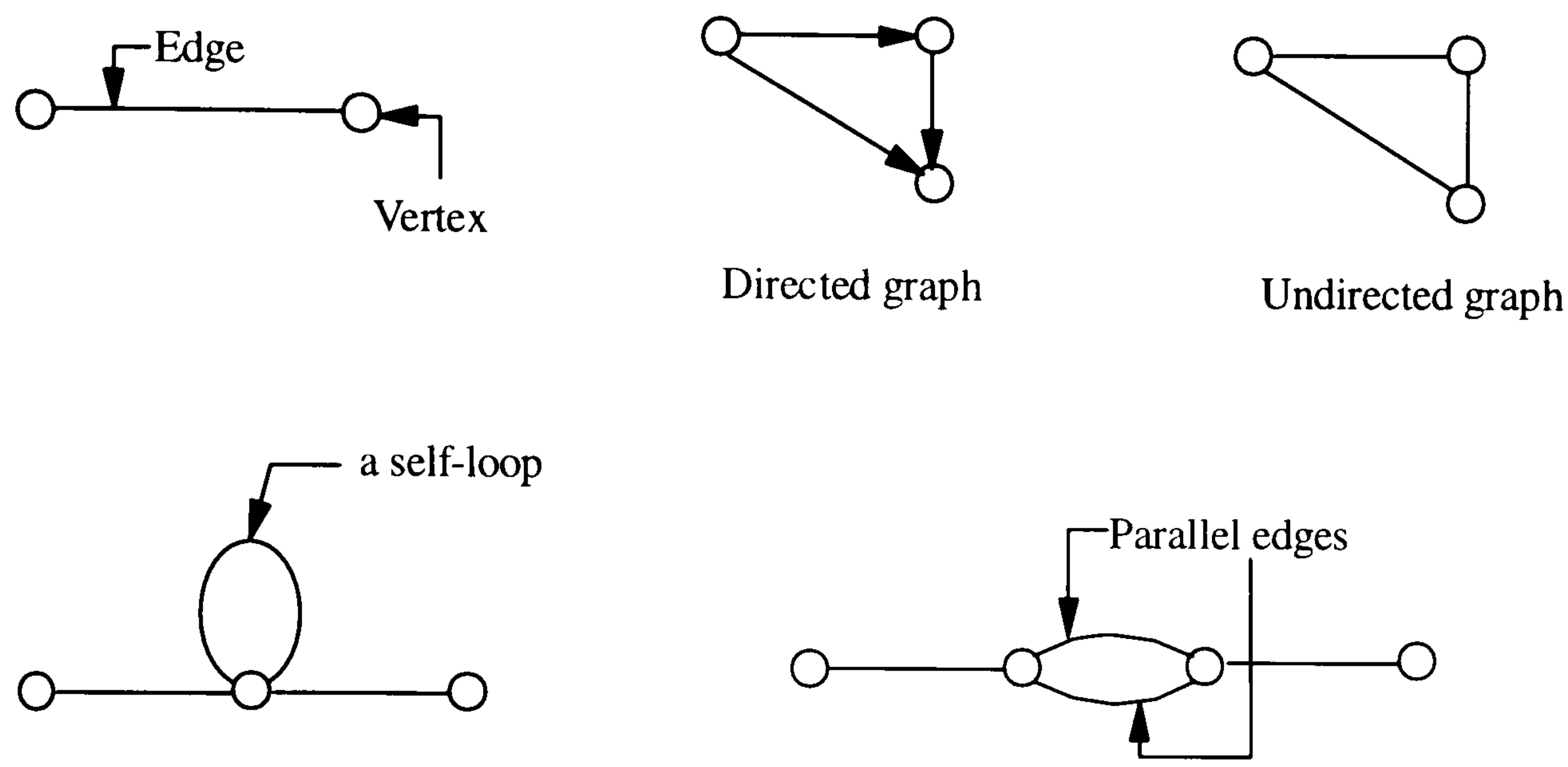


Figure 3.2 Examples of graph definitions

3.3.2 **Incidence and Adjacency Matrices**

If an edge e has a vertex v as an end-point, then e is said to be *incident* with v, and vice versa. Also, if $(v_i, v_j) \in E$, then v_i and v_j are *adjacent*.

The incidence and adjacency of a graph can be described by means of matrices. The incidence matrix of a graph M(G) = [m_{ij}] is a $v \times e$ matrix in which m_{ij} (0, 1 or 2) is the number of times that v_i and e_j are incident. The adjacency matrix A(G) = [a_{ij}] is a $v \times v$ matrix in which a_{ij} is the number of edges joining v_i and v_j.

The incidence and adjacency matrices are just a mathematical way of expressing the graph. For a simple graph, the elements in both matrices are binary.

For the example graph G1 in Figure 3.1,

$$
M(G_1) = \begin{array}{c|cccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\
\hline
v_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_2 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
v_3 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
v_5 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1
\end{array}
$$

and

$$
A(G_1) = \begin{array}{c|ccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 \\
\hline
v_1 & 0 & 1 & 0 & 0 & 0 \\
v_2 & 1 & 0 & 1 & 1 & 2 \\
v_3 & 0 & 1 & 1 & 1 & 0 \\
v_4 & 0 & 1 & 1 & 0 & 1 \\
v_5 & 0 & 2 & 0 & 1 & 0
\end{array}
$$

For the example graph G2 in Figure 3.1,

$$
M(G_2) = \begin{array}{c|cccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\
\hline
v_1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
v_2 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
v_3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
v_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1
\end{array}
$$

and

$$
A(G_2) = \begin{array}{c|ccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 \\
\hline
v_1 & 0 & 1 & 1 & 0 & 1 \\
v_2 & 1 & 0 & 1 & 1 & 1 \\
v_3 & 1 & 1 & 0 & 1 & 0 \\
v_4 & 0 & 1 & 1 & 0 & 1 \\
v_5 & 1 & 1 & 0 & 1 & 0
\end{array}
$$

3.3.3 **Degree of Vertex**

The *degree* of a vertex v, written as $d(v)$, is the number of edges incident with v. It can be calculated by adding up the elements of each row in adjacency matrix A(G).
For example graph G_2 in Figure 3.1,

$$D(G_2) = \begin{array}{c|c|} d(v_1) & 3 \\ d(v_2) & 4 \\ d(v_3) & 3 \\ d(v_4) & 3 \\ d(v_5) & 3 \end{array}$$

3.3.4 **Subgraphs**

A graph G' = { *V'*, *E'* } is said to be the *subgraph* of G, if $V' \in V$ and $E' \in E$. And G is called the *supergraph* of G'. We use $G' \subseteq G$ to show the relationship.

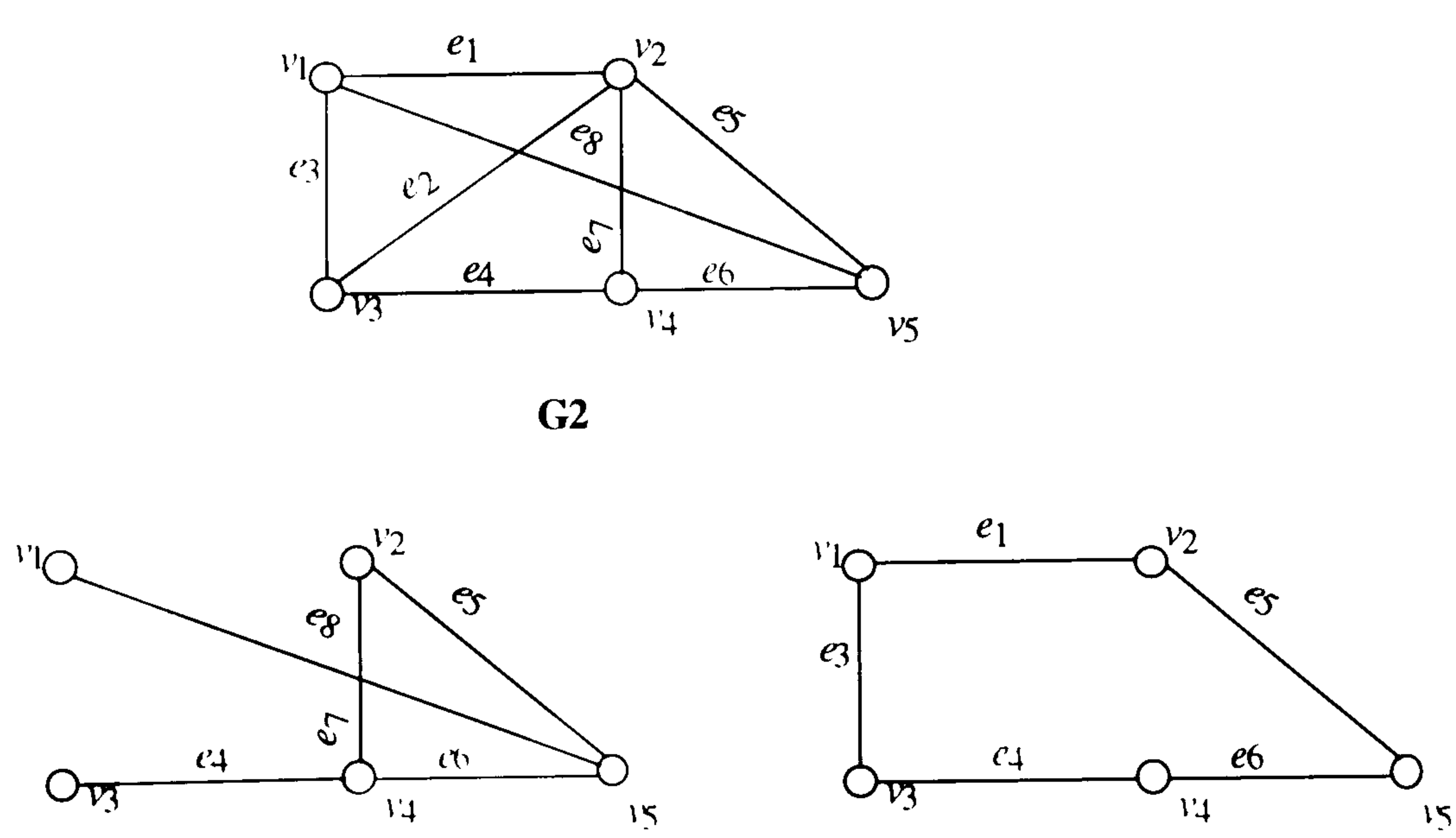
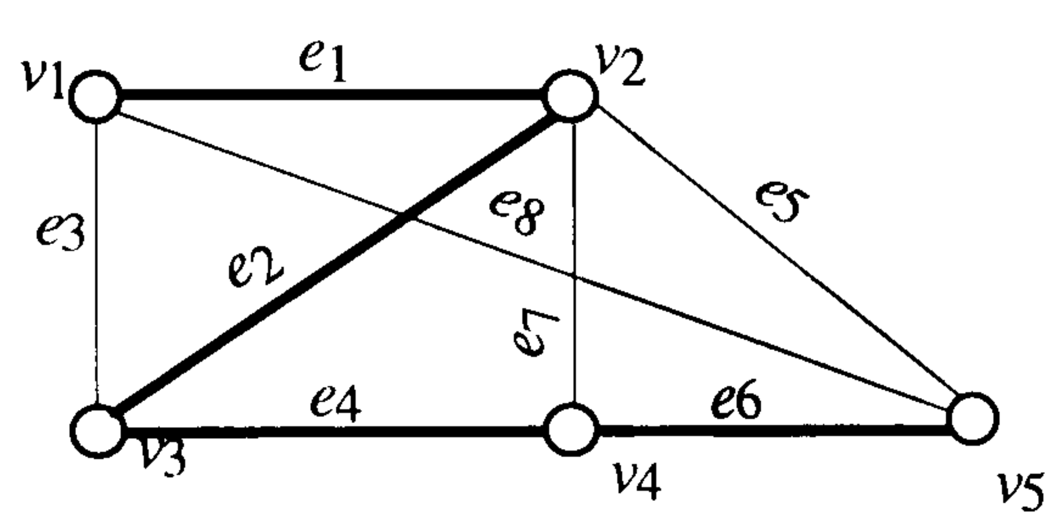


Figure 3.3 Examples of subgraphs

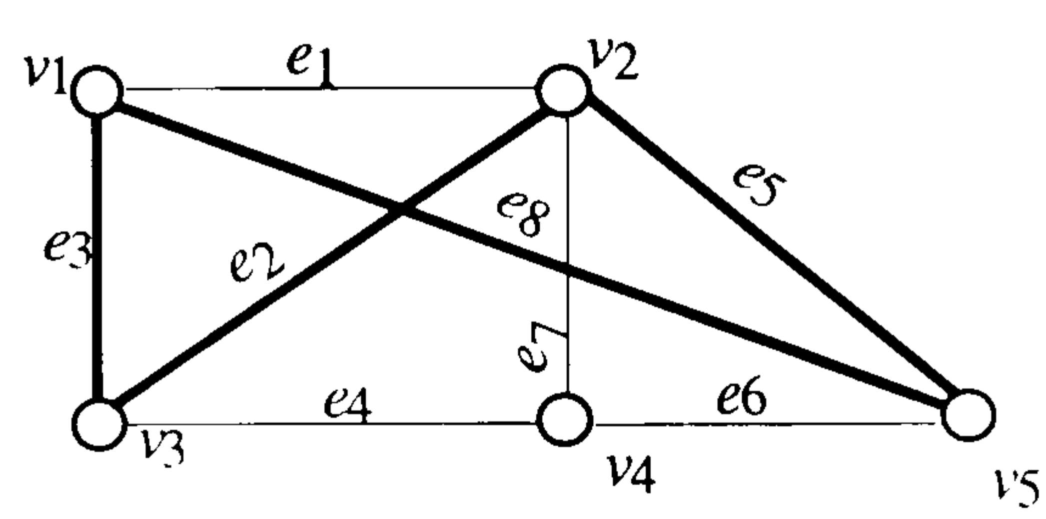
3.3.5 **Paths and Cycles in a graph**

Concepts of path and cycle are very important for the further application of this research. A *path* (P) in a graph from v_1 to v_i is a sequence of alternating vertices and edges. P = { v_1, e_1, v_2, e_2, ... e_{i-1}, v_i }. In the path, for $1 \leq j < i$, e_j is incident with v_j and v_{j+1}. In a path, each vertex and edge can only appear once. If $v_i = v_1$, then P is said to be a cycle. The *length* of a path of a cycle is the number of the edges it contains. For a simple graph, a path or a cycle can be specified by the sequence of vertices, since there is only one edge between a pair of vertices.



A path in G2

P = { *v1,e1,v2,e2,v3,e4,v4,e6,v5* } or
P = { *v1,v2,v3,v4,v5* }



A cycle in G2

P = { *v1,e3,v3,e2,v2,e5,v5,e8,v1* } or
P = { *v1,v3,v2,v5,v1* }

Figure 3.4 Examples of a path and a cycle in a graph

3.3.6 **Connectivity of Graphs**

A graph is *connected* if there is a path between any two vertices in the graph. Otherwise it is *disconnected.* A graph is *complete* if there is an edge assigned to every pair of vertices.

The *connectivity* of a graph is often defined as the number of cut-edges in a graph. In a graph G, if the removal of an edge *e* will disconnect G, then *e* is said to be a *cut-edge*.

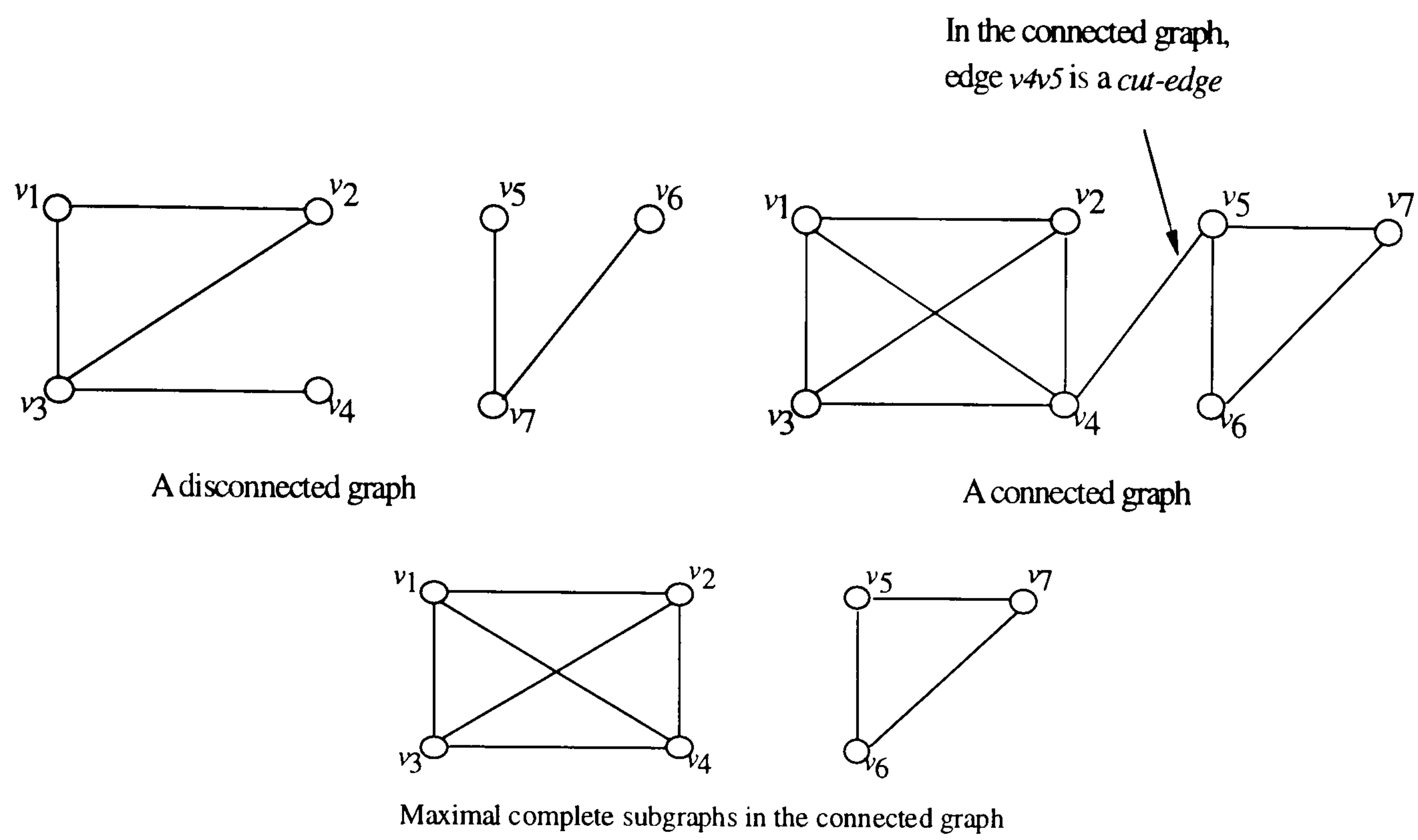


Figure 3.5 Examples of disconnected and connected graphs

3.3.7 Weighted Graphs

In many practical applications, a number $w(e)$ is assigned to each edge e of a graph which is called the *weight* of an edge. A graph with such edges is called *weighted* graph. Otherwise, it is called *unweighted* graph. The weight of a graph/subgraph is equal to the sum of the weight of its edges.

Weighted graphs are used frequently in applications of graph theory. When in a friendship graph, weights might represent the intensity of association, or in communications graph, they could indicate the costs of various links. Often in path or cycle finding applications, it is referred to as *length* rather than *weight*. This should not be confused with the length of a path or a cycle in an unweighted graph.

3.4 The Graph Representation of a Structural System

The purpose of introducing graph theory in the previous section is to develop a graph model representing a structural system. The objective of the research is to assess the vulnerability in the form of the structural system. Therefore, the graph model to be developed must be able to capture the principal characteristics of the structural system in relation to that purpose.

3.4.1 Structural systems

A structure, in the civil engineering context, is defined as "a body capable of resisting applied loads" (Marshall and Nelson, 1977). To certain degree, almost everything in our everyday life needs to sustain some loads without being broken. The word "structure" can describe most things in the natural world.
The function of a structure is to transmit forces from one point in space to another. In the case of building structure, it is to transmit the applied forces to the ground.
The main type of structures are:

a) Framed structures --- those which resist applied loads by virtue of their geometry
b) Mass structures --- those which resist applied loads by virtue of their weight.
c) Others, e.g. plated structures, cable structures, etc.

The present research is confined to the first type of structures only.
To fulfil the function of withstanding loads, the materials which construct the structure must have proper strength. However, the strength of materials is only a partial property of the structure. The way that the materials are distributed and arranged to resist the loads, i.e. the structural form, is also very important. The form of the structure determines the stiffness and deformations of the structure, and can significantly affect the efficiency of the materials used in the structure.

The term "system" is used widely with different meanings under different context. The general definition of a *system* is that a structured set of interrelated objects or attributes (Wilson, 1984). It is not a simple assembly of elements, but a *whole* which has the emergent properties of its components (Checkland, 1984). In a system, a *holon* is a key concept. It is both a part and a whole (Koestler, 1968). It is a part of a larger system, and meanwhile, it is a system itself. The systems concept and approach will be further discussed in later chapters.

A *structural system* is a set of inter-related elements with interrelationship to fulfil functions as a civil engineering structure. The elements include joints and members in a frame structure. The structural system is a holon. It is an element in a large system which encompasses the physical, environmental and even social conditions of the structural system.

3.4.2 **Representing a structural system with a graph**

A structural system can be represented with a graph **S** = { ***M, J*** }, which consists of a finite set of joint objects *J* and a finite set of member objects *M*. (Wu, 1991)

A *joint* object *j* is a vertex in the graph model. It is the reference point in the structural system where member objects connect to each other. The features of a joint object include its geometrical position, type of joint and its degree of freedom.

A *degree of freedom* (DOF) is defined as the capacity of an object in **S** (a joint object or a member object) to permit the transmission of force in a principal co-ordinate direction. In the case of a planar structure, an object can have up to three degrees of freedom, corresponding to longitudinal, transverse and rotational forces respectively.

A joint object can be of a few types, i.e. *Fixed, Pinned, Roller* or *Cut.* The type of a joint object is related to its numbers of degrees of freedom.

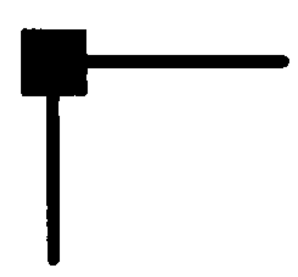
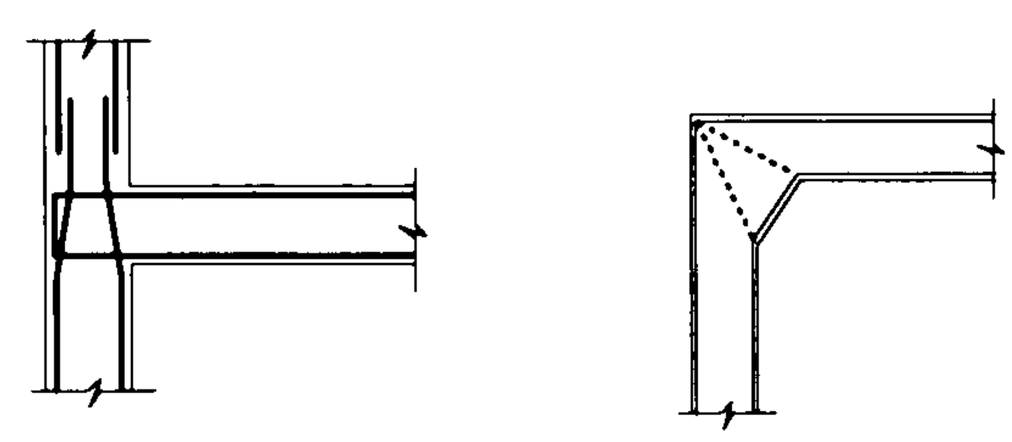
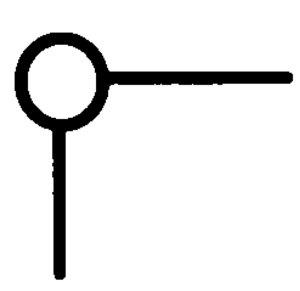
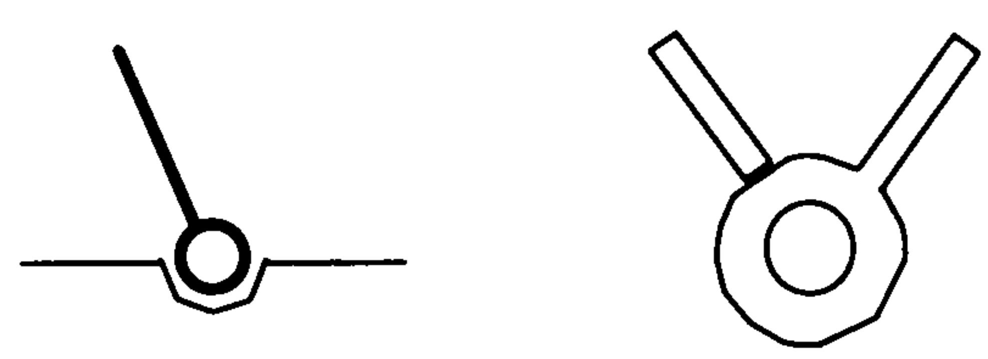
Corresponding to those used in structural engineering, the type of joints is shown in Table 3.1.

Table 3.1 Type of joint in graph model

In graph model	**In structural engineering**
Fixed	Fixed-end, stiff, encastre, built-in
Pinned	Pinned,
Roller	Roller bearing
Cut	Open-end

Different type of joints in the graph model and in real-life structures are shown in Table 3.2.

Table 3.2 Joints in graph model and in real-life

In the graph model	**In actual structures**	**DOF**
 Fixed type joint		3
 Pinned type joint		2

Note: The degree of freedom is as defined in Chapter 1.

A *member* object m is defined by a pair of joint objects. $m = j_i j_j$. The pair of joint object is not ordered, therefore, m = $j_i j_j = j_j j_i$. A member object is a link, or a communication channel between joint objects. The features of a member object include its degree of freedom, and geometrical and physical properties such as length $\boldsymbol{l}$, area of cross-section $\boldsymbol{A}$, second moment of area $\boldsymbol{I}$ and Young's Modulus $\boldsymbol{E}$, etc. These features make the graph a weighted graph. Member objects may have different "weight", which is the well-formedness, Q(m). Well-formedness is a measure of the quality of the form of the structure. For a member object, it is a function of all these features listed above. The detailed definition of well-formedness will be given in later sections in this chapter.

3.4.3 **The association and fixity matrices**

The configuration of the structural system can be described with two matrices, the *association matrix* and the *joint fixity matrix*.

If n is the total number of joint objects in S, then the association matrix $C(S) = [\ c_{ij}\ ]$ is a $n \times n$ matrix in which:

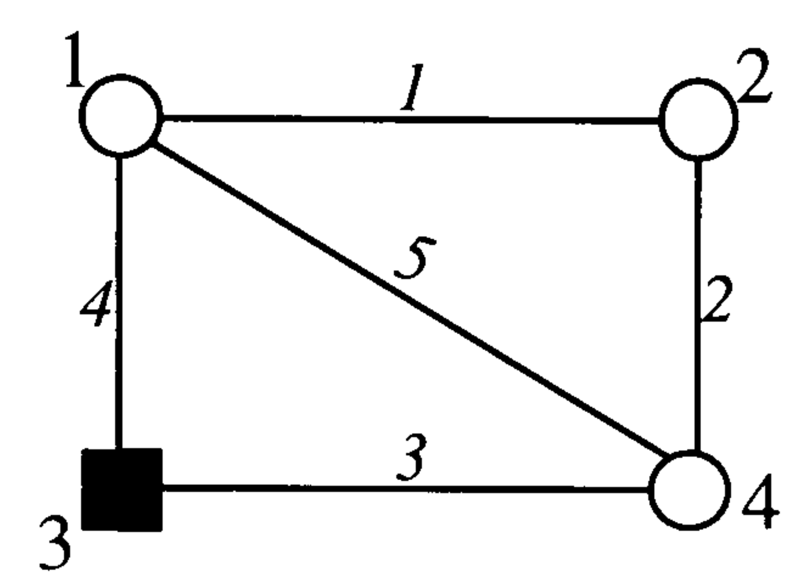
$c_{ij} = 1$ if $(j_i j_j) \in M$

$c_{ij} = 0$ otherwise

If l is the total number of member objects in S, then the joint fixity matrix $F(S) = [\ f_{ij}\ ]$ is a $n \times l$ matrix in which:

f_{ij} = DOF of the end-node, if joint i is an end-node of member j or,

$f_{ij} = 0$ otherwise.



A structure S

A structure S which has 4 joint objects and 5 member objects as shown above can be expressed by the matrices:

$$C = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} \qquad F = \begin{vmatrix} 2 & 0 & 0 & 2 & 2 \\ 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 \\ 0 & 2 & 2 & 0 & 2 \end{vmatrix}$$

3.4.4 **Structural paths and loops**

Wu (Wu, 1991) defined the structural paths and loops. A *structural path* is a sequence of adjacent joint objects in which the same joint object can only appear once. A *structural loop* is a closed structural path beginning and ending with the same joint object.

Structural paths and loops are not only a description of the connectivity between joint and member objects, but also having structural characteristics. The degree of freedoms maintained in a structural path or loop determine its behaviour and feature. It is the structural loop and path from which the important concept of structural ring is evolved.

3.4.5 Degree of joints

The degree of a joint *j*, denoted as *d(j)*, is the number of member objects incident with it. The degree of all joint objects can be written in a column matrix *D(j)*. It can be calculated by adding together the elements of each row in the association matrix C.

$$D(j) = \sum_{x=1}^{n} C_{jx} \quad (\text{ for } j = 1,\ 2,\ ...,\ n\ ) \qquad (3.1)$$

From the degree of joints, we define the *a*th degree of joints $D(j)^a$ as:

$$D(j)^a = C \times D(j)^{a-1} \quad (\text{ for } j = 2,\ ...,\ n\ ) \qquad (3.2)$$

$D(j)^a$ is the total number of *a*-link structural paths starting from joint *j*. For example, the element which is on the *ith* row and *jth* column in $D(j)^2$ will be the numbers of 2 link path beginning from the *ith* joint and ending at the *jth* joint.

3.4.6 Comparison of terms

To summarise the concepts discussed so far, a comparison of the terms describing a structural system and those used in graph theory is shown in Table 3.3:

Table 3.3 Comparison of terms in graph model and graph theory.

Graph Model of Structure Systems	**Mathematics (Graph Theory)**
A structure (S)	A graph (G)
Sub-structures (S')	Sub-graphs (G')
Members (M)	Edges (E)
Joints (J)	Vertices (V)
Association matrix (C(S))	Adjacency Matrix (A (G))
Fixity matrix (F(S))	Incidence matrix (M (G))
Structural paths	Paths
Structural loops	Cycles
Degree of joints ($D(j)$)	Degree of Vertices ($d(v)$)
Well-formedness (Q(m))	Edge Weight ($w(e)$)

Thus, a structural system can be represented by a simple, undirected and weighted graph. So far, we have only discussed the geometrical construction of the graph model. However, there are properties and characteristics of the graph model which make it to function as a *structure* we defined earlier. In the following sections, the concept of ***structural rings*** and the ***well-formedness*** will be discussed fully.

3.5 Structural Rings

3.5.1 Concept of a structural ring

The function of a structure is to transmit force from one point to another in space. Without dynamic effects, a structure is a body in static equilibrium. A structural ring is a structural path/loop which is equivalent to a structure.

A *structural ring* is a minimum structural path/loop which has sufficient degrees of freedom to maintain equilibrium. Therefore it can withstand an arbitrary equilibrium set of forces. Here, the word "minimum" indicates that it is the shortest structural path or loop which can satisfy the requirement.

A structural ring must possess a sufficient number of DOF (as previously defined in Chapter 1) in its joint and member objects. Depending on the total number of DOF, a structural ring can be *just-stiff* or *over-stiff*.

For a just-stiff structural ring, the removal of single one DOF from its joint or member objects will cause the transformation of the structural ring into a mechanism. For an over-stiff structural ring, there are more than one DOF to be removed before such a transformation may occur.

A structural ring must be at least just-stiff.

In this graph model of a structural system, a structural ring is a concept. It is the pattern in the graph modal which makes the graph/part of the graph a valid structure. It is also a "holon". Thus a basic component in a structural system can be a structural ring, as well as the whole structure. At different levels of description, the structural system can be described either as one structural ring (at the current level) or a set of interconnected structural rings (at the lower level).

At different levels of description of the structure, a structural ring can be a sequence of alternating complex joint objects and sub-structures (structural clusters). It will be discussed in Chapter 4 & 5.

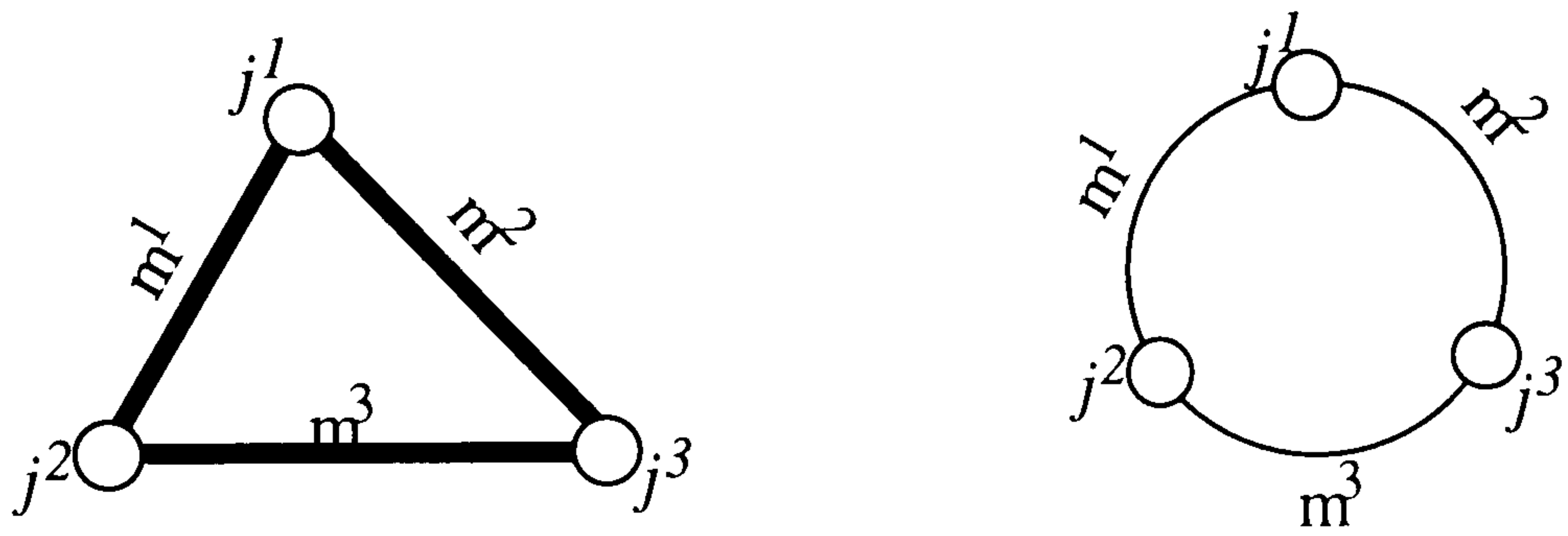
3.5.2 **Types of structural rings**

The shortest structural path which can maintain equilibrium is related to the type of joint objects in the path. Depending on the number of member objects and type of joint objects in the structural path/loop, there are two basic type of structural rings: *a 3-link-ring* and *a 2-link-ring*.

3-link-rings:

Since a pinned joint is not able to transmit any rotational force, a structural path can not transmit any rotational force too if all its node objects are pinned type of joints.

Therefore, a structural ring with all pinned joints must be a structural loop. When a simple graph is concerned, i.e. when there is no parallel link and self-loop found in the graph, the shortest structural loop in the graph model is a 3-joint-3-member loop, which is a maximal complete sub-graph. This type of structural ring is designated as a *3-link-ring* and denoted with the symbol shown below.



A structure with pinned joints

Represented as a *3-link-ring*

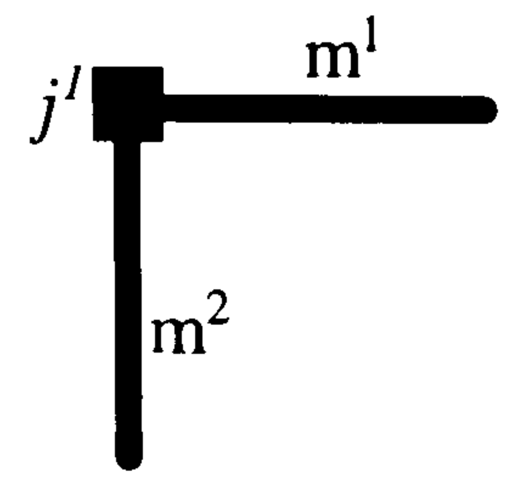
Figure 3.6 Illustration of a *3-link-ring*

A 3-link-ring is a just-stiff structural ring.

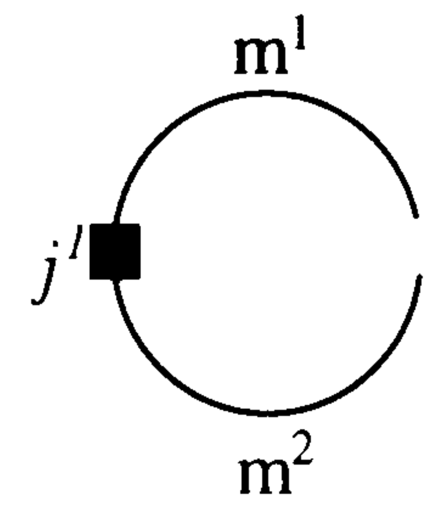
The case in which the graph has parallel links will be discussed in the 2-link-ring section.

2-link-rings:

The case of the structural ring with fixed type joints is rather different. Similar to a member object, a fixed joint can transmit three degrees of freedom, hence a fixed joint can be treated as part of a member object. Therefore, the shortest structural path in the graph model is a 1-joint-2-member path. This structural path can also be described as that the two member objects are connected with two joint objects: one fixed joint (j^1) and one cut joint. In this way, the 1-joint-2-member path can be generalised as a 2-joint-2-member loop. This type of structural ring is designated as a *2-link-ring* and denoted with the symbol shown below.



A structure with a fixed joint.



Represented as a *2-link-ring*.

Figure 3.7 Illustration of a *2-link-ring*

The 2-link-ring shown above is a just-stiff structural ring. However, in higher levels of description of the structure, there can be parallel links in the graph model. (See Chapter5). Depending on the type of joint objects, the 2-link-ring can be over-stiff structural ring in the following forms:

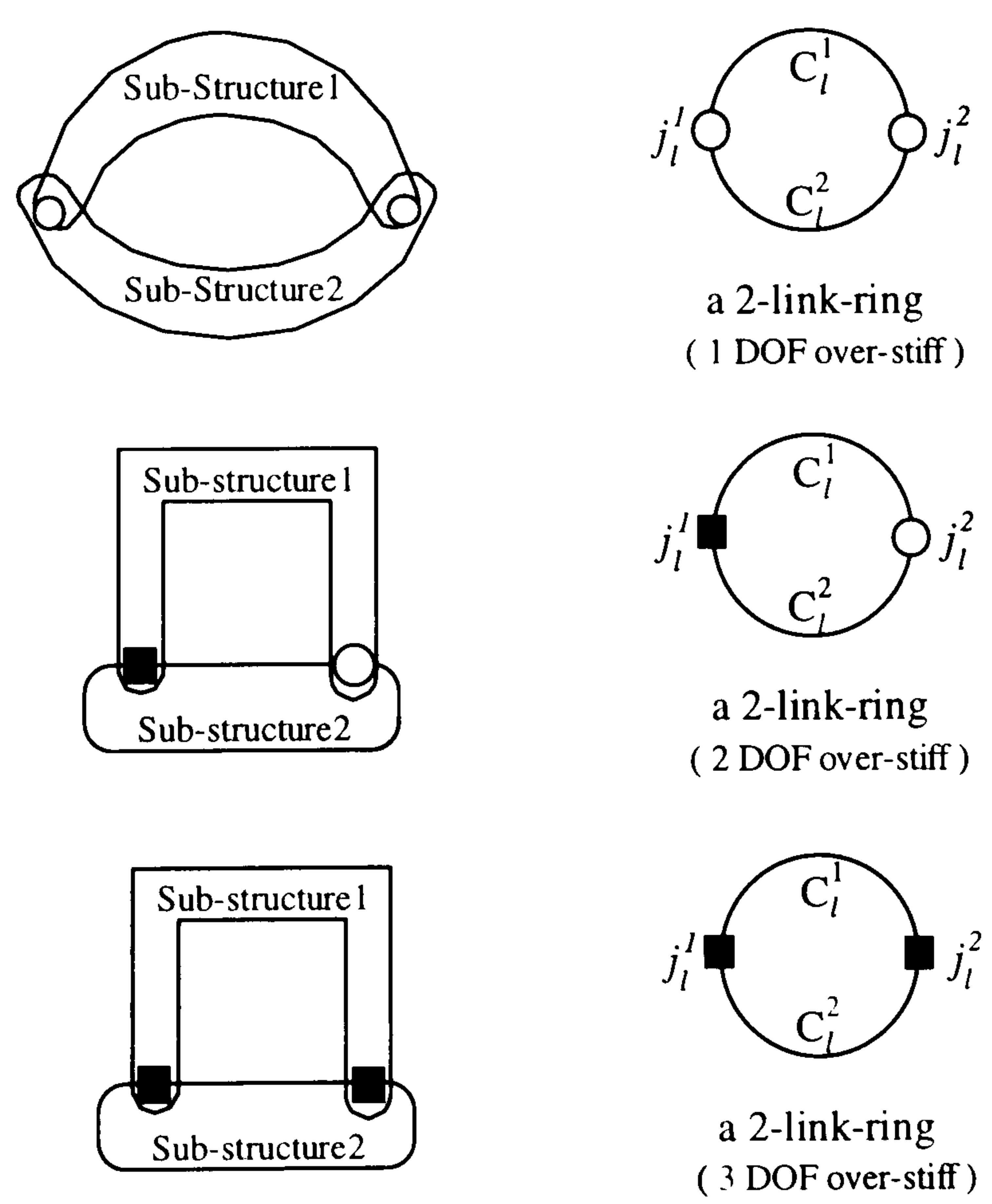
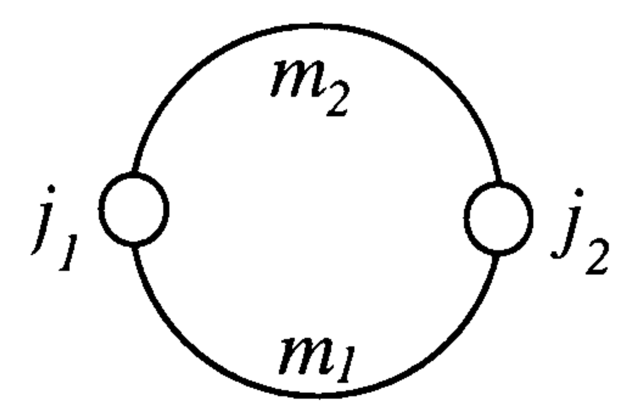


Figure 3.8 Different forms of a *2-link-ring*

3.5.3 String patterns of structural rings

A structural ring can be presented as a sequence of member and joint objects. For a 2-link-ring with two member objects (m_1 and m_2) and two pinned type joint objects (j_1 and j_2), the ring (R) can be described as:



$\boldsymbol{R} = \{\ m1, j1, m2, j2\ \}$

Instead of using the symbols to represent the member and joint objects in a ring, the detailed degree of freedom in each object can be used. For any object (member or joint), a string pattern of $\{\mu,\nu,\theta\}$ is used. Each of the elements in the string pattern is a degree of freedom corresponding to one of the three principle co-ordinate directions: the lateral (μ), vertical (ν) and rotational (θ). The value of them is either 1, if the object is capable to transmit force along the specific principal co-ordinate, or otherwise be 0.

Thus, the string pattern for a member object and various types of joint objects are shown in Table 3.4:

Table 3.4 String pattern for objects in a structural ring.

Objects	**String pattern**
member	{ 1, 1, 1 }
joint (fixed)	{ 1, 1, 1 }
joint (pinned)	{ 1, 1, 0 }
joint (roller)	{ 1, 0, 0 } or { 0, 1, 0 }
joint (open)	{ 0, 0, 0 }

For the 2-link-ring shown above, the string patterns for its member and joint objects are:

$j_1 = \{1, 1, 0\}, \qquad m_1 = \{1, 1, 1\},$

$j_2 = \{1, 1, 0\}, \qquad m_2 = \{1, 1, 1\}.$

Therefore, using the string pattern, the same 2-link-ring is represented as:

$$\boldsymbol{R} = \{\ \{1, 1, 1\}, \{1, 1, 0\}, \{1, 1, 1\}, \{1, 1, 0\}\ \}$$

3.5.4 **Redundancy, deterioration and failure of a structural ring**

As discussed in previous sections, a structural ring can be over-stiff or just-stiff. In the case of an over-stiff structural ring, there are *redundant DOFs* in the ring. The number of redundant DOF (Red.) is a measure of the degree of redundancy of the structural ring. It can be calculated using the following equation:

$$\text{Red.} = \left(\sum_{j} \sum_{n=\mu,\nu,\theta} D_j^n - \sum_{m} \sum_{n=\mu,\nu,\theta} D_m^n\right) + 3 \qquad (3.3)$$

m --- Number of member objects

j --- Number of joint objects

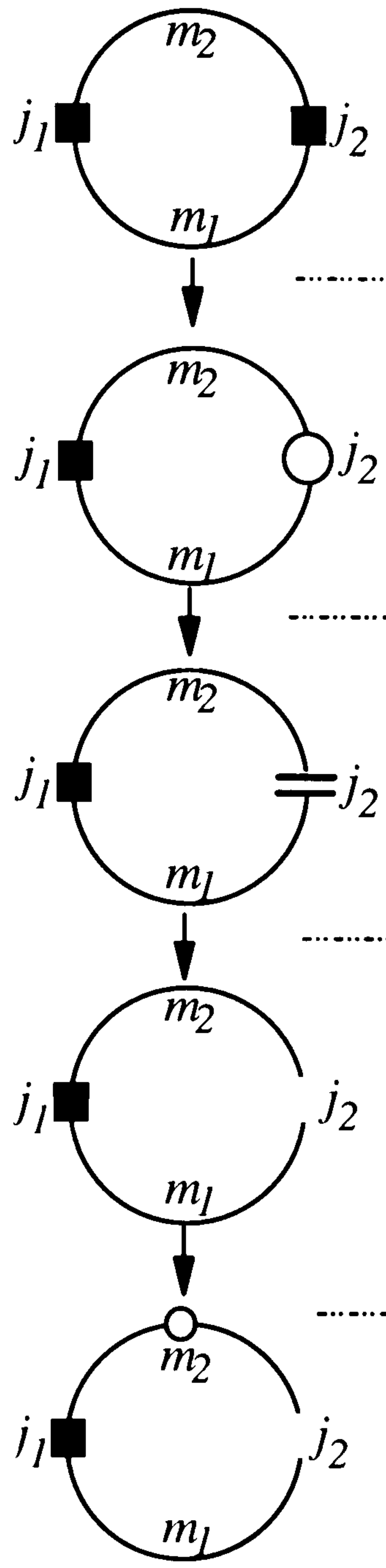
D^n--- Degree of freedom of an object at **n***th* principal direction.

Equation-3.3 has been specifically derived for the two structural ring models, i.e. 2-link-ring and 3-link-ring. The equation has been tested against all cases of these two types of structural ring and proven to be correct.

A structural ring can be transformed into a mechanism by releasing some of the DOF. A structural ring is in a state of failure if it becomes a mechanism.

As each DOF is released, there is a process of damage leading to failure. Each step is called a ***deteriorating event***. A deteriorating event can occur either adjacent to a joint object or in a member object.

An over-stiff ring deteriorates into a just-stiff ring when all the redundant DOFs are lost. For a just-stiff ring, the loss of one more DOF will bring it into its failure state. The following example demonstrates the deterioration of an over-stiff 2-link-ring:



A 2-link-ring

{ *{1, 1, 1}, {1, 1, 1}, {1, 1, 1}, {1, 1, 1}* }

Red = 3

One DOF is released at joint *j2*, the joint deteriorated from a fixed joint to a pinned joint.

{ *{1, 1, 1}, {1, 1, 1}, {1, 1, 1}, {1, 1, 0}* }

Red = 2

Another one DOF is released at joint *j2*, the joint deteriorated from a pinned joint to a roller joint.

{ *{1, 1, 1}, {1, 1, 1}, {1, 1, 1}, {0, 1, 0}* }

Red = 1

Another one DOF is released at joint *j2*, the joint deteriorated from a roller joint to an open joint.

{ *{1, 1, 1}, {1, 1, 1}, {1, 1, 1}, {0, 0, 0}* }

Red = 0

The 2-link-ring is now a just-stiff ring.

One DOF is released at joint *m2*, the ring then becomes a mechanism, i.e. the ring failed.

{ *{1, 1, 1}, {1, 1, 1}, {1, 1, 0}, {0, 0, 0}* }

Figure 3.9 Deterioration of a *2-link-ring*

3.5.5 Deterioration hierarchy of structural rings (DHSR)

In the process of deterioration of a structural ring, one degree of freedom is released at each step. However, depending on how and where a degree of freedom is released in a structural ring, a structural ring can deteriorate in many different ways. Therefore, the

Deterioration Hierarchy of Structural Rings (DHSR) has been developed (Wu, 1991) to include all possible patterns in which an over-stiff ring deteriorates into a mechanism. At each step, only one degree of freedom is released, either at a joint object or in a member object, and the ring degenerates into a new ring.

At the top level in the DHSR, the structural ring is a maximum over-stiff ring which is a 2-link-ring with two fixed joint objects. At each lower level in the hierarchy, one degree of freedom is released from the ring at the immediate upper level and the ring degenerates into new rings. In the DHSR, the structural ring at a higher level is more tightly connected than those at lower levels. At the bottom level, all structural rings become mechanisms.

At the second lowest level, all structural rings are just-stiff rings, including 3-link-ring. They are one step away from failure.

The DHSR illustrates all possible ways in which an maximum over-stiff ring can deteriorate into a mechanism. A path through the DHSR is a failure scenario. A failure scenario is corresponding to a specific way in which a structural ring deteriorates and fails. When a structural system is modelled with the graph model as a structural ring, using the DHSR, all possible failure scenarios can be studied and the vulnerability of the structural system can then be analysed. (Chapter 6)

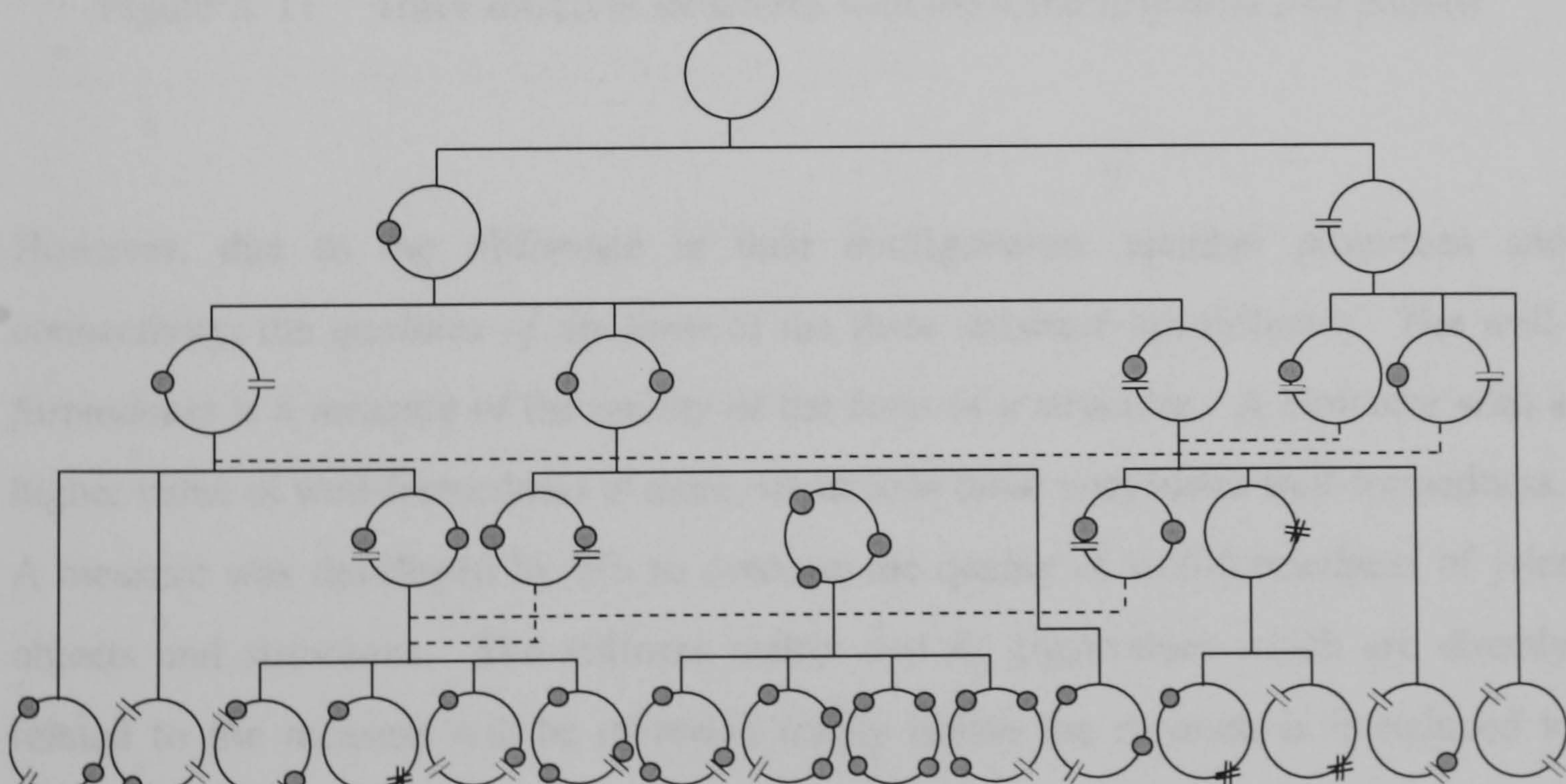


Figure 3.10 A partial deterioration hierarchy of structural ring (DHSR)

3.6 Well-formedness of a structure

3.6.1 Concept of Well-formedness

In the graph model, a structural ring can represent a type of structure with the same structural attributes. For example, the following three structures can all be denoted by the same structural ring:

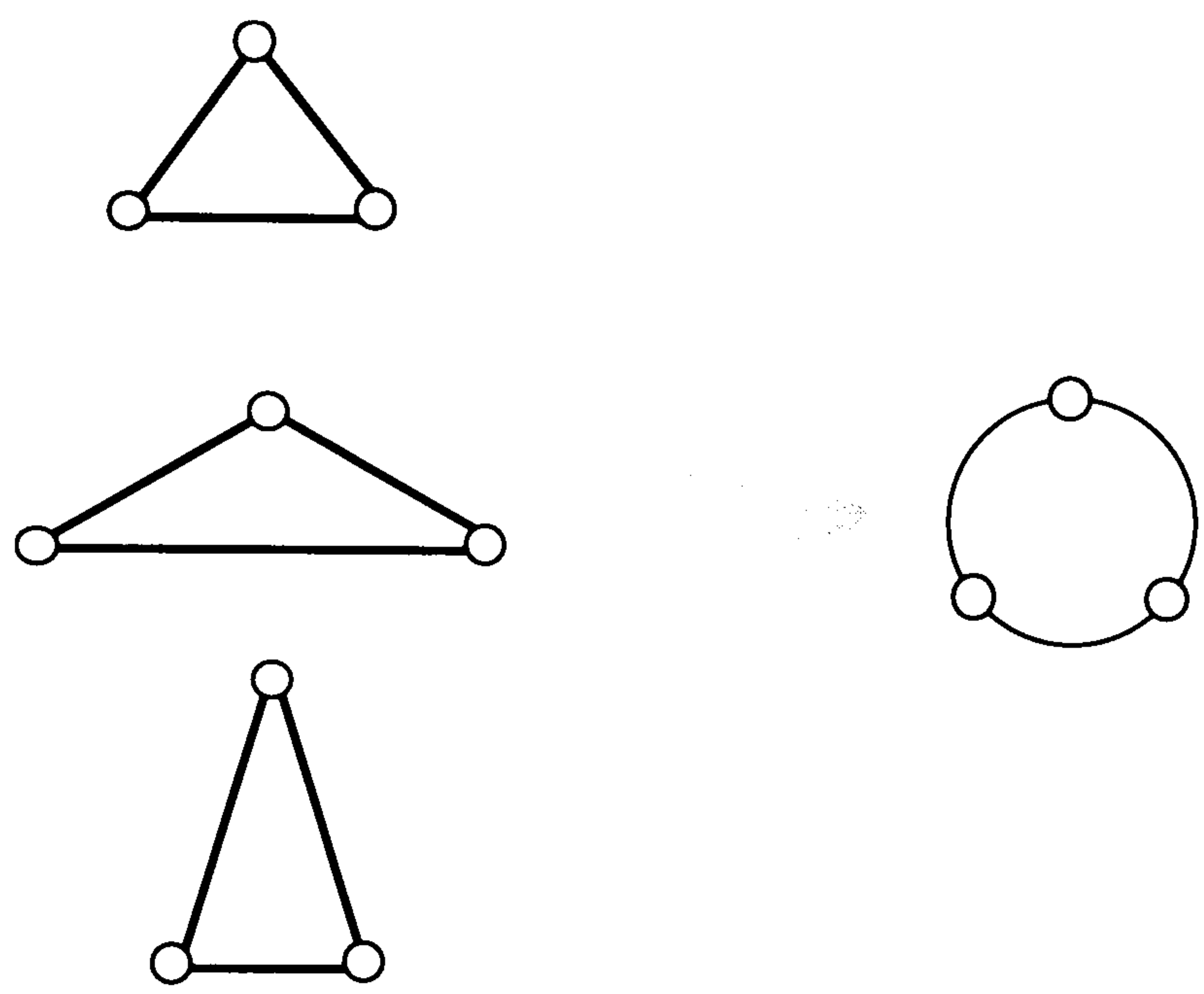


Figure 3. 11 Three different structures with the same structural ring pattern

However, due to the difference in their configuration, member properties and connectivity, the *qualities of the form* of the three structure are different. The *well-formedness* is a measure of the quality of the form of a structure. A structure with a higher value of well-formedness is more robust than those with lower well-formedness. A measure was developed by Wu to evaluate the quality of well-formedness of joint objects and structures. The stiffness matrix and its eigenvalues which are directly related to the measure will be reviewed briefly before the measure is introduced in coming sections.

3.6.2 Stiffness Matrix and Principal Stiffness Coefficients

In the matrix formulation of elastic structural analysis, for a structure, the relationship between the force and the displacement vector is expressed by the stiffness matrix:

$$F = KX \tag{3.4}$$

where: **F** --- global force vector

K --- structural stiffness matrix

X --- structural displacement vector.

If n is the total number of node in the structural system, the structural stiffness matrix K is a $n \times n$ symmetric matrix of local stiffness sub-matrices. According to the theorems in linear algebra, (Anton, 1984) (William, 1976) we have:

$$det(\mathbf{K} - \lambda\mathbf{I}) = 0 \tag{3.5}$$

where **I** is a $n \times n$ unit matrix and λ is an eigenvalue of **K** corresponding to one eigenvector.

There should exist an orthogonal matrix **P** such that:

$$\mathbf{H} = \mathbf{P}^{-1}\mathbf{K}\mathbf{P} \tag{3.6}$$

where **H** is a diagonal matrix with the eigenvalues of **K** (λ_1, λ_2, ... λ_n) on its leading diagonal:

$$\mathbf{H} = \begin{vmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & \lambda_n \end{vmatrix}$$

The sum of all eigenvalues is a constant.

From equation-3.6, we also have:

$$det(\mathbf{K}) = det(\mathbf{PHP}^{-1}) = det(\mathbf{H}) = \lambda_1 \times \lambda_2 \times, \dots, \lambda_n \qquad (3.7)$$

From equation-3.4 and equation-3.6, we have:

$$\mathbf{F} = \mathbf{KX} = \mathbf{PHP}^{-1}\mathbf{X} \qquad (3.8)$$

$$\text{or} \qquad (\mathbf{P}^{-1}\mathbf{F}) = \mathbf{H}(\mathbf{P}^{-1}\mathbf{X}) \qquad (3.9)$$

If $\mathbf{F'} = \mathbf{P}^{-1}\mathbf{F}$ and $\mathbf{X'} = \mathbf{P}^{-1}\mathbf{X}$, then:

$$\mathbf{F'} = \mathbf{HX'} \qquad (3.10)$$

Let X' be the unit displacement vector, that is $x_1' = x_2' = \dots = x_n' = 1$, equation-3.10 represents a set of linear equations:

$$\begin{aligned} F'_1 &= \lambda_1 \\ F'_2 &= \lambda_2 \\ &\dots \\ F'_n &= \lambda_n \end{aligned} \qquad (3.11)$$

i.e. the value of force F_i' is equal to the eigenvalue λ_i when given the unit displacement.

The eigenvalue problem in matrix analysis has many application in solving engineering problems. In matrix structural analysis, the eigenvalue λ_i is called the *principal stiffness coefficient* and the eigenvector corresponding to λ_i defines the *principal displacement axis*. F_i' and x_i' are force and displacement along the principal displacement axis.

All principal displacement axes are linearly independent. Therefore a principal stiffness coefficient indicates the capacity of the structure to resist loading along the corresponding principal displacement axis.

3.6.3 Well-formedness of a structural joint

Having introduced the stiffness matrix and the principal stiffness coefficients, a measure of well-formedness of structural joints can be derived from them.

In structural analysis, when a global co-ordinate system is set up, the global stiffness matrix of a structure (K_S) can be constructed by assembling all member stiffness matrices (K_{11}', K_{22}', K_{12}' *or* K_{21}') into it.

$\mathbf{K_S}$ is a $n \times n$ matrix of sub-matrices, where n is the total number of joints in the structure.

$\mathbf{K_S}$ can be written as $\mathbf{K_S} = \{\mathbf{K}_{Sij}\}$, where i = 1,2,...,n; and j = 1,2,...,n.

$\mathbf{K_{11}}$', $\mathbf{K_{22}}$', $\mathbf{K_{12}}$' and $\mathbf{K_{21}}$' are the member stiffness matrices after co-ordinate transformation, where *1* and *2* represents the two end nodes of a member.

The global stiffness matrix is constructed such that

- the element submatrix on the leading diagonal (when i = j = J) is the sum of $\mathbf{K_{11}}$' or $\mathbf{K_{22}}$' of all the members which meet at joint J in the structure. It is called the submatrix associated with joint J.
- the element submatrix in the off-diagonal position (when i $\neq$ j) contains either $\mathbf{K_{ij}}$' (if there is a member between joint i and j), or a zero matrix (if there isn't one).

The following points may be noted from the submatrix associated with a joint:

⇒ The dimension of the submatrix associated with a joint J ($\mathbf{K}_{Sjj}$) depends on the degree of freedom of that joint. If J is a pinned joint, the submatrix is a 2×2 matrix. If J is a fixed joint, the submatrix is a 3×3 matrix.

⇒ The dimension of the submatrix associated with a joint is independent of the number of members meeting at the joint.

⇒ The submatrix is symmetric.

Referring to previous section, for a square symmetric matrix $\mathbf{K}_{Sjj}$, we have:

$$det(\mathbf{K}_{Sjj} - \lambda \mathbf{I}) = 0 \qquad (3.12)$$

Depending on the dimension of the matrix, there may be two or three eigenvalues.

For a pin joint:

$$det(\, Ks_{jj}\,) = \lambda_1 \times \lambda_2 \tag{3.13a}$$

For a fix joint:

$$det(\, Ks_{jj}\,) = \lambda_1 \times \lambda_2 \times \lambda_3 \tag{3.13b}$$

where $det(Ks_{jj})$ is simply called the determinant of joint J, and λ_i is called the eigenvalues of the joint.

Wu has drawn the following conclusions from a close study of the submatrix, its determinant and eigenvalues:

- The eigenvalues and determinant of a joint are independent of the global co-ordinate system.
- The eigenvalues, i.e. principal stiffness coefficients, of a joint is related to the stiffness of all members which meet at the joint.
- The eigenvalues of a joint depend upon the angles between members which meet at the joint.
- The eigenvalues of a joint also depend on the structural characteristics of the joint, i.e. type of the joint.

From above conclusions, the determinant of a joint has been chosen as the measure of the well-formedness of that joint. The well-formedness of a joint J_i, denoted as q_i, is the determinant of the stiffness submatrix associated with joint J_i:

$$q_i = det(\, \mathbf{Ks}_{ii}\,) \tag{3.14}$$

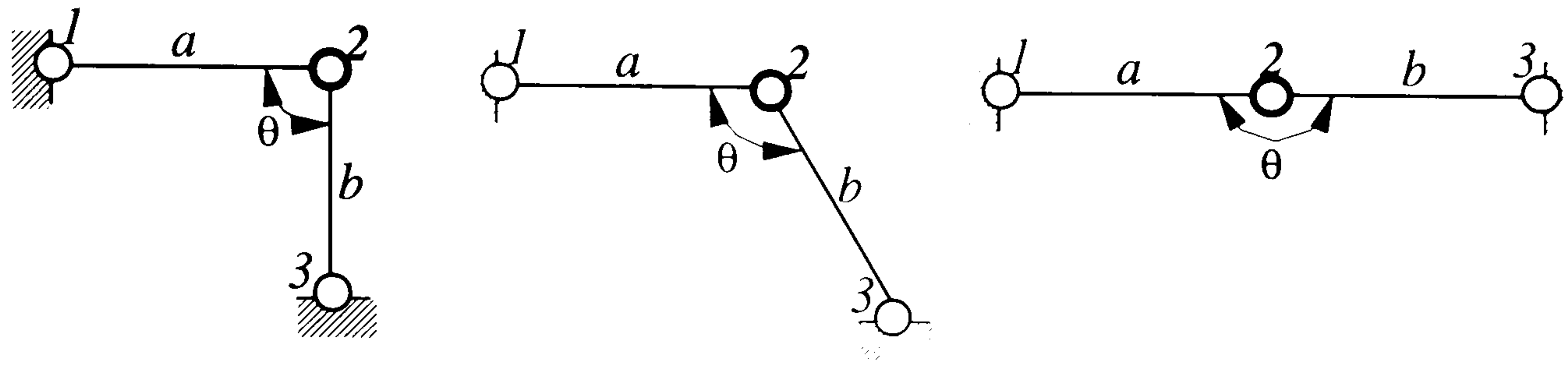
where $\mathbf{Ks}_{ii}$ is the stiffness submatrix associated with joint J_i.

Since the measure q_i is the product of the eigenvalues of the stiffness submatrix, i.e. principal stiffness coefficients, it is a measure of quality of the *form* of the structural joint. The measure is independent of co-ordinate system, and only related to

- the type of the joint (pinned or rigid)
- the stiffness of the members meeting at the joint
- configuration of the members at the joint.

The following example shows the computation of the well-formedness of a joint, and comparison can be made between the form of different joints.

Example 3.1



(1) (2) (3)

The structure in example-3.1 has 2 members (*a, b*) and 3 joints (*1, 2, 3*). θ is the angle between *a* and *b*.

Let α_a and α_b be the angle between the *x*-axis in a global co-ordinate system and member *a* and *b* respectively. $\alpha_b = \alpha_a + \theta$.

Assume l_a and l_b and the length of member *a* and member *b*,

$\mathbf{k_a}$ and $\mathbf{k_b}$ are the stiffness matrix of member *a* and member *b*.

Elastic modulus E and the area of cross section A are the same for both members.

When lateral displacement is ignored, then:

$$k_a = \begin{vmatrix} EA/l_a & 0 \\ 0 & 0 \end{vmatrix} \quad \text{and} \quad k_b = \begin{vmatrix} EA/l_b & 0 \\ 0 & 0 \end{vmatrix}$$

The structural stiffness matrix is:

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \mathbf{K}_s = & \left|\begin{array}{c|c|c} \mathbf{k}_{a11} & \mathbf{k}_{a12} & \mathbf{0} \\ \hline \mathbf{k}_{a21} & \mathbf{k}_{a22} + \mathbf{k}_{b22} & \mathbf{k}_{b23} \\ \hline \mathbf{0} & \mathbf{k}_{b32} & \mathbf{k}_{b33} \end{array}\right| \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

If we choose joint 2, the submatrix associated with joint 2 is:

$$\mathbf{K}_{s22} = \mathbf{k}_{a22} + \mathbf{k}_{b22} = \left|\begin{array}{c|c} \mathbf{k_a}\cos^2\theta + \mathbf{k_b}\cos^2(\theta+\alpha_a) & \mathbf{k_a}\cos\theta\sin\theta + \mathbf{k_b}\sin(\theta+\alpha_a)\cos(\theta+\alpha_a) \\ \hline \mathbf{k_a}\cos\theta\sin\theta + \mathbf{k_b}\sin(\theta+\alpha_a)\cos(\theta+\alpha_a) & \mathbf{k_a}\sin^2\theta + \mathbf{k_b}\sin^2(\theta+\alpha_a) \end{array}\right|$$

The well-formedness q of joint 2 is:

$$q_2 = det(\,\mathbf{K}_{S22}\,) = \lambda_1 \times \lambda_2 = \mathbf{k_a} \times \mathbf{k_b} \times \sin^2\theta \qquad (3.15)$$

Using the measure of the well-formedness, we can compare the form of the joint **2** in three different cases in example 3.1:

In case (1): $\theta = 90°$, $q_2 = \mathbf{k_a} \times \mathbf{k_b}$

In case (2): $\theta = 120°$, $q_2 = 0.75\mathbf{k_a} \times \mathbf{k_b}$

In case (3): $\theta = 180°$, $q_2 = 0$

The well-formedness of joint **2** reaches its maximum value when member a and member b are in a right angle. In this form, the joint is most fit to withstand an arbitrary set of forces. When the angle between the two member changes, the well-formedness of the joint decreases accordingly. q_2 becomes 0 when member a and member b are in a straight line, i.e. when the joint has no stiffness along one principal displacement axis.

This can also be illustrated with the change in principal stiffness coefficients.

From Equation-3.13, we have

$$det(\,\mathbf{K}_{S22} - \lambda\mathbf{I}\,) = 0 \qquad (3.16)$$

or

$$\lambda^2 - (\,\mathbf{k_a} + \mathbf{k_b}\,)\lambda + \mathbf{k_a}\mathbf{k_b}\sin^2\theta = 0 \qquad (3.17)$$

The solution of equation-3.17 is

$$\lambda_i = \frac{(k_a + k_b) \pm \sqrt{(k_a + k_b)^2 - 4k_a k_b \sin^2\theta}}{2} \qquad (\,i = 1,\,2\,) \qquad (3.18)$$

Thus,

when $\theta = 0°$ or $\theta = 180°$: $\lambda_1 = \mathbf{k_a} + \mathbf{k_b}$, $\lambda_2 = 0$,

$q_2 = \lambda_1 \times \lambda_2 = 0$;

when $\theta = 90°$: $\lambda_1 = \mathbf{k_a}$, $\lambda_2 = \mathbf{k_b}$,

$q_2 = \lambda_1 \times \lambda_2 = \mathbf{k_a} \times \mathbf{k_b}$

3.6.4 **Well-formedness of a structure**

If R is the structural ring representing the structure S, the well-formedness of a structure (Q_S) is defined as the sum of the well-formedness of all joints in the ring divided by the total number of the joints in the ring (Wu, 1991).

$$Q_S = \sum_{i=1}^{N_j(R)} q_i \Big/ N_j(R) \tag{3.19}$$

where Q_S is the well-formedness of the structure S,

$N_j(R)$ is the total number of joint in ring R, and

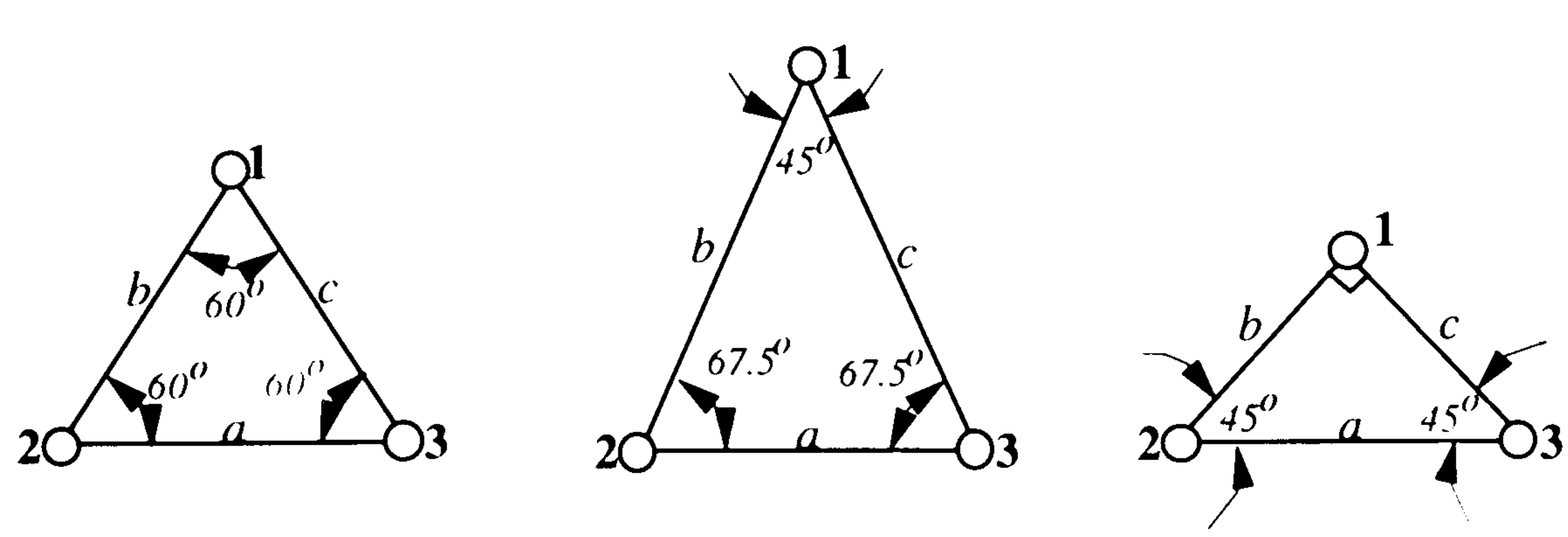
q_i is the well-formedness of the ith joint in ring R.

Q_S is a measure of the form of the structure. Again, it is independent of any co-ordinate system. It is only related to

- the type of the joint (pinned or rigid) in the structure,
- the stiffness of the members in the structure
- configuration of the members in the structure.

The following example illustrates the computation and use of well-formedness of structures.

Example 3.2



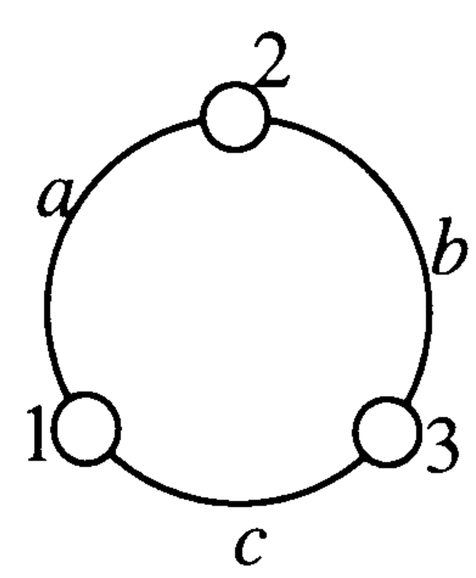
(1) (2) (3)

The length of members are l_a, l_b and l_c.

The elastic module of members are E_a, E_b and E_c.

The area of cross section of members are A_a, A_b and A_c.

Apparently, the above three structures can all be represented by the same structural ring:



(a) Assume in case (1), (2) and (3), the stiffness of members are

$\mathbf{k_a} = E_aA_a/l_a = \mathbf{k_b} = E_bA_b/l_b = \mathbf{k_c} = E_cA_c/l_c = \boldsymbol{k}$

The well-formedness of the structure and its joints are shown in Table 3.5:

Table 3.5 Well-formedness for structures (a)

Structure	well-formedness of joints q_i ($i = 1, 2, 3$)			well-formedness of the structure Q(R)
	q_1	q_2	q_3	
(1)	$0.75\boldsymbol{k}^2$	$0.75\boldsymbol{k}^2$	$0.75\boldsymbol{k}^2$	$0.75\boldsymbol{k}^2$
(2)	$0.5\boldsymbol{k}^2$	$0.85\boldsymbol{k}^2$	$0.85\boldsymbol{k}^2$	$0.73\boldsymbol{k}^2$
(3)	$\boldsymbol{k}^2$	$0.5\boldsymbol{k}^2$	$0.5\boldsymbol{k}^2$	$0.67\boldsymbol{k}^2$

The results shows that if the member stiffness is kept constant, then

- the form of the unit depends on the angles between its members,
- a truss unit is in its best form as an equilateral triangle.

(b) Assume in case (1), (2) and (3), l_a is constant. EA for all members are constant.

$\mathbf{k_a} = EA/l_a$; $\mathbf{k_b} = EA/l_b$; $\mathbf{k_c} = EA/l_c$.

According to the angles given in the example,

for case (1): $l_a = l_b = l_c$;

for case (2): $l_b = l_c = 1.3\ l_a$

for case (3): $l_b = l_c = 0.707\ l_a$

The well-formedness of the structure and its joints are shown in Table 3.6:

Table 3.6 Well-formedness for structures (b)

Structure	well-formedness of joints q_i ($i = 1, 2, 3$)			well-formedness of the ring
	q_1	q_2	q_3	Q(R)
(1)	$0.75k_a^2$	$0.75\ k_a^2$	$0.75\ k_a^2$	$0.75\ k_a^2$
(2)	$0.42\ k_a^2$	$0.71\ k_a^2$	$0.71\ k_a^2$	$0.61\ k_a^2$
(3)	$2k_a^2$	$0.71\ k_a^2$	$0.71\ k_a^2$	$1.14\ k_a^2$

Normally, E and A of a member are kept unchanged, hence the stiffness of a member changes with the length of the member. The form of a truss unit is then dependent of both the angle between members and the stiffness of the members. When the length of a member in a truss unit is constant, the form of the truss is in its best when the other two members joint in a right angle with each other.

3.7 Conclusions

In this chapter, the basic concepts of graph theory have been briefly reviewed and used to introduce a graph model of the structural system --- the structural ring.

A structural ring is a concept of structure in the graph model. Therefore, it is capable of transmitting force along arbitrary direction in space. It is a sequence of joint objects and member objects in alternate order and must be either just-stiff or over-stiff, i.e. the redundancy of the ring must be equal to or greater than zero ($red[R] \geq 0$).

A structure is failed when it becomes a mechanism, i.e. the redundancy of the corresponding structural ring is less than zero ($red[R] < 0$).

There are many different ways in which a structural ring can deteriorate into a mechanism. A deterioration hierarchy of structural ring (DHSR) is used to include all possible ways a fully fixed structural ring deteriorates into a mechanism. Each path in the DHSR is a failure scenario (See chapter 6). Therefore the DHSR is essential for identifying failure scenarios in vulnerability analysis.

For a structure, the well-formedness is a measure of the quality of its form. The measure is independent of the co-ordinate system. It is only related to the type of joint, the stiffness of the members and the configuration of the members in the structure. It is closely related to the principal stiffness coefficients of the joints, indicating the ability of the structural ring to resist loads from arbitrary direction. Using this measure, comparison can be made on forms of different structures.

Chapter 4

Structural Clusters

4.1 Objectives

The objectives of this chapter are to:

- review briefly techniques of cluster analysis;
- define the concept of a structural cluster;
- define four different types of structural cluster;
- develop a set of criteria to evaluate the tightness of structural clusters;
- introduce the principles of cluster formation;
- illustrate the process of cluster formation with an example.

4.2 Introduction

In the previous chapter, we have defined the concept of a structural ring and concluded that any structural system can be represented as a set of interconnected structural rings. However, in the graph model of a large complex structural system, there are a large number of member and joint objects and the number of possible structural rings formed among them can be enormous.

In this chapter, we will apply cluster analysis to classify the vast number of objects in the graph model into groups of manageable size. The concept of structural clusters will be introduced and different types of structural cluster will be defined.

The conventional techniques of cluster analysis have their merits and problems in various field of application. The technique used in this work is based on hierarchical technique and has adopted a unique relationship imposed on structural objects to form a structural cluster. A set of criteria for cluster formation will be discussed in detail.

A hierarchy will be generated at the end, which consists of structural objects in the form of interconnected structural rings. The hierarchy represents the structural system at various level of description, in which the level indicates the intensity of the structural tightness.

The properties of the hierarchy will be discussed further in later chapters.

Finally, an example will be used to demonstrate the process of cluster formation.

4.3 Cluster Analysis

Cluster analysis is a methodology which optimizes intra-group homogeneity in a given population (Lance & Williams, 1966). As a powerful tool for classification and data compression, it has been widely used in various fields of research such as zoology, biology, botany, sociology and psychology. It has also become popular in applications in engineering network, environmental analysis, artificial intelligence and information technology to deal with the complexity in large scale systems. (Shekar *et al*, 1989, Banerjee & Rosenfeld 1993).

The literature shows that workers from different fields use cluster analysis for different purposes. This explains why there are great many cluster analysis techniques in use.

In some applications, the problem is to find the disjoint groups in a population in which the individuals of a multivariable samples bear greater similarity when compared with individuals not inside the same group. In other applications, the problem may be to separate subsystems in a large system in a way that the elements in the same subsystem are strongly interconnected, whereas the elements in different subsystems are not.

However different in detail, all these techniques deal with the same problem in general, that is:

Given a set of N objects or individuals, each of which has an attribute set (the attribute set is the same for all objects),

devise a classification scheme for grouping the objects into g classes.

The number of classes and the characteristics of the classes to be determined.

4.3.1 Techniques of cluster analysis

In practice, techniques for cluster analysis aim to separate a set of data or individuals into groups or clusters. A wide range of clustering techniques have been developed and used over the years in different applications to achieve the same goal, while the interest and focus in the data varies.

Cluster analysis techniques may be classified into the following types: (Everitt, 1974)

- Hierarchical techniques
- Optimisation partitioning techniques
- Density or mode-seeking techniques
- Clumping techniques
- Others

It should be stated that some techniques can be classified in more than one type listed above, hence these types are not necessarily mutually exclusive.

Generally, **hierarchical techniques** include two distinct methods --- *agglomerative method* and *divisive method.* When employed in cluster analysis of a set of N entities, the former tends to fuse N individual entities successively according to certain predifined criteria and finally form a single cluster containing all the entities. Whereas the latter is to partition the set of all entities into finer groups and finally split the entire set into N groups each containing a single entity. In both methods, divisions or fusions once made are irrevocable.

Partitioning techniques aim to partition a set of entities to predetermined number of groups. In the process of clustering, relocation of an entity is admitted, therefore allowing initial clustering to be refined in the process.

Density or mode-seeking techniques are those which search for natural sub-grouping in the set of entities. This type of method searches in the predifined space of entities for those parts which are highly dense and separated by parts of low density.

Most of the techniques search for disjoint or distinct grouping of entities, however, in some area of application, overlap between groups must be considered. **Clumping Techniques** include those which allow overlapping clusters.

Various other techniques have been used by workers from many fields. They vary on choice of variables, measures of distance and similarity between entities and even definition of clusters.

Each of these techniques has its own merits and problem. Very often the relative merits of them is difficult to judge. The choice of technique is highly dependent of the objectives of the investigator.

4.3.2 Clusters

No matter which technique is used, a clear definition of a cluster must be given in accordance to what requirements the cluster analysis is to fulfil.

In general, a *cluster* can be defined as a group of contiguous elements of a statistical population in a specific space.

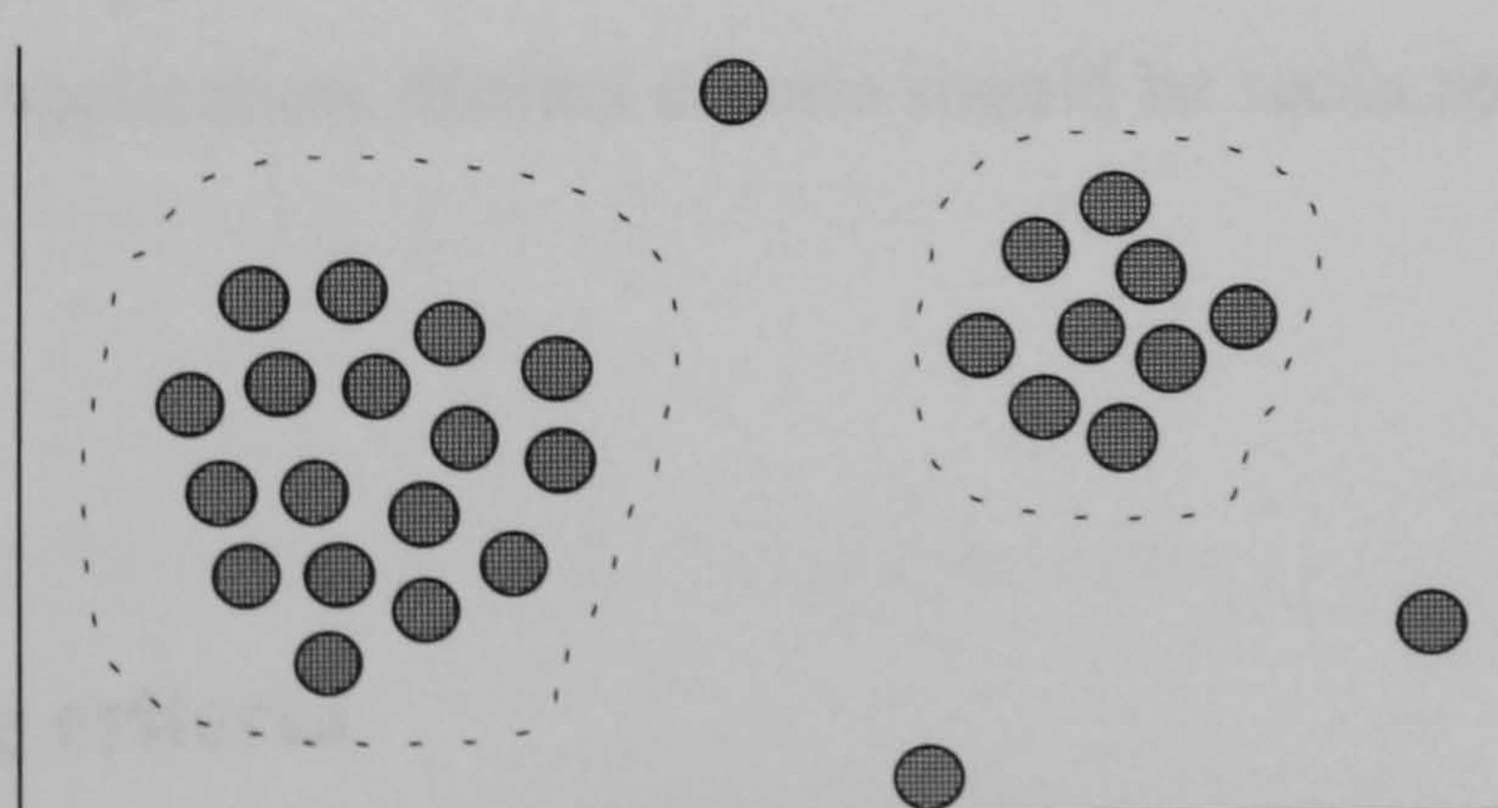


Figure 4.1 Example of a set of two-dimensional data

To illustrate the concept of a cluster, let us plot the location of residential dwellings in a two-dimensional space (Figure 4.1). This simplest example reveals the concept of

natural clusters. According to the closeness in distance between any pair of these dwellings, the population under investigation can be clearly divided into two clusters.
In some applications, the objects are presented in a graph model. In Figure 4.2, the graph can be divided into three obvious clusters according to the connectivity between its vertice.

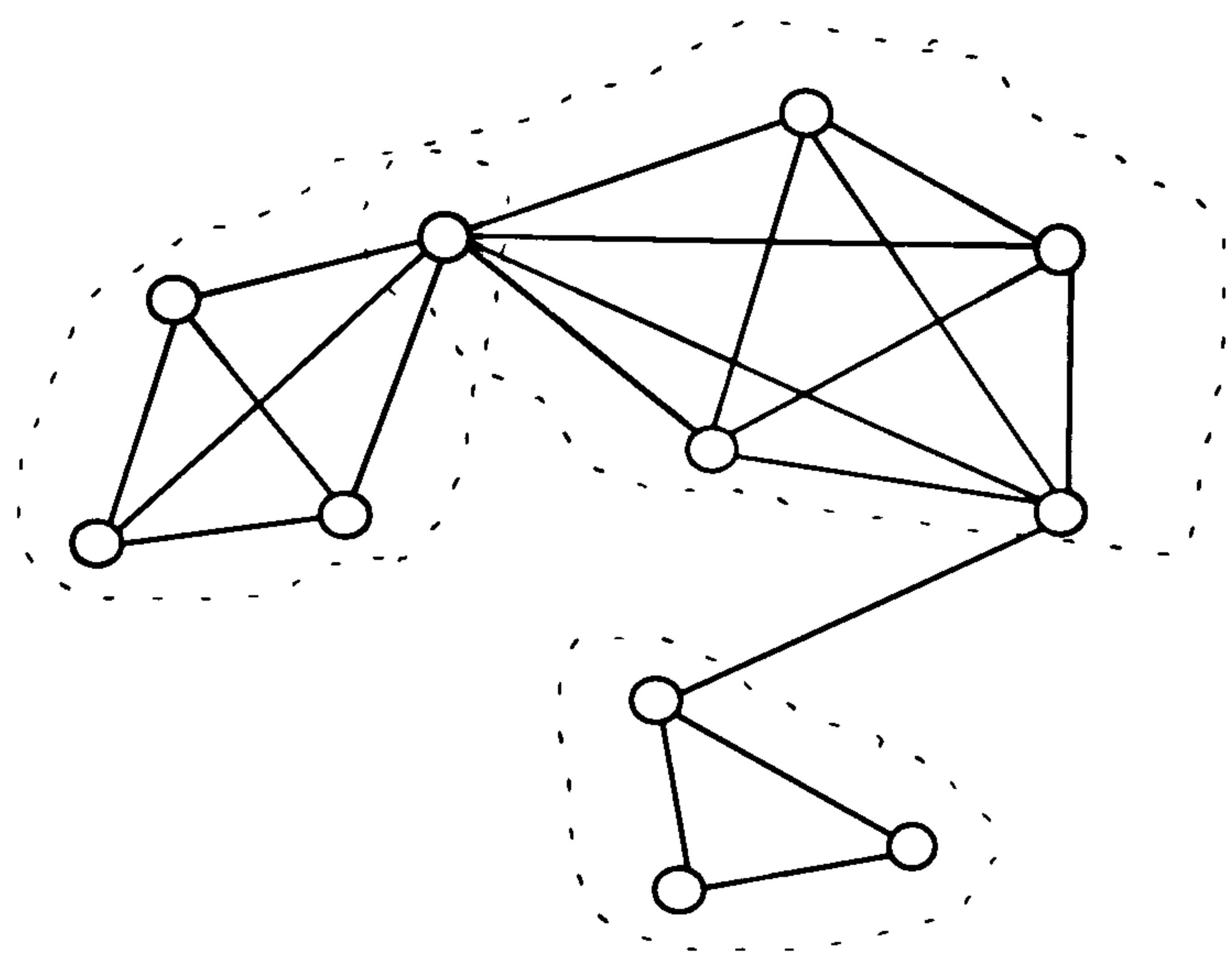


Figure 4.2 Clusters in a graph

In majority of the applications, clusters may not be so easily perceivable as in the two examples used. The relationship considered within the data set may not be just simply the pairwise relationship. Complicated formulas may be employed, involving several specifically defined variables, to work out a measure of the similarity or closeness among the set of data.
Therefore, for each application, distinct criteria should be set in advance of the process of clustering.

4.3.3 **Clustering criteria**

In cluster analysis, the basic data is a set of entities on which we choose to have certain variables as clustering criteria. The choice of the particular set of variables constitutes a frame of reference within which to establish the clusters. The choice also reflects the investigator's judgement of relevance for the purpose of the classification.

The input criteria ultimately determines the clusters found. Thus it is essential to choose the correct and most relevant variables.

Once the set of variables is chosen, the other consideration is the weighting of variables. Variables may be given differential weighting in setting up clustering criteria, if they are thought to be different in priority for the purpose of classification.

The measures can be in the form of binary, qualitative and quantitative data, depending on what the investigator decides the best for the application.

4.4 Structural Clusters

So far in this chapter, general aspects of cluster analysis have been reviewed. Now, we will move on to introduce a method to perform cluster analysis on our graph model of a structural system.

A graph model of a structural system consists of large number of member and joint objects at the elementary level. All these objects are highly interconnected in a complex structure. In practice, the interest of engineers in a structural system is not only confined at the elementary level, sometimes different levels of detail may become important. Analysis of a structure at various levels can be carried out only if we rearrange all the member and joint objects into *clusters* which can themselves be used, in consistency, as elements at a higher level of description.

4.4.1 Method of clustering structural systems

The goal of the cluster analysis here is to produce a model with which the structural system can be assessed at different levels of detail. The level of description indicates the intensity in terms of the connectivity between clusters and the form of the structure. The level of description ranges from the elementary level at which each of the structural objects are as individuals to the structural level where all objects are considered as one.

According to the requirements, the agglomerative method of hierarchical technique is most fit for the purpose. Clustering starts from the elementary level and fuses objects to form clusters at a higher level of description. When a cluster at a higher level of description is created, it captures the information of those objects which formed it and becomes an individual object at that level. All objects at the elementary level will finally evolve into, through different clusters at a finite number of levels of description, a single cluster which represents the whole structure.

The term structural cluster and several types of cluster are to be defined according to the method adopted.

4.4.2 Definition of a structural cluster

In our graph model, a structural system is presented as $\boldsymbol{S} = \{ \boldsymbol{M}, \boldsymbol{J} \}$.

A structural cluster $\boldsymbol{C}_i^l$ at a level of description l is, first, a subset of $\boldsymbol{S}$, thus:

$$\boldsymbol{C}_i^l = \{ \boldsymbol{M}_i^l, \boldsymbol{J}_i^l \} \qquad (4.1)$$

where, $\boldsymbol{M}_i^l \in \boldsymbol{M}$, $\boldsymbol{J}_i^l \in \boldsymbol{J}$

However, it is not *any* subset of $\boldsymbol{S}$. The subset itself must be a valid structural system. That is to say: the subset can be represented with a structural ring or a set of overlapping structural rings.

Finally, a structural cluster is a subset of S in which the objects are more strongly connected to each other inside the subset than to those outside the subset.

To summarise the points discussed so far:

a structural cluster $\boldsymbol{C}_i^l$ at a level of description l is a subset of the graph model of a structural system $\boldsymbol{S}$, the objects in which must be (1) able to form one structural ring or a set of overlapping structural rings, and (2) more strongly connected to each other than to those objects not inside C_i^l.

Because of the structural characteristics of the structural cluster, it can be treated as a unit in the structural system. It can be used together with other member and joint objects in $\boldsymbol{S}$ to construct further structural clusters at a higher level of description.
A single member object in $\boldsymbol{S}$ is a structural cluster at the first level (elementary level) of description because, as discussed in Chapter 3, a single member object can form a 2-link-ring on its own.
On the other hand, a structural system $\boldsymbol{S}$ itself is a structural cluster at the highest level of description.

4.4.3 **Types of structural clusters**

Having defined a structural cluster in general, we further define four different types of structural clusters:

A *leaf/primitive cluster* is a structural cluster which contains a single member object.

A *branch/intermediate cluster* is a structural cluster which contains more than one member object. It may also contain clusters which are of the same type but from a lower level of description.

A *root/complete cluster* is a structural cluster which contains the entire set of member and joint objects in the structural system. A root cluster is the entire structural system, i.e. $\boldsymbol{S}$.

A *reference cluster* is a cluster which is specially specified for the individual purpose and interest of the investigator to carry out vulnerability analysis. For example, when

analysing a building structure, the ground is often treated as a reference for the rest of the structure. In such a case, the ground will be specified as the reference cluster.
A reference cluster may contain one or more leaf clusters. A leaf cluster which belongs to a reference cluster is termed as a *reference leaf cluster*.

The leaf, branch and root clusters are corresponding to different levels of description. Together, they define the space in which our hierarchical clustering method is applied to classify the structural objects. In the next Chapter when a hierarchical representation of a structural system is developed, these different types of clusters will be used to construct such a hierarchy with which our vulnerability analysis can be performed.
The importance of a specially defined reference cluster will come to light when the failure scenarios are discussed.

4.4.4 Structural rings and structural clusters

The relationship between structural rings and structural clusters needs to be clarified to distinguish the two terms.
In previous work, a structural cluster at a level of description is defined as "a group of overlapping structural rings", and structural rings were treated as the basic objects for clustering. (**Wu**, 1991)
However, the definition has used the concept of a structural ring in a way which may cause confusion. By organising structural objects into different structural rings and then clustering the structural rings, the analysed problem S, i.e. the structural system, has been classified or clustered before further classfication. In that case, the concept of a structural ring *is* used as a structural cluster.
In fact, the two concepts are rather distinct from each other. In the graph model, the concept of a structural ring is about relationships. It is the *relationships* within a set of member and joint objects which defines a structure. If the set of member and joint objects can form a structural ring, then the combination of them is a structure. Otherwise, it is a mechanism.

The essential concept of a structural cluster is that, first of all, it is a subset or a set of structural objects. What makes a collection of structural objects a structural cluster is the relationship among those objects, which is structural rings. *If* and *only if* those objects can form a structural ring or more than one overlapping structural rings, *then* the set of them is a structural cluster.

Now, structural rings are no longer treated as the basic objects in clustering. The basic objects in our cluster analysis are structural member and joint objects. A structural ring is a relationship criterion imposed upon structural objects in defining structural clusters.

4.4.5 Measures of a structural cluster

So far, we have defined structural clusters. The next stage is to choose a set of suitable measures. These measures will allow evaluation and comparison to be made on clusters.

As in any cluster analysis, the choice of variables is directly related to the purpose of the analysis. We focus our attention on the state of the form of the structure and the process in which the structure may deteriorate and finally fail. Thus the following measures are attributed as the characteristics to a structural cluster:

- the well-formedness of a structural cluster
- the minimum damage demand of a structural cluster
- the nodal connectivity of a structural cluster
- the distance from the reference of a structural cluster

4.4.5.1 Well-formedness of structural clusters

As introduced in Chapter 3, the well-formedness of a structure, $Q(S)$, is a measure of the form of the structure. It is a function of *a)* type of the joint objects, *b)* stiffness of the member objects and *c)* the configuration of the member objects in the structure.

A structural cluster C is a subset of S. Extended from the definition of the well-formedness of a structure, the well-formedness of a structural cluster, $Q(C^l)$, is therefore the sum of the determinant of the stiffness sub-matrix associated with the joint objects in the structural cluster divided by the total number of joint objects in the cluster.

$$Q(C^l) = \delta\{ \sum_{i=1}^{N_j(C^l)} q_i \Big/ N_j(C^l)\} \qquad (4.2)$$

where δ is the validity factor.

$\delta = 1$, if $Red(C^l) \geq 0$, or $\delta = 0$, otherwise,

$Q(C^l)$ is the well-formedness of the structural cluster C^l,

$N_j(C^l)$ is the total number of joint in cluster C^l, and

q_i is the well-formedness of the ith joint in cluster C^l.

For a structural cluster, the well-formedness $Q(C^l)$ is the most important measure in terms of the form of the structure. However, the ultimate goal of our clustering analysis is to produce a model with which the possible deterioration and failure of the structure can be systematically studied. Thus we need several other measures to assess the damage potential of a structural cluster.

4.4.5.2 **Minimum Damage demand of structural clusters**

In Chapter 3, we have introduced the concept of a deteriorating event. The *Damage Demand* is a measure associated with a deteriorating event. It is defined as the effort which is required to achieve the deteriorating event. (**Wu**, 1991)

If we denote $f^l_{i,j,k}$ as the deteriorating event which is adjacent to a joint object and $g^l_{i,j,k}$ as the one which occurs in a member object, then the damage demand for a deteriorating event $f^l_{i,j,k}$ or $g^l_{i,j,k}$ is denoted as $e(f^l_{i,j,k})$ and $e(g^l_{i,j,k})$ respectively,

where

l is the level of description,

i is the index of the joint object in $f^{l}_{i,j,k}$ and the member object in $g^{l}_{i,j,k}$.

$j = \{ \mu,\nu,\theta \}$ is the string pattern of the joint object in $f^{l}_{i,j,k}$ and the member object in $g^{l}_{i,j,k}$, and

k is the level in a failure scenario or in the DHSR.

Let $d^{l}_{i,j,k}$ and $s^{l}_{i,j,k}$ be the degree of freedom of the joint and member objects respectively, we have:

$$f^{l}_{i,j,k} = d^{l}_{i,j,k} - d^{l}_{i,j,k+1} \qquad (4.3)$$

and

$$g^{l}_{i,j,k} = s^{l}_{i,j,k} - s^{l}_{i,j,k+1} \qquad (4.4)$$

in which $d^{l}_{i,j,k}$ or $s^{l}_{i,j,k}$ is the degree of freedom of the joint or member object *before* the event and $d^{l}_{i,j,k+1}$ or $s^{l}_{i,j,k+1}$ is that *after* the event.

It is logical to assume that the damage demand of a deteriorating event is proportional to the principal stiffness coefficient corresponding to the specific degree of freedom which is lost after the deteriorating event. Hence the damage demand can be quantitatively described as:

$$e(f^{l}_{i,j,k}) = w^{l}_{i,j,k} \; (\lambda^{l}_{i,j,k} \times d^{l}_{i,j,k} - \lambda^{l}_{i,j,k+1} \times d^{l}_{i,j,k+1}) \qquad (4.5)$$

or

$$e(g^{l}_{i,j,k}) = w^{l}_{i,j,k} \; (\lambda^{l}_{i,j,k} \times s^{l}_{i,j,k} - \lambda^{l}_{i,j,k+1} \times s^{l}_{i,j,k+1}) \qquad (4.6)$$

in which $w^{l}_{i,j,k}$ is a constant, $\lambda^{l}_{i,j,k}$ is the principal stiffness coefficient which is corresponding to the lost degree of freedom before the event and $\lambda^{l}_{i,j,k+1}$ is that after the event.

The minimum damage demand of a structural cluster $e_{\min}(C^{l})$ is the smallest damage demand among those of all objects in the cluster.

$$e_{\min}(C^l) = min\{\ e(f^l_{i,j,k}),\ e(g^l_{i,j,k})\ \} \qquad (4.7)$$

where

$i = 1,2,\ldots,N_j(C^l)$ or $N_m(C^l)$ — $N_j(C^l)$ is the total number of joint objects in the cluster, and $N_m(C^l)$ is the total number of the member objects in the cluster.

$j = \mu,\nu,\theta$

$k = 1$

The minimum damage demand of a structural cluster is an indicator of the toughness of the cluster in terms of *any* damage which may happen in it. Notice that here we do not consider the scale and consequence of the damage and the nature of the actions that may have caused it.

In a sense, the higher the minimum damage demand of a cluster, the tighter the cluster. In the clustering process, it may come to a point that using the well-formedness measure alone is not sufficient to produce a satisfactory result. The minimum damage demand becomes the next important measure to use, when, and only when, using well-formedness criterion fails to select an unique candidate.

4.4.5.3 **Nodal connectivity of structural clusters**

One of the commonly known problems with hierarchical clustering techniques is that they contain no provision for reallocation of entities as the clustering process goes on. There is no possibility of correcting a decision made in an early stage of clustering. In our cluster analysis, the selected candidate according to the clustering criteria at a level of description naturally forms a base for further clustering at the next level of description. When there are more than one equally qualified candidates, decision must be made to choose one for the specific level of description. However, if we choose one randomly, it may appear in later stage, or at higher levels of description, that the selected one was not the best choice. As we are dealing with a finite search space in cluster formation when given a structure, ideally, a full back-tracking technique can be

used to solve this problem. However, the search space can grow to be enormous for a large scale structure. Because of the limitation of time and resource in the research, such a technique is difficult to implement and unnecessary.
Nodal connectivity of a cluster can be used as a criterion for dealing with multi-candidate situation.

The *nodal connectivity* of a cluster $\eta(C^l)$ is defined as:

$$\eta(C^l) = \sum_{j=1}^{N_J(C^l)} D(J_j) \qquad (4.8)$$

where $N_J(C^l)$ is the total number of joint objects in the cluster
$D(J_j)$ is the degree of joint for jth joint object. (see 3.4.4 in Chapter 3)

The physical meaning of the degree of a joint object is the number of member objects connecting into it. The nodal connectivity of a cluster is then the total number of member objects connecting to the joint objects in the cluster. It is the indication of the potential capacity of the cluster to form further structural ring with other structural clusters. The cluster of the higher nodal connectivity has more connections or overlaps with external clusters, and therefore is capable of forming tighter clusters in higher levels of description.

This measure is effective when both well-formedness and minimum damage demand do not produce a unique candidate.

4.4.5.4 **Distance from the reference**

The purpose of our cluster analysis is to build up a foundation upon which our vulnerability analysis can be performed. As discussed in Chapter 2, the vulnerability of a structure cannot be isolated from its context. It is related to the form of the structure, damage caused by actions and the consequences of damage.

This last measure, distance from the reference, is chosen because it is related to the consequence of damage.

In the graph model, the reference cluster is not only an integral part of the structural system, but also a reference point from which the scale of damage is quantitatively evaluated (see Chapter 6). The spatial position of the structural clusters determines that some clusters (say C_a, C_b) only connect to the reference via connecting to other clusters (say C_c). These clusters, C_a and C_b, which are apparently further away from the reference cluster, can be disconnected from the reference without any internal damage, but by the failure of C_c, i.e. those clusters which connect them to the reference cluster.

Distance from the reference of a cluster $\Delta(C^l)$ is a measure related to the potential damage consequence of a cluster. It is calculated as:

$$\Delta(C^l) = \sum_{j=1}^{N_J(C^l)} \delta(J_j) \Big/ N_J(C^l) \qquad (4.9)$$

where J_j is the jth joint object in the cluster

$N_J(C^l)$ is the total number of joint objects in the cluster, and

$\delta(J_j)$ is the shortest distance between the jth joint object and the reference cluster

4.5 Cluster Formation

Having defined the structural cluster and having chosen the set of measures for evaluating and comparing clusters, we are now ready to transform a structure system into a set of structural clusters at a finite number of levels of description.

In this section, the general principle of cluster formation and the clustering criteria will be introduced, however, a detailed algorithm of cluster formation will be discussed later in Chapter 7.

4.5.1 Principles of cluster formation

In general, a complete clustering system may involve the following processes: (Lance & Williams)

a). A method to initialise clusters.

b). A method to fuse existing clusters.

c). A method to determine when further allocation of element may be regarded as unprofitable.

d). A method to reallocate some or all of the elements to existing clusters when the main classificatory process is completed.

All systems involve a) & b), but in any particular problem, c) or d), or both, may be lacking.

The general principle of cluster formation when using graph theoretic techniques can be summarised in the following sentence:

Given a set of n objects,

find the clusters according to the attributes of objects

from a collection of clusters characterized in the manner of internal coherence or external isolation

with levels of cohesive intensity.

Dealing with a structural system which is represented in the graph model, the specific problem is to find a set of structural clusters from the given complete set of structural objects, such that the objects within a cluster are more densely connected to each other than to other objects outside the cluster, hence a cluster is a subset of structural objects which is *tighter* and *better formed*.

The initialisation of structural clusters is based on the fact that the objects selected can form a structural ring together. When selecting the best formed cluster, the judgement is based on the clustering criteria. The clustering criteria are also used to determine whether further allocation of element is necessary.

4.5.2 **Criteria**

At a level of description, the cluster formation process includes two tasks:

1) to identify all possible ways to fuse elements into clusters, the elements may themself be clusters which have been formed from previous levels,
2) to select the one which can maximize the structural tightness criteria, the elements in the combination will merge into one cluster at next higher level of description.

In task 2), when comparing various options identified by task 1), the following five criteria are used for the cluster selection in the following order:

1. Maximum *well-formedness* of a cluster
2. Maximum *minimum damage demand* of a cluster
3. Maximum *nodal connectivity* of a cluster
4. Maximum *distance from the reference* of a cluster
5. Choose randomly

The first criterion is the well-formedness of structural clusters. The fusion of the elements which can give the maximum increase in the value of well-formedness of cluster will be selected as the tightest cluster at the level.
However, the first criterion does not necessarily produce a single result from the population of candidates. Such situation is very common when dealing with structural systems which are uniformly shaped. The second criterion is used to distinguish between multiple results from the first criterion. The same principle applies to all the

criteria down the line, and that is, from the second criterion onwards, a criterion is in effect only when the previous criterion/criteria fail to distinguish the candidates.

The second criterion is the minimum damage demand of a structural cluster. When this criterion is applied, the aim is to select the fusion of elements which has the maximum value of minimum damage demand. The result will be the tighest cluster at the level upon which it is most difficult to cause any damage.

The third criterion is the nodal connectivity of a structural cluster. The cluster which has a higher value of nodal connectivity may have more connections with objects in other clusters, therefore is more likely to form the tighter cluster in future levels.

The fourth criterion is the distance from the reference of a structural cluster. This criterion is from the perspective of damage and consequence of failure of a cluster. As discussed in the previous section, the further away a cluster is from the reference, the less consequence it will cause when being disconnected from the reference, because there will be less other clusters spatially depend on it. Hence, in a sense, it has the better form.

The above four criteria can significantly reduce the presence of the migration problem in the clustering technique used in the research. However, further refinement in the clustering criteria or improvement in the clustering algorithm will be desirable.

Finally, if it is still unable to distinguish the candidates after using all four criteria, a candidate will be chosen randomly. At this stage, the risk of migration problem is considered as negligible.

4.5.3 Initial, secondary and reference clustering

In the process of cluster formation, the reference cluster is treated specially. When a cluster is defined as the reference cluster, the damage and failure state of the rest of the structure will be assessed according to it, and the failure or damage of that particular cluster is not of any interest, so that we assume the reference cluster is undamageable. Since our attention is focused on the internal form of the structure, it is logical to divide the cluster formation process into three distinct stages:

- the initial clustering,
- the secondary clustering,
- the reference clustering.

In the beginning of cluster formation, each of the structural components is defined as a primitive cluster. The first stage is to construct clusters using all the primitive clusters. This initial clustering stage does not involve the reference cluster. At this stage, a cluster will be initialised using primitive clusters, and at each step, only primitive clusters will be allocated to the existing cluster. When there is no more primitive cluster that can be added according to the criteria, the cluster formation process will initialise another cluster using the remaining primitive clusters, and perform the same process until it is not possible to find any more valid structural clusters. This is the end of the initial clustering stage. At this point we have a set of intermediate clusters, each of which has the maximum internal tightness, the reference cluster and the remaining primitive clusters, if any. Those remaining clusters are a special set of primitive clusters because they cannot be allocated in the initial clustering stage.

The next stage still excludes the reference cluster. The task of the secondary clustering stage is to construct clusters using those clusters formed in the initial clustering stage. When intermediate clusters fuse into a single cluster, it is necessary to modify the clustering criteria. As in the initial clustering stage, we aim to maximise those selected measures in the criteria. However, unlike the initial clustering stage in which the value of well-formedness is always increased when other clusters are allocated, it may decrease at the secondary stage. Therefore, the criteria to terminate the process is that it is unable to form any valid structural clusters using any clusters except reference cluster.

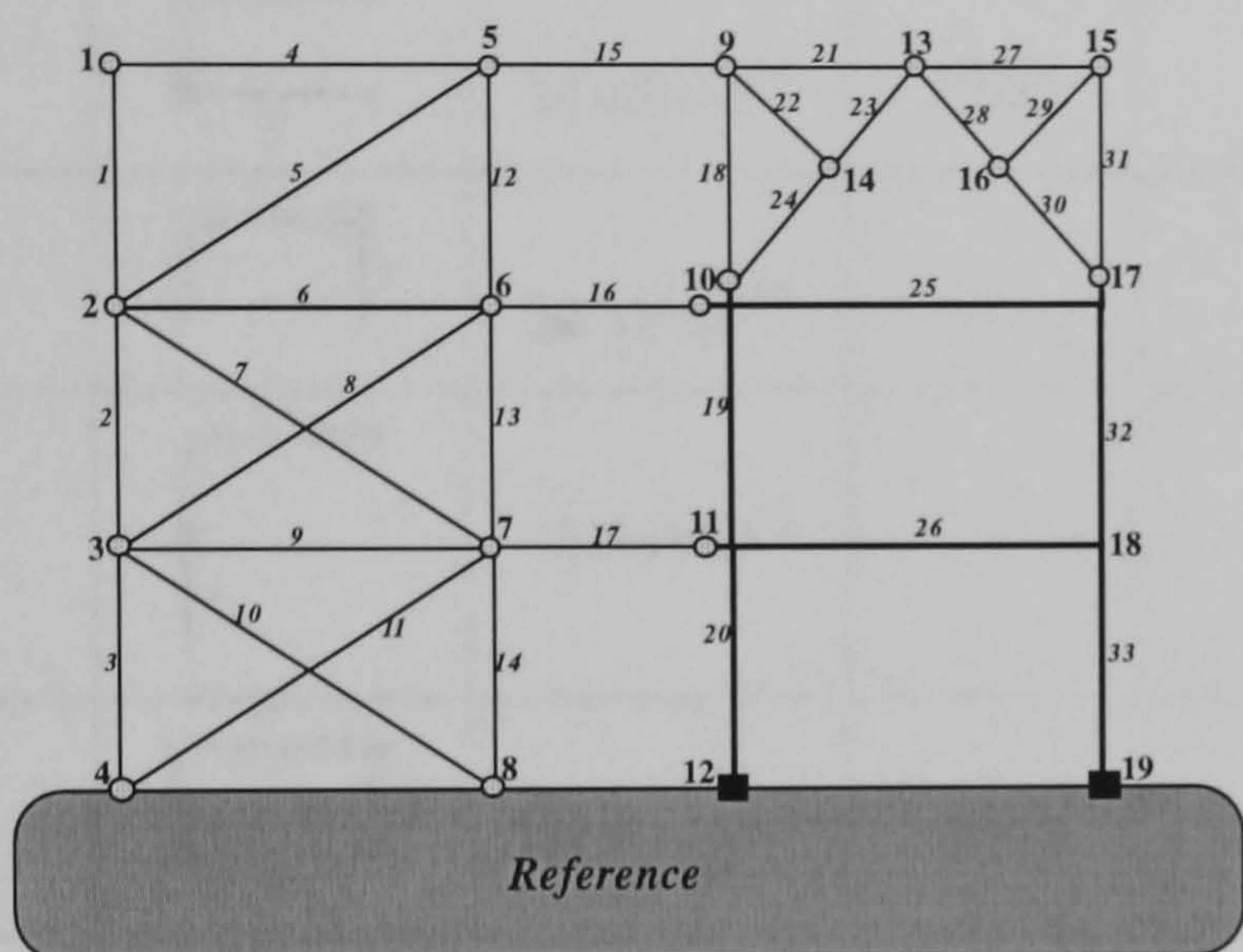
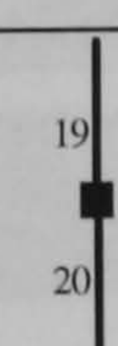
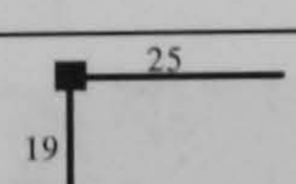
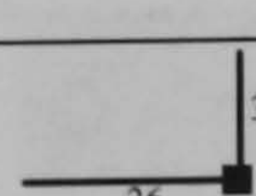
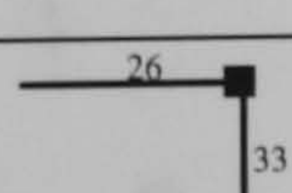
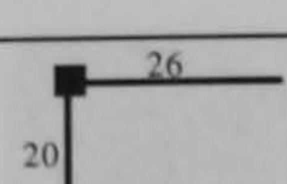
The reference cluster will be included in the final stage of cluster formation --- the reference clustering stage. The clustering criteria are the same as the secondary clustering stage. The reference clustering will come to an end when the complete cluster is found, which is also the end of the whole cluster formation process.

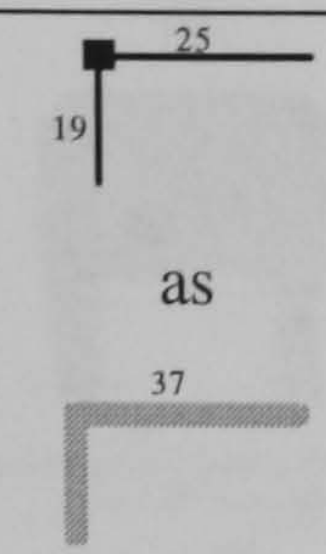
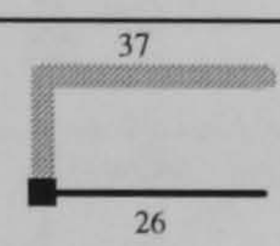
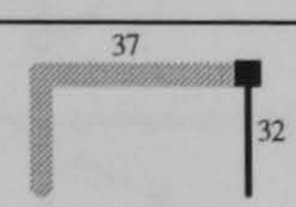
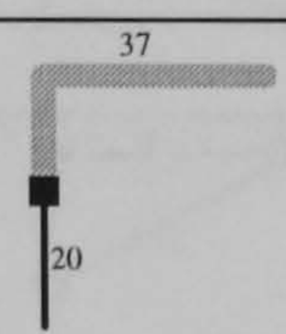
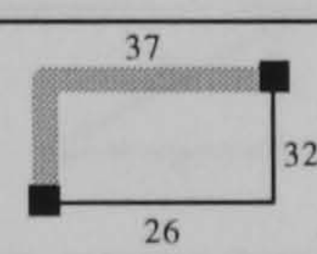
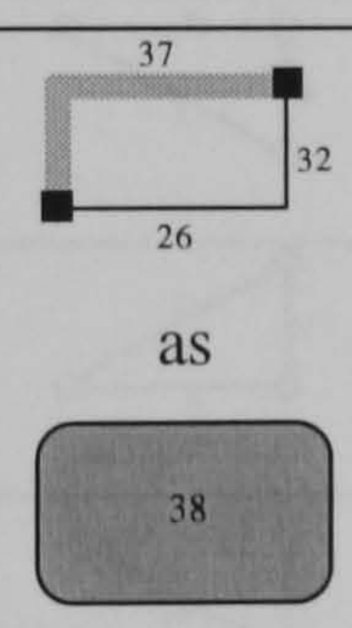
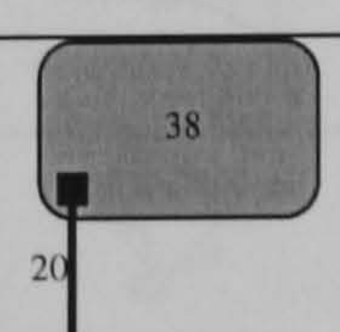
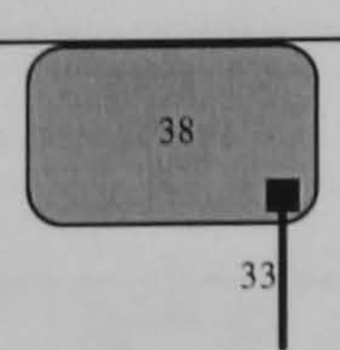
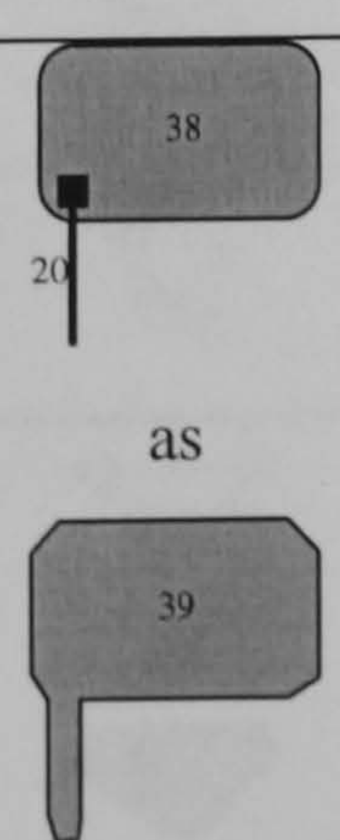
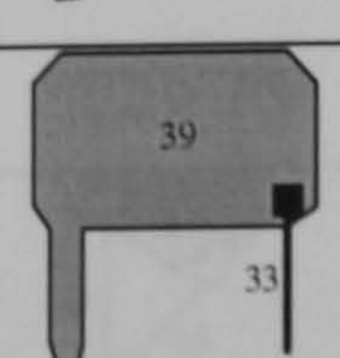
4.6 Example

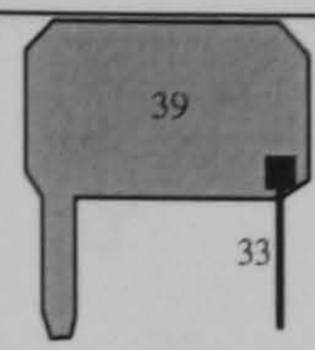
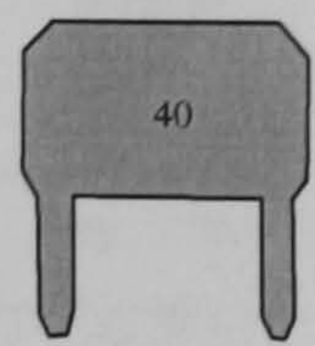
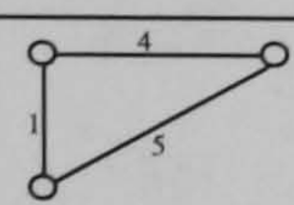
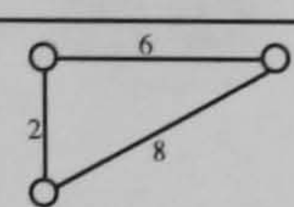
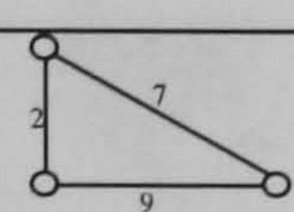
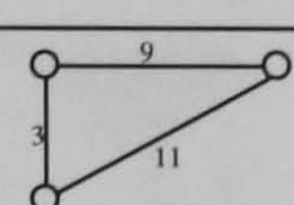
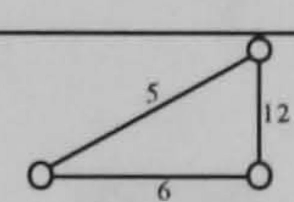
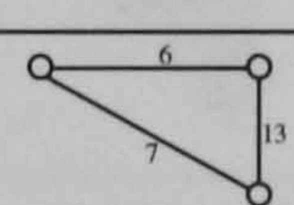
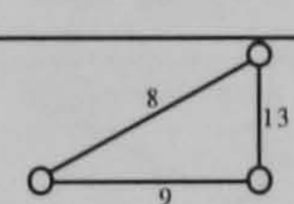
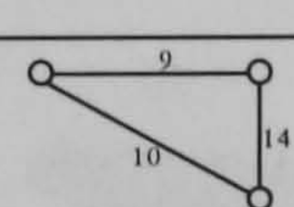
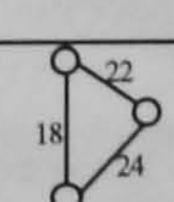
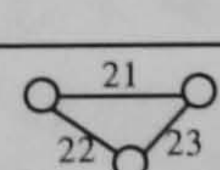
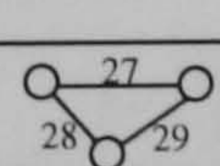
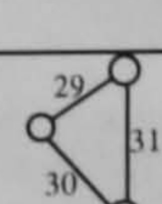
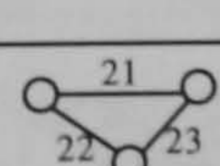
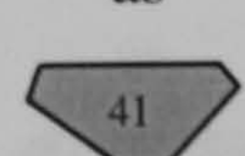
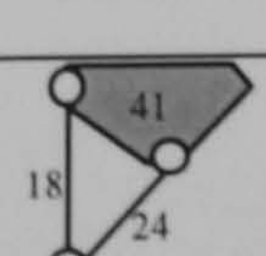
In this section, we will use an example to illustrate the full process of cluster formation.

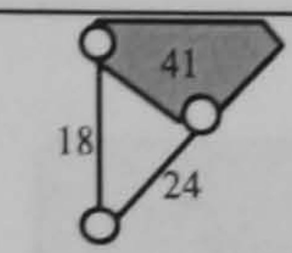
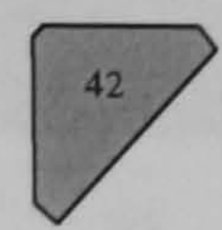
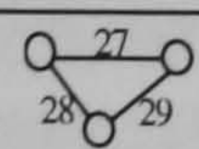
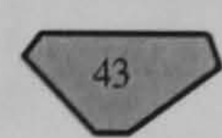
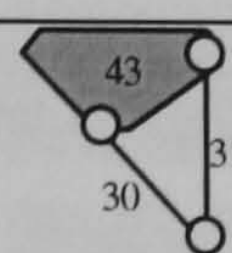
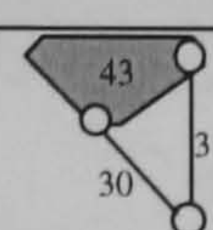
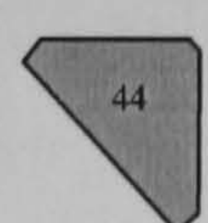
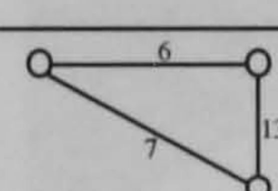
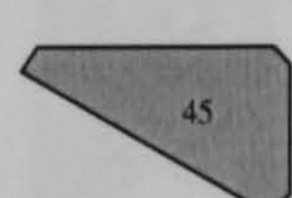
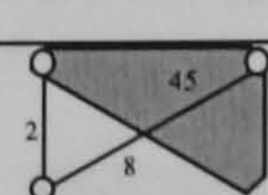
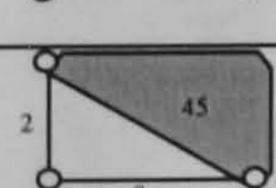
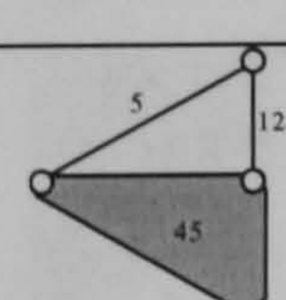
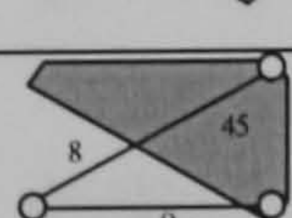
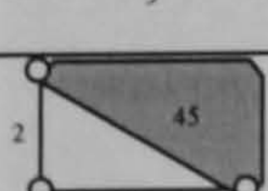
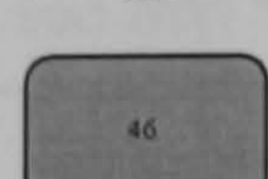
The example being used here is illustrated in Figure 8.8 in Chapter 8. Full detail of the structure in the example is given in Figure 8.8, Table 8.21 - 8.23.

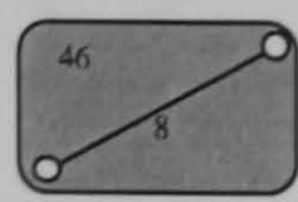
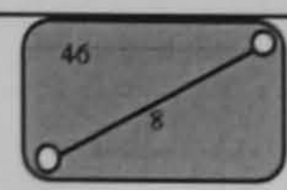
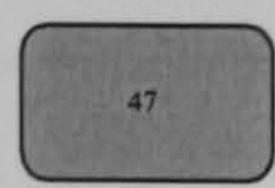
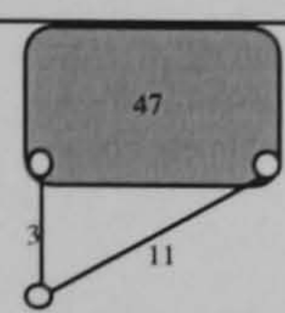
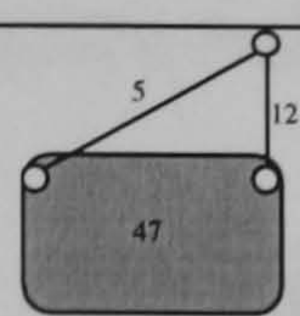
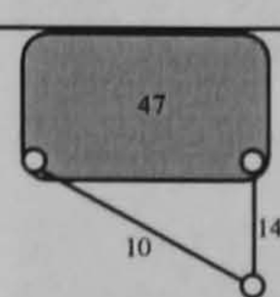
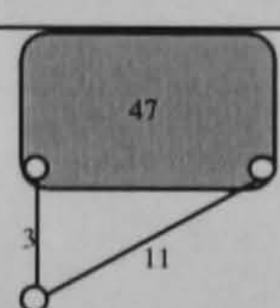
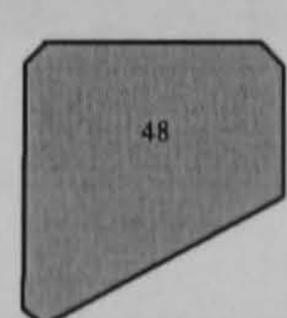
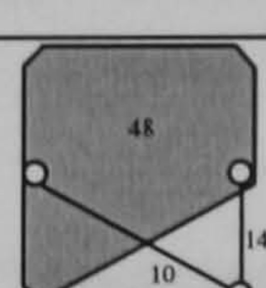
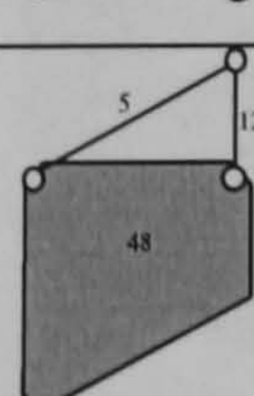
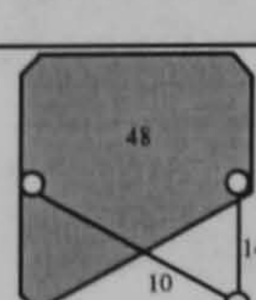
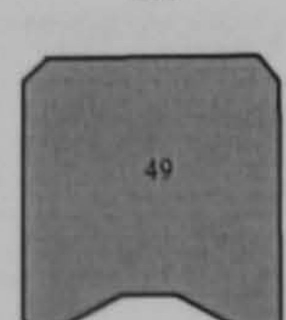
Table 4.1 Step by step cluster formation of a structure

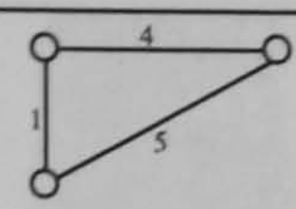
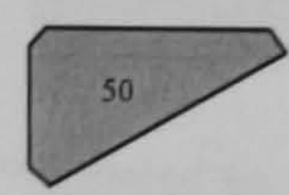
Steps	Components	Cluster Formed	Well-formedness	Damage demand	Nodal degree	Distance
The structure: 						
----- Initial Clustering Stage -----						
Step1	19+20		0.74×10^{14}	39.03	10	3.0
	19+25		16.18×10^{14}	39.03	**13**	9.0
	19+26		16.18×10^{14}	39.03	12	-
	26+32		16.18×10^{14}	39.03	11	-
	26+33		16.18×10^{14}	39.03	8	-
	20+26		16.18×10^{14}	39.03	8	-
	25+32		16.18×10^{14}	39.03	12	-

	32+33		0.74×10^{14}	-	-	-
	Forming cluster 37		*Selection Criteria*: Higher nodal degree			
Step 2	26+37		24.25×10^{14}	39.03	16	6.0
	32+37		24.23×10^{14}	-	-	-
	20+37		12.66×10^{14}	-	-	-
	20+26+37		$\mathbf{48.39\times10^{14}}$	39.03	16	9.0
	Forming cluster 38		*Selection Criteria*: Higher well-formedness			
Step 3	20+38		56.28×10^{14}	39.03	17	3.0
	33+38		56.28×10^{14}	39.03	17	3.0
	Forming cluster 39		*Selection Criteria*: Random choice			
Step 4	33+39		61.54×10^{14}	39.03	18	0.0

	Forming cluster 40	 as 	*Selection Criteria*: One choice, increased well-formedness.			
Step 5	1+4+5 *		2.21×10^{10}	9.31	11	21.0
	2+6+8 *		3.73×10^{10}	15.14	15	12.0
	2+7+9 *		3.73×10^{10}	15.14	16	9.0
	3+9+11 *		3.73×10^{10}	15.14	13	-
	5+6+12 *		2.68×10^{10}	-	-	-
	6+7+13 *		3.73×10^{10}	15.15	16	12.0
	8+9+13 *		3.73×10^{10}	15.14	16	9.0
	9+10+14 *		3.73×10^{10}	15.14	13	-
	18+22+24 *		5.76×10^{10}	23.37	12	19.57
	21+22+23 *		9.91×10^{10}	43.51	11	**24.13**
	27+28+29 *		9.91×10^{10}	43.51	11	24.13
	29+30+31 *		5.76×10^{10}	-	-	-
	Forming cluster 41	 as 	*Selection Criteria*: Greater distance from Ref.			
Step 6	18+24+41		13.67×10^{10}	23.37	16	19.57

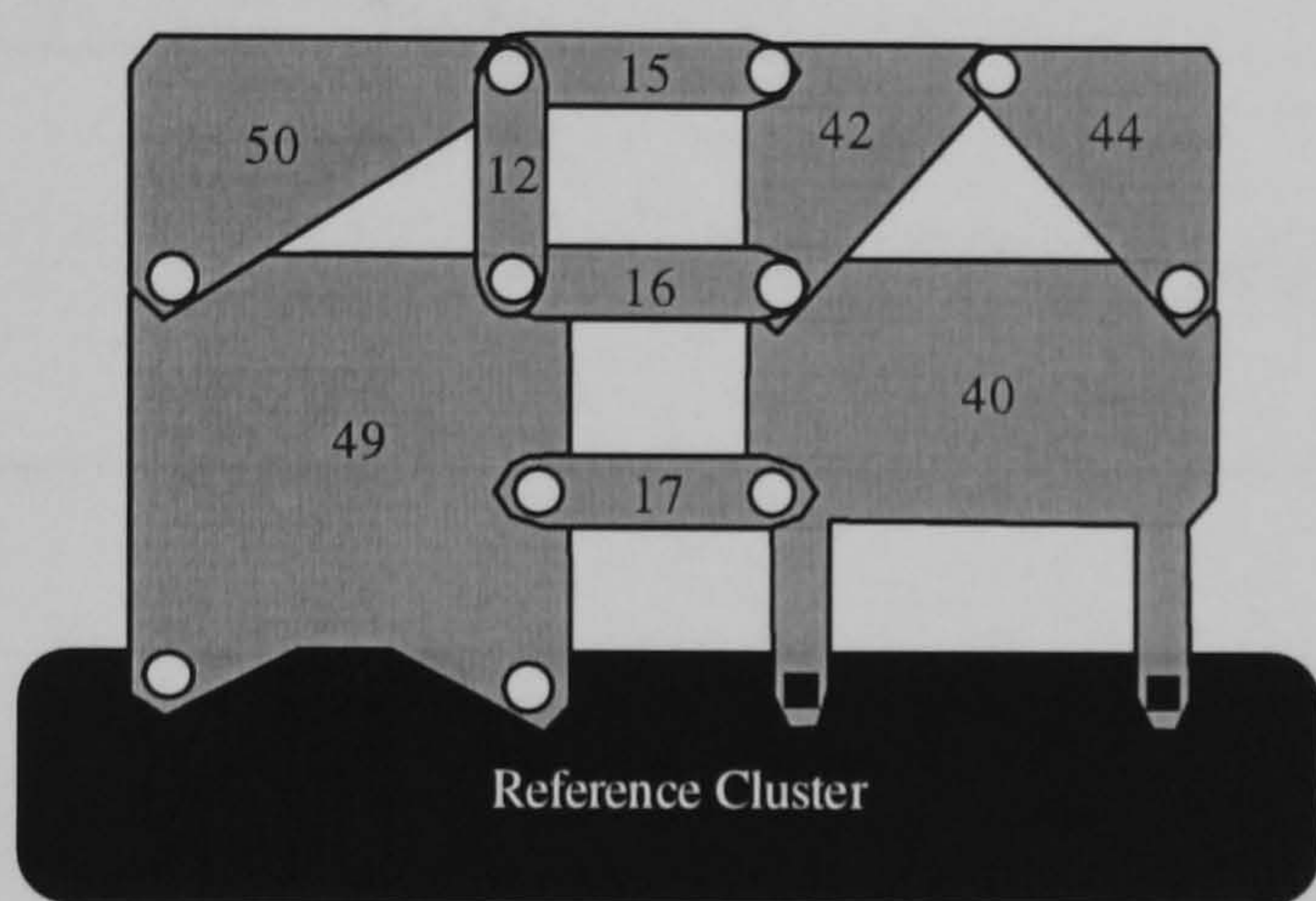
	Forming cluster 42	 as 	*Selection Criteria*: One choice, increased well-formedness.			
Step 7	*	See Step 5				
	Forming cluster 43	 as 	*Selection Criteria*: Higher well-formedness.			
Step 8	30+31+43		13.67×10^{10}	23.37	16	19.57
	Forming cluster 44	 as 	*Selection Criteria*: One choice, increased well-formedness.			
Step 9	*	see step 5				
	Forming cluster 45	 as 	*Selection Criteria*: Greater distance from Ref.			
Step 10	2+8+45		7.26×10^{10}	15.14	21	9.0
	2+9+45		$\mathbf{8.84\times10^{10}}$	15.14	21	9.0
	5+12+45		5.36×10^{10}	-	-	-
	8+9+45		7.26×10^{10}	-	-	-
	Forming cluster 46	 as 	*Selection Criteria*: Higher well-formedness.			

Step 11	8+46		**11.20×10^{10}**	15.14	21	6.0
	Forming cluster 47	 as 	*Selection Criteria*: Higher well-formedness.			
Step 12	3+11+47		13.1×10^{10}	15.14	23	3.0
	5+12+47		12.03×10^{10}	-	-	-
	10+14+47		13.1×10^{10}	15.14	23	3.0
	Forming cluster 48	 as 	*Selection Criteria*: Random choice between two identical clusters.			
Step 13	10+14+48		**15.47×10^{10}**	15.14	25	0.0
	5+12+48		13.48×10^{10}	-	-	-
	Forming cluster 49	 as 	*Selection Criteria*: Higher well-formedness.			
Step 14	*	see step 5				

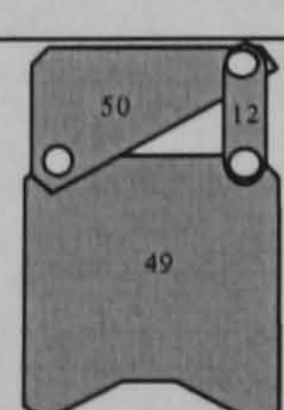
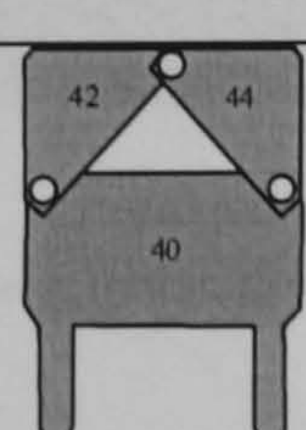
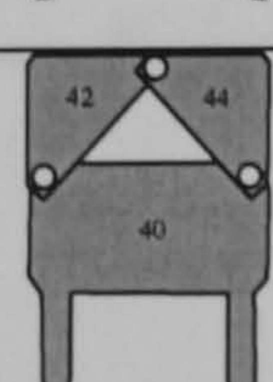
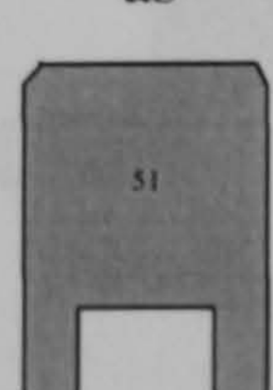
	Forming cluster 50	 as 	*Selection Criteria*: Only choice left.			

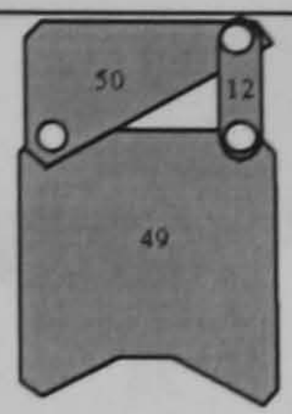
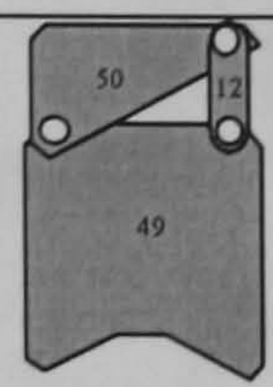
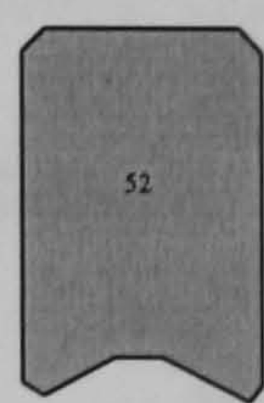
End of Initial Clustering Stage

<u>The structure at the end of ***Initial*** *Clustering Stage*:</u>

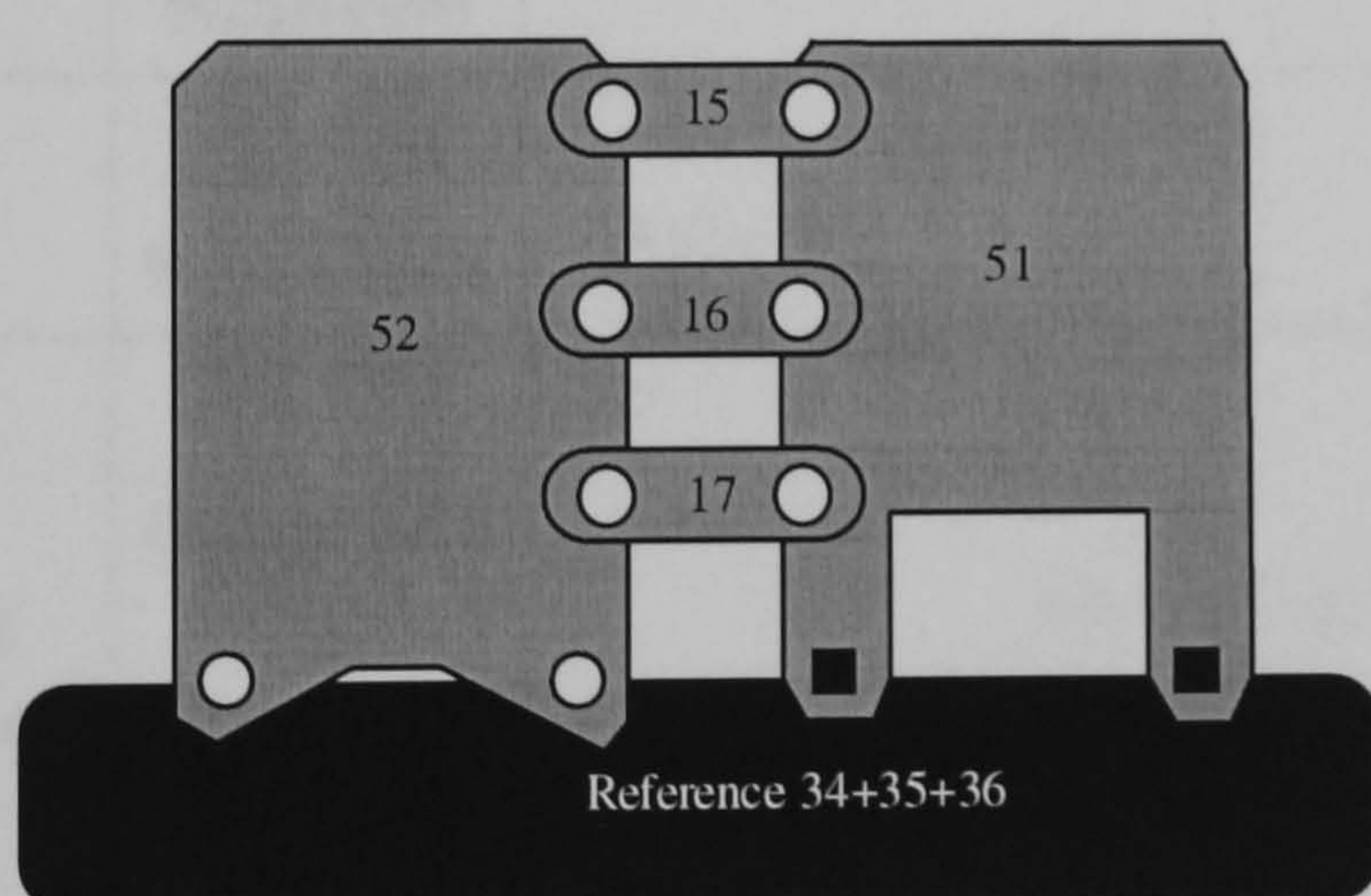


----- Secondary Clustering Stage -----

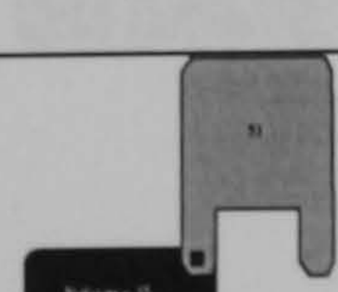
Step 15	12+49+50		15.77×10^{10}	9.31	31	12.0
	40+42+44		$\mathbf{51.14 \times 10^{14}}$	23.37	35	12.0
	Forming cluster 51	 as 	*Selection Criteria*: Higher well-formedness.			

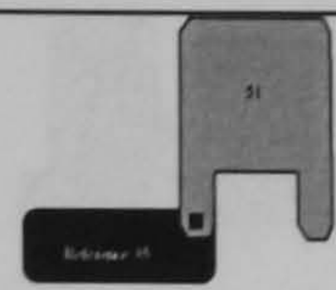
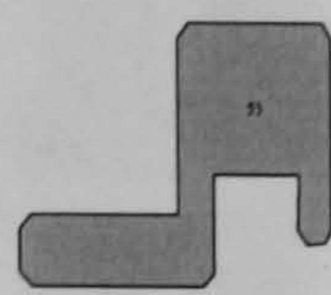
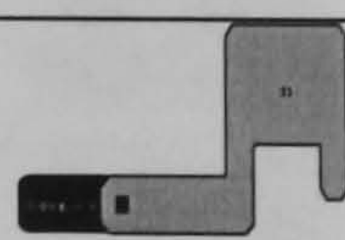
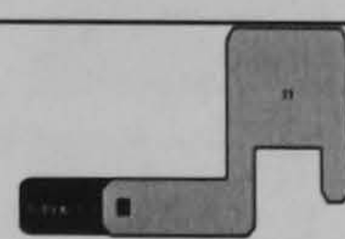
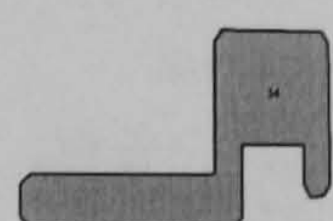
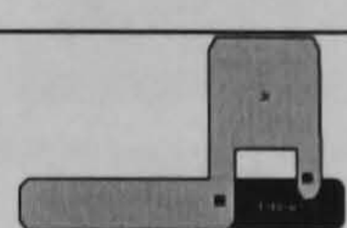
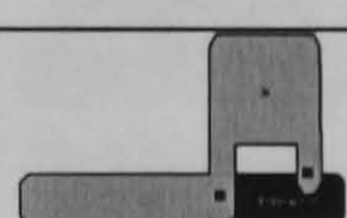
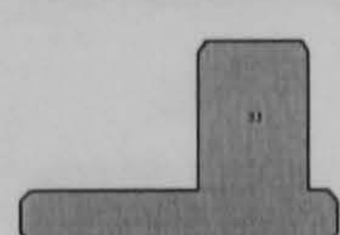
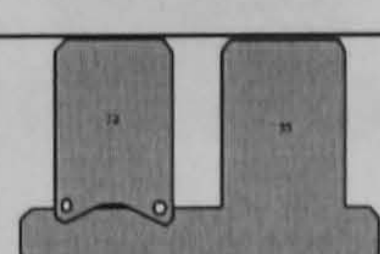
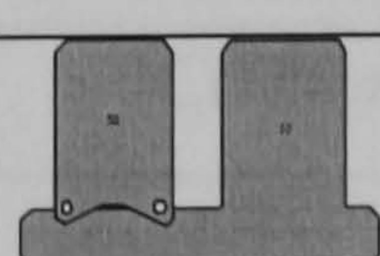
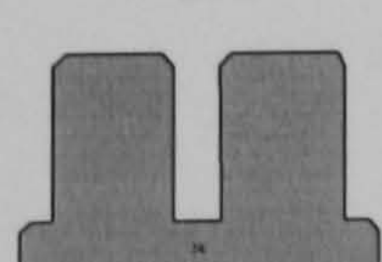
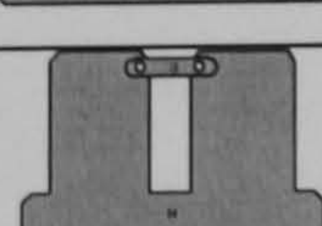
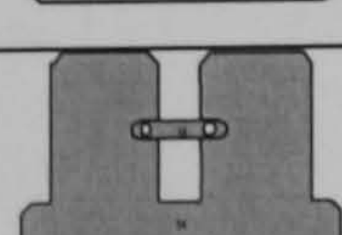
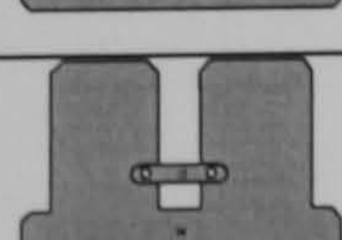
Step 16	12+49+50		**15.77×10^{10}**	**9.31**	**31**	**12.0**
	Forming cluster 52	 as 	*Selection Criteria*: Only choice.			
End of Secondary Clustering Stage						

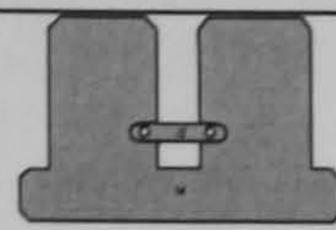
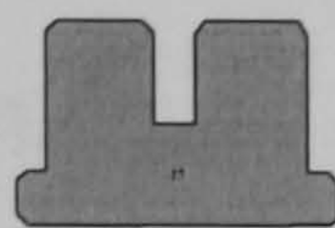
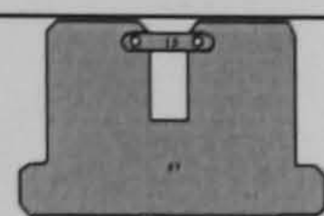
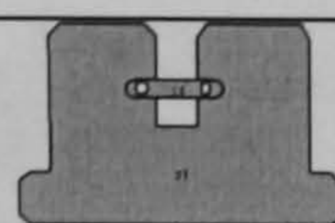
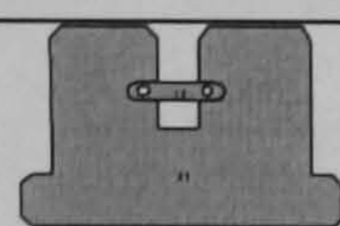
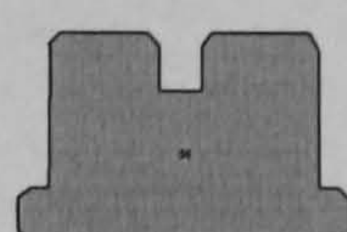
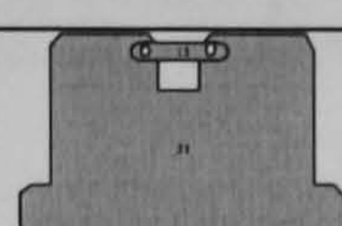
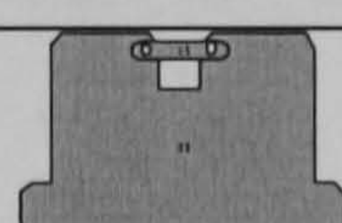
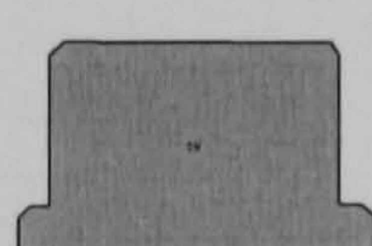
The Structure at the end of ***Secondary** Clustering Stage*:



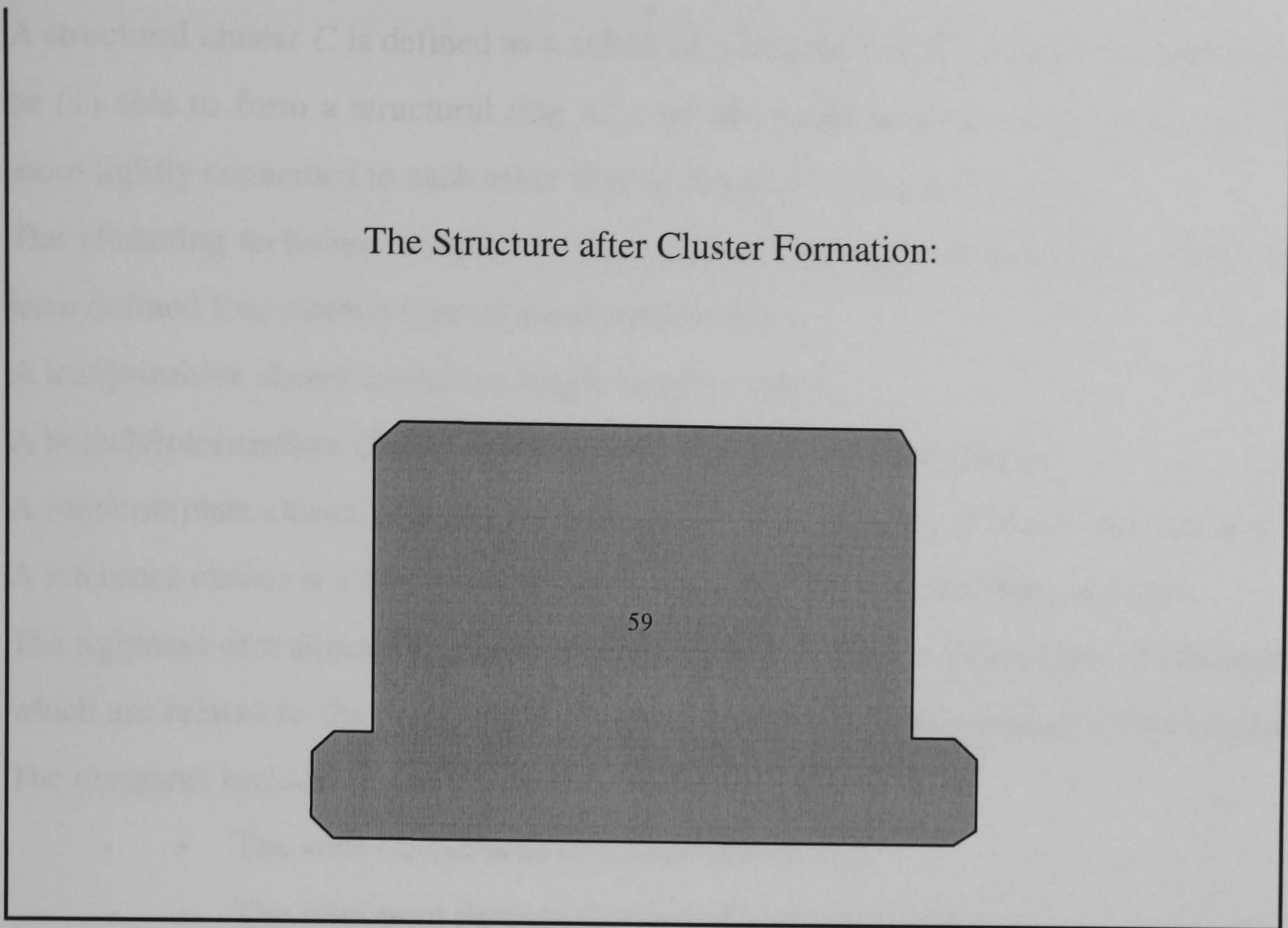
----- Reference Clustering Stage -----

Step 17	35+51		**51.13×10^{14}**	23.36	35	0.0

	Forming cluster 53	 as 	*Selection Criteria*: Maximum well-formedness.			
Step 18	34+53		**51.13×10^14**	23.36	35	0.0
	Forming cluster 54	 as 	*Selection Criteria*: Maximum well-formedness.			
Step 19	36+54		**51.13×10^14**	23.36	35	0.0
	Forming cluster 55	 as 	*Selection Criteria*: Maximum well-formedness.			
Step 20	52+55		**29.61×10^14**	9.31	66	0.0
	Forming cluster 56	 as 	*Selection Criteria*: Only choice.			
Step 21	15+56		29.6×10^14	32.68	66	9.0
	16+56		35.46×10^14	32.68	66	6.0
	17+56		**36.19×10^14**	32.68	66	3.0

	Forming cluster 57	 as 	*Selection Criteria*: Higher well-formedness.			
Step 22	15+57		39.2×10^{14}	43.51	66	6.0
	16+57		$\mathbf{42.05\times10^{14}}$	43.51	66	6.0
	Forming cluster 58	 as 	*Selection Criteria*: Higher well-formedness.			
Step 23	15+58		42.052×10^{14}	43.51	66	3.0
	Forming cluster 59	 as 	*Selection Criteria*: Only choice.			
Cluster formation complete						

The Structure after Cluster Formation:



4.7 Conclusions

In this chapter, cluster analysis has been introduced and different techniques which are commonly used were reviewed. Cluster analysis is a methodology of classification. Given a population of N objects, the problem is to find the scheme for grouping them into g classes according to the attribute of objects and the characteristics of the classes. There are many different clustering techniques in use and they can be classified into four main groups: hierarchical techniques, optimisation partitioning techniques, density or mode-seeking techniques and clumping techniques. The choice of clustering technique is dependent on the objectives of the analysis. However, the definition of a cluster is necessary and the clustering criteria must be clearly set no matter what technique is used.

A structural cluster C is defined as a subset of a structure S, the objects of which must be (1) able to form a structural ring or a set of overlapping structural rings, and (2) more tightly connected to each other than to those not inside the cluster.

The clustering technique adopted in this study is a hierarchical technique. Thus, we have defined four distinct type of structural clusters:

A leaf/primitive cluster contains a single member object,

A branch/intermediate cluster contains more than one member objects,

A root/complete cluster contains the entire set of objects in the structural system, and

A reference cluster is a specified cluster for the purpose of vulnerability analysis.

The tightness of a structural cluster can be evaluated using an ordered set of measures which are related to the form of the structure and the damage potential of the cluster. The measures include:

- The well-formedness of a structural cluster.
- The minimum damage demand of a structural cluster.
- The nodal connectivity of a structural cluster.
- The distance from the reference of a structural cluster.

The measures also set the clustering criterion which is to maximise each of the measures when selecting the best cluster. Based on the criteria and the concept of structural clusters, the cluster formation process will transform the set of objects in the structural system into a hierarchy of clusters, with the level of description ranging from the elementary level to the system's level. The cluster formation is a recursive process which starts with all primitive clusters (the initial clustering stage), then intermediate clusters (the secondary clustering stage) and finally the reference cluster (the reference clustering stage).

The full process of cluster formation has been illustrated step-by-step in an example.

Chapter 5

Hierarchical Representation of Structural Systems

5.1 Objectives

The objectives of this chapter are to:

- introduce systems concepts and methodology for modelling complex systems;
- discuss hierarchy as the underlying structure of complex systems;
- represent a structural system with a hierarchy;
- examine the process of hierarchy formation with an example.

5.2 Introduction

In this chapter, we will introduce the systems concepts for modelling complex systems. With the systems approach, the graph model of a structural system and well-formedness introduced in Chapter 3 and the clustering analysis in Chapter 4 are brought together, to form a hierarchical representation of a structural system. The aim is to generate a hierarchical model of the structural system which will lead to a vulnerability analysis of the structural system in future chapters.

5.3 Systems Approach

The systems approach holds a holistic view which is captured in the following general thesis:

A whole is more than the sum of its parts.

This is in contrast to the "traditional scientific approach" which is reductionist. The scientific approach involves selective inattention and seeks to explain behaviours and characteristics of the whole by examining its parts in isolation (Dias & Blockley, 1994).

Systems thinking emerged first from the modern science of biology. The development of biology and practical advances in the science of living things lead to realisation of emergence and hierarchy in living organism. It was the biologist L. von Bertalanffy who, in mid-1940s, first proposed a general theory of systems the central concept of which is the idea of wholeness. Since then, contributions to the development of systems methodology have come from many different fields, such as electrical communication and control engineering, cosmology and psychology, etc. (Checkland, 1984).

The systems paradigm is concerned with wholes and parts and interactions between them, their properties and their hierarchical arrangement. The characteristics and behaviours of a system are due to its nature of wholeness. A system consists of organised parts where the parts do not participate by means of simple aggregation.

Developments in systems thinking also lead to a recognition that the architecture of complexity is one of hierarchical organisation. The parts in a system can be considered at various hierarchical levels. At each level there may be emergent properties, properties which are only associated with a particular level and not valid for any other levels. For instance, pressure, as one of the properties of gas, is not valid at a molecular level, but a result from the interaction of the molecules.

5.4 Hierarchy

5.4.1 The structure of a complex system

Complexity can be modelled in the form of hierarchy (Simon 1965). Complete and detailed models cannot be made of truly complex systems. Since a model is not the reality, it lacks some characteristics of the reality and is therefore necessarily incomplete (Blockley, 1992b). Modelling is therefore about choosing the attributes and properties which are needed and appropriate for problem solving. The dilemma in modelling is between simplicity for understanding and the need to take into account the numerous behavioural aspects of a complex system. Hierarchical description is the resolution of the dilemma.

Hierarchy, as a method of conceptualisation, discriminates entities, relationships, processes, and levels as the ingredients of the structure of a complex system, whether natural, physical or abstract.

In his paper, Simon used the example of Hora and Tempus, the watchmakers, to demonstrate the advantages of modularization. Hora makes watches in modules while Tempus assembles watches element by element. Hora prospers but Tempus eventually goes out of business. The reason is because when disrupted, Tempus has to resume from scratch while Hora need not. Also, Hora's products are more resistant to damage and much easier to maintain. From this tale, Koestler even suggests that life is possible only if hierarchically organised (Koestler, 1967). Both Simon and Koestler argue that complex systems evolve far more rapidly when hierarchically organised.

Hierarchical structures are also important in representing knowledge at varying levels of detail which are appropriate to different problems. De Bono argues that hierarchies with continuing selective loss of detail up the hierarchy are vital for efficient reasoning (De Bono, 1971).

Hierarchy is also a powerful tool for explaining complex systems. Dawkins states that any complex system, whether biological or mechanical, may be described in terms of components in a hierarchy (Dawkins, 1986). When investigating the system, the objective is *"to explain the behaviour of a component at any given level, in terms of interactions between subcomponents whose own internal organisation for the moment, is taken for granted"*.

Complex systems can be structured hierarchically because they have some common properties that are independent of their specific content. A general characteristics of hierarchical structures is the repeated classification and neglect of detail at successive transitions to higher levels of the hierarchy (Comerford, 1989).

5.4.2 **Holons**

A hierarchical system is composed of interrelated subsystems, each of the subsystems being in turn hierarchical in structure until some the elementary or lowest level subsystem is reached. Each entity in a hierarchy can be considered as either a part or a whole.

The common terminology of "part" and "whole" become inappropriate when discussing property of an entity in a hierarchy. An entity is a part with respect to the entities above it in the hierarchy but a whole with respect to those below it. Koestler designated these dual-natured or "Jenus-faced" entities by the term *holon*. The word is from the Greek *holos*, meaning whole, plus the suffix *-on* as in proton or neutron suggesting a part.

A holon is both a whole and a part.

Each holon is complete in itself and consists of parts from the levels below in the hierarchy, yet is also a constituent of some entities of higher levels. Blockley states (Blockley, 1990):

if we think of all concepts as holons in a hierarchically structured knowledge base then by looking upwards towards the infinite vague unity of the universe any concept (holon) is a part, and looking downwards, to the precise infinitesimal of the universe any concept (holon) is a whole.

The significance of the term holon lies beyond the representation of a class of particular entities. Thinking about holons is a view of the world.

5.4.3 **Levels**

The concept of a hierarchy is that of a structure with multiple levels; each level is a model of the system made of the interacting parts at that level. Each part at a level is made of parts at a lower level, thus each part in a hierarchy is a holon. Hierarchical structures are a type of multi-level representation or description in the form of a tree or graph (Frost, 1986). A hierarchy can describe a system by a set of models each of which is concerned with the behaviour of the system as viewed from a different level of abstraction. For each of these levels, there is a set of relevant features and principles in terms of which the system's behaviour is described.

How a system can be described by a hierarchy of models is illustrated by the example of a system which produces a spoken literary composition (Mesarovic and Macko, 1969). The system produces the literary text. It can be described from at least four different levels:

Level 1: The system generates letters as a sound-making machine,

Level 2: The system generates letters in a way that are acceptable as words in a given language as a word-producing machine, Level 3: The system generates words and constructs them according to the given syntax and semantic rules to form sentences to express certain statements,

Level 4: The system generates sentences that the quality and literary value of its composition is assessed by some literary aesthetic standards.

A system may be considered as a structure of many levels of organisation. There are attributes and types of behaviours at each level which may not exist at any other levels. Specific problems and phenomena *emerge* at each level of description. These are peculiar to that level. This enables multiple descriptions, models or theories of a system with each level having its own concepts and language. Hence the system and its behaviours may be understood and explained in many different ways all equally valid but of different levels of detail or complexity.

The description at different levels in a system is necessary because a certain level of description may fail to yield adequate explanations. Then it becomes important to design descriptions at a deeper level. It is important to choose an appropriate level of description in an investigation of a problem. The appropriateness of definition ensures that a level and the models being adequate for the problem requirements.

The multi-leveled description of a hierarchy allows complex systems to be analysed from simpler subsystems (bottom-up). In turn, it also allows complex systems to be broken up into their component parts (top-down). The levels in a hierarchy must be determined by the nature of the relations between the holons which constitute it in the way that useful and dependable explanations of the problem can be provided.

5.5 Hierarchical Representation of a Structural System

In Chapter 3, a graph model of structural systems was as an interconnected set of member and joint objects. The member and joint objects must be connected in the pattern as a structural ring in order to construct a valid structural system.

In Chapter 4, a clustering technique was applied to identify structural clusters C_i^l in the structural system at various levels of description (l).

The application of clustering analysis and introduction of the concept of structural clusters are the basis of a hierarchical representation of a structural system.

5.5.1 Internal and external connectivity of structural clusters

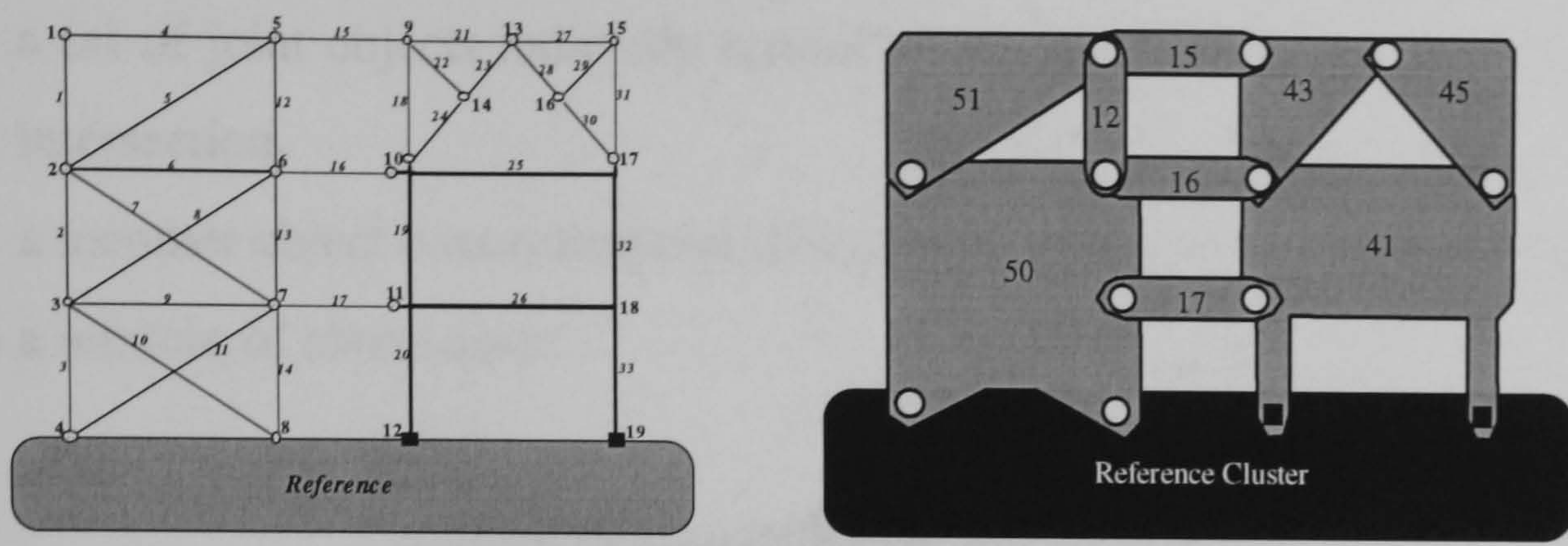
In Chapter 4, a structural cluster C^l, at a level of description l, is defined as a *subset* of the structural system S in which the objects are *more strongly connected* to each other inside the subset than to those outside the subset.

Internal to a structural cluster C^l, the member and joint objects must satisfy two requirements:

- they are connected to form a set of structural rings,
- their construction results in an increased value of structural well-formedness.

The above requirements ensure that in the process of cluster formation the internal form and connectivity of a structural cluster is maximised at any level of description.

A structural system may be represented, at certain level of description, as a set of structural clusters, some or each of them consists of primitive clusters which give them the maximum internal connectivity. One example is the structure in example 4.1 at the end of initial clustering stage: (see page 4-26)



original *at the end of initial clustering*

Figure 5.1 A structural system represented at two different levels of description

Apparently, at this higher level of description, the model of the structural system is much simpler compared with the original one. At this point, the internal connectivity of each of the structural clusters are stronger than the external connectivity between them. Because each cluster is treated as a whole at that level of description, the detail of internal structure of each of the clusters can be ignored or taken for granted.

5.5.2 Complex joints

Wu has defined the *complex joint* to distinguish between the joint objects as a constituent in the structural system and the joint between structural clusters at various levels of description. (Wu, 1991)

A *complex joint* is defined as the intersection of any two connected structural clusters:

$$j^l = C_i^l \cap C_j^l \qquad (5.1)$$

Two structural clusters are said to be connected if they share at least a single joint object.

Therefore, a complex joint may be either

(a) a single joint object ,

(b) a set of joint objects indirectly connected through the clusters which form the intersection,

(c) a member object connecting two shared joint objects by both of the clusters,

(d) a mixture of above cases.

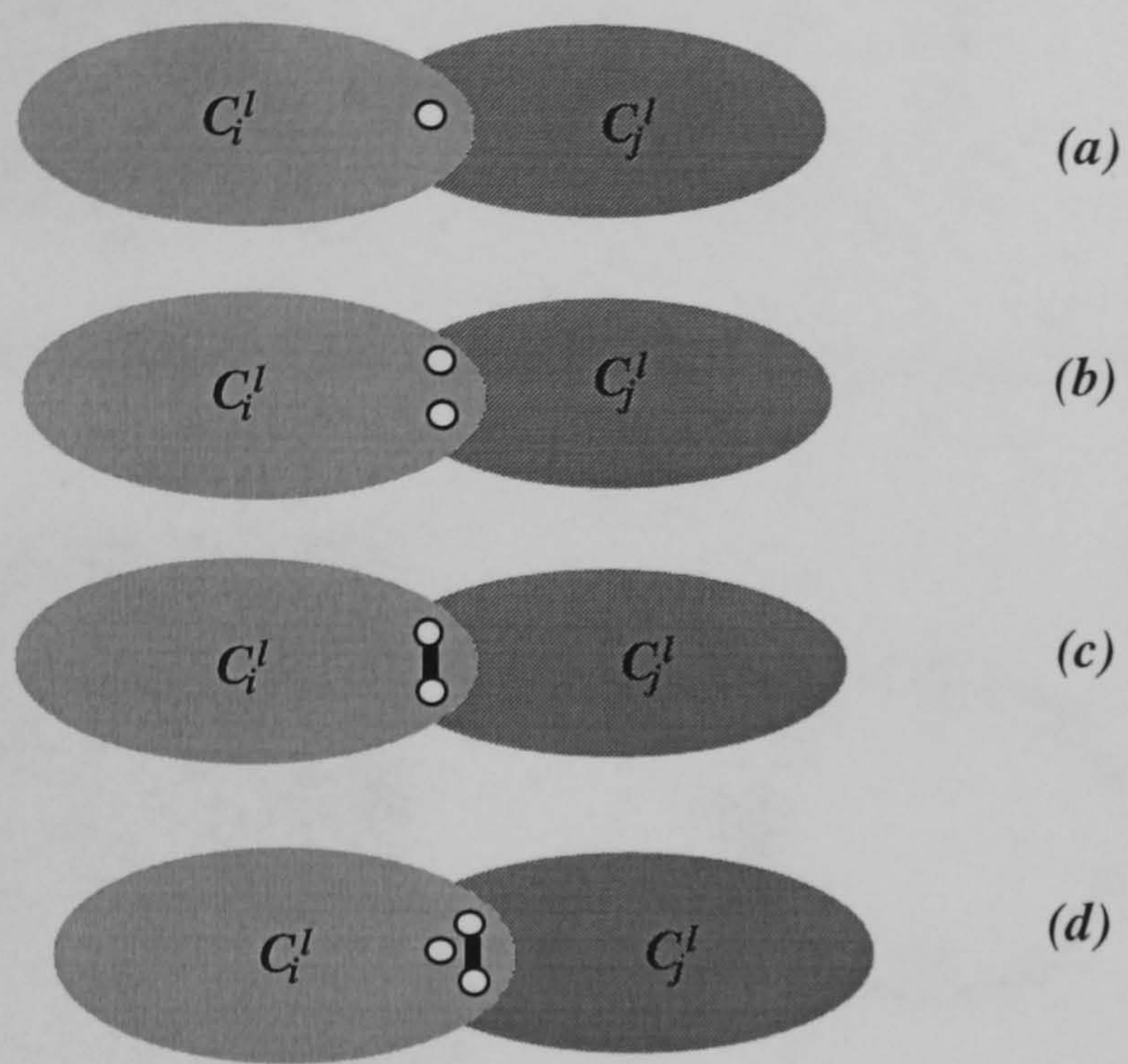


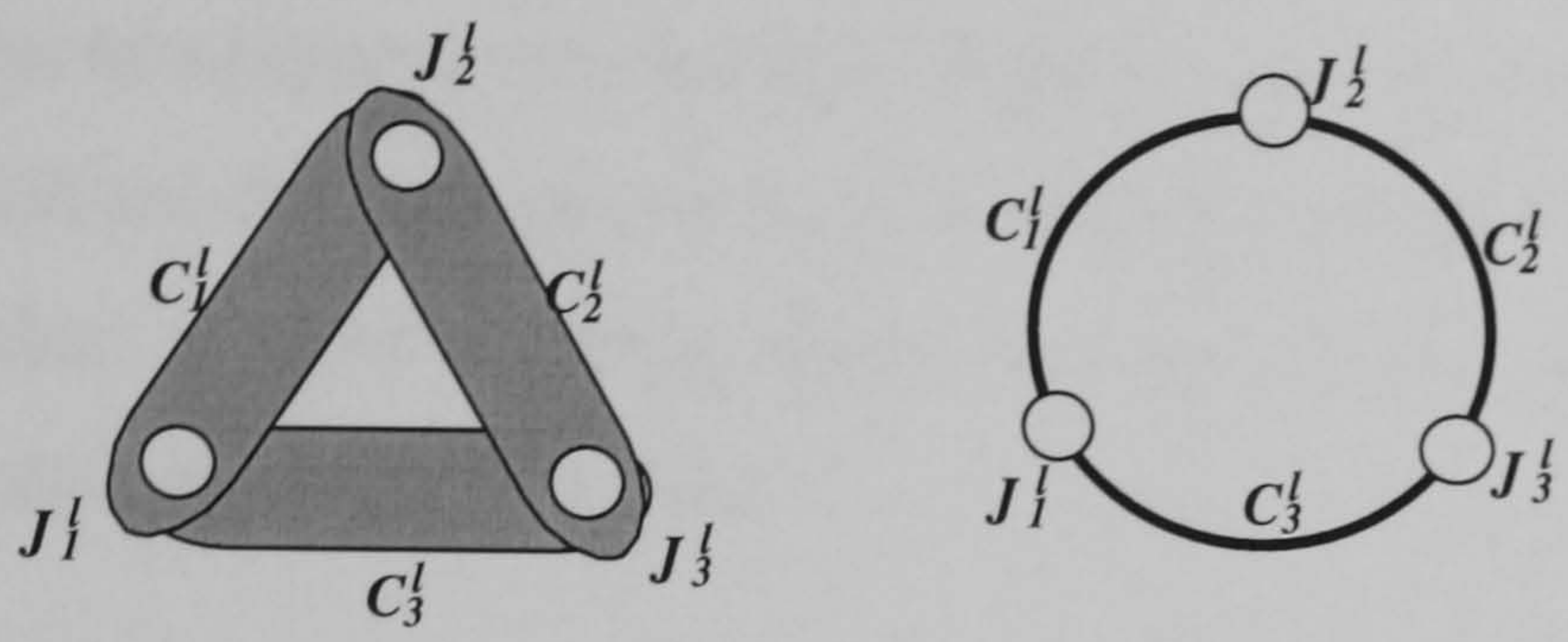
Figure 5.2 Complex joints

5.5.3 Forming a structural ring at a higher level of description

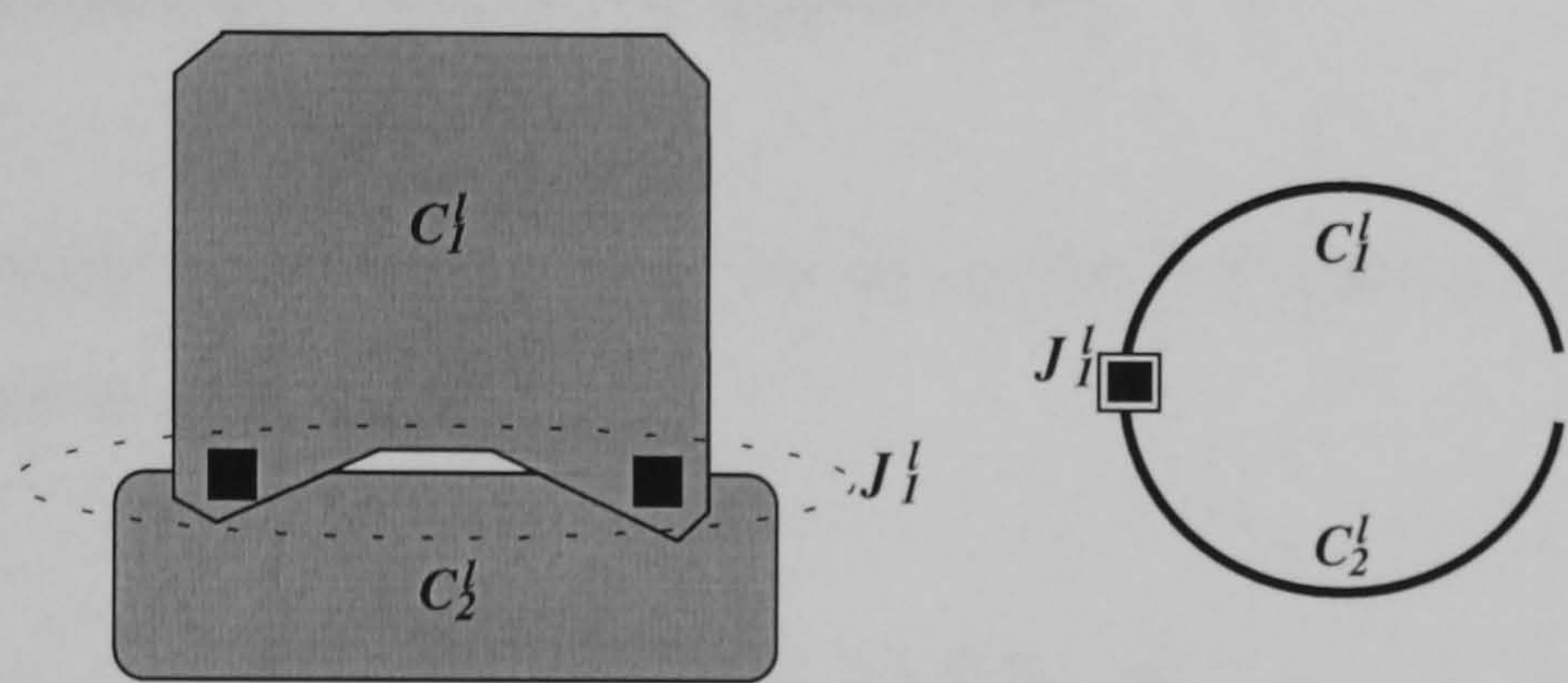
At a higher level of description, the graph model of the structural system is transformed into a set of interconnected structural clusters and complex joints. In this simplified graph, there exist many structural paths and loops just as in the original graph.

A structural ring at a higher level of description in the graph model is a set of alternating structural clusters and complex joints which, as a whole, possesses a sufficient number of degrees of freedom to maintain equilibrium.

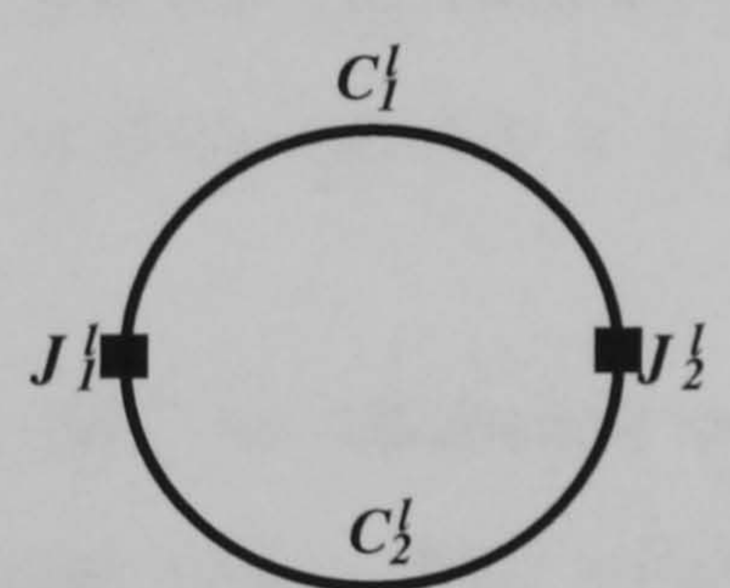
As discussed in Chapter 3, (section 3.4.2) a structural ring at a higher level of description can also be any of two types: 2-link-ring or 3-link-ring. Depending on the number of DOF of the structural clusters and complex joints, a higher level structural ring can be just-stiff or over-stiff.



(a) A set of structural clusters and complex joints form a just-stiff 3-link-ring



(b) Two clusters and a complex joint form an over-stiff 2-link-ring, or



(c) Alternative representation for case (b).

A structural ring at a higher level of description can be presented using its string pattern. The redundancy of a structural ring at a higher level of description can be computed using equation (3.3).

As for the lower level structural rings, with its string pattern and redundancy, the deterioration process and failure of a higher level structural ring can be traced by pattern matching using the DHSR, the deterioration hierarchy of structural rings.

It should be noted that in the process of deterioration of a structural ring at a higher level of description, loss of a DOF may have a different meaning comparing to those structural rings at lower levels of description. To achieve the loss of a single DOF in a structural cluster in a higher level ring may require loss of more than one DOF in lower level rings which are the constituents of that particular structural cluster. This will be illustrated further in later chapters when the process of identification of failure scenarios and algorithms are discussed.

5.5.4 The structural system as a hierarchy

Based on the systems concepts discussed in section 5.3 and 5.4, a structural system can be presented as a hierarchy:

$$S = \{S_i^l \mid l = 1,2,\ldots h;\ i = 1,2,\ldots n\} \qquad (5.2)$$

where:

h is the total number of levels, and

n is the number of sub-structure/structural clusters at level l.

A structural system can be described at various levels of detail as a set of interconnected structural rings. The structural characteristics and functions of the structural system at each level of description is peculiar to that level.

The complexity of the structural system becomes more manageable with the hierarchical representation. In a large structural system, there are often large numbers of member and joint objects and the complicated interconnection to be dealt with. By classifying them into a set of structural clusters with which the general attributes and feature of the structural objects are indicated, the amount of detailed information is reduced and the organisation of the structural system can be revealed. At each higher level of description, the detail of the information about each of the structural objects is hidden inside the structural clusters and becomes irrelevant to the higher levels. As the level ascending, the number of structural objects in the system become smaller and the interaction between them become easier to study.

Each sub-structure/structural cluster S_i^l is a holon. It is a part with respect to the cluster at next higher level, and a whole with respect to the clusters which are its constituents. At the lowest level of description in the hierarchy, where further detailed behavioural and structural aspects are thought to be irrelevant to the purpose of the model, the structural system consists of its basic component objects, as in structural members and joints. At the highest level of description in the hierarchy, where the involvement of any other meta-system is outside the concern of the model for the time being, all components in the structural system form a complex structural cluster as a single object.

Generally speaking, a structural system can be represented by a set of sub-structures/structural clusters at successively subordinate levels of description in the form of a hierarchy. Structural clusters at lower levels in the hierarchy have more detailed information of the structure than those at higher levels of description.

5.6 Hierarchy Formation

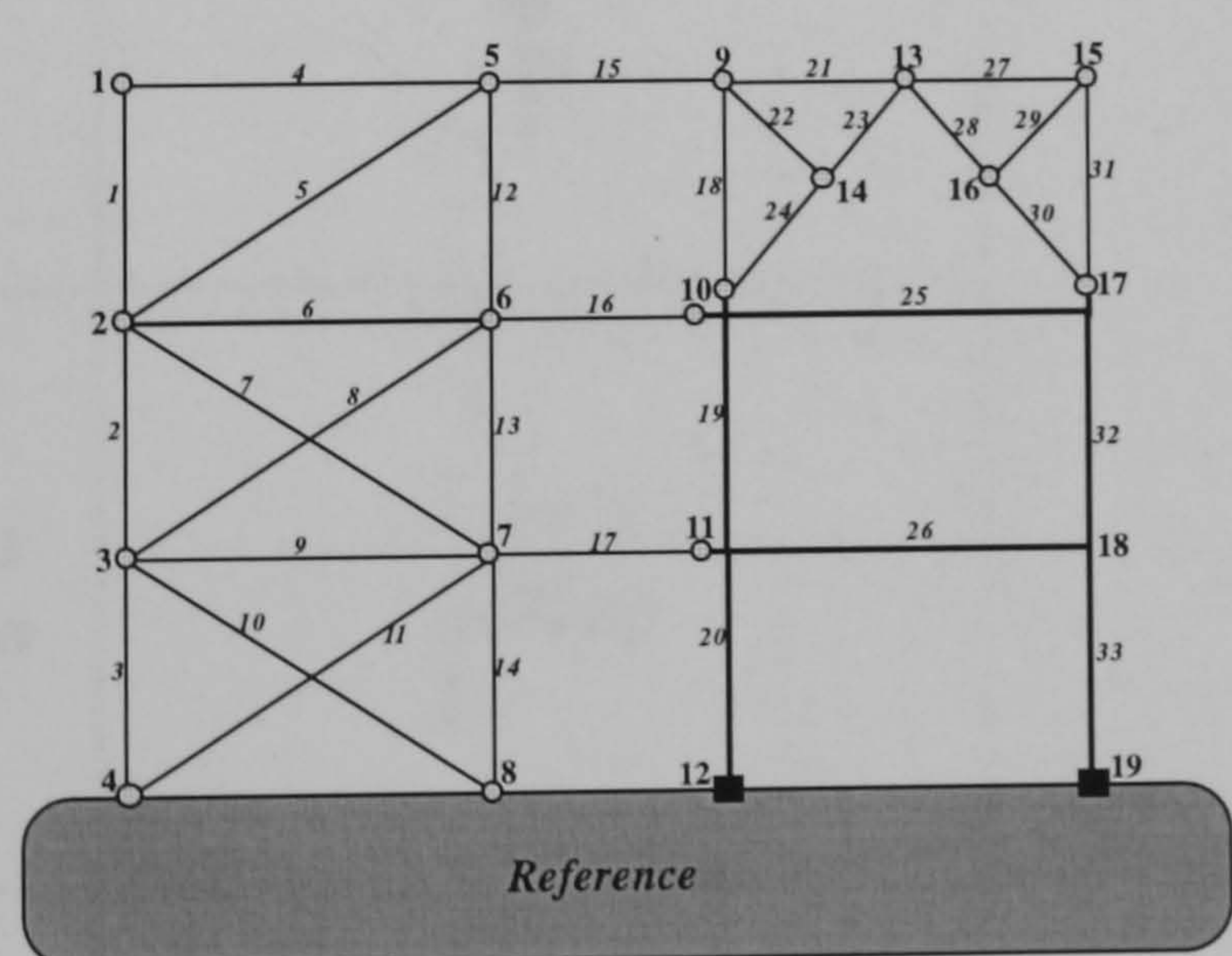
In Chapter 4, the principles and criteria for cluster formation have been introduced. The technique for cluster formation used in Chapter 4 is hierarchical clustering. Therefore, the hierarchy formation process is based on cluster formation.

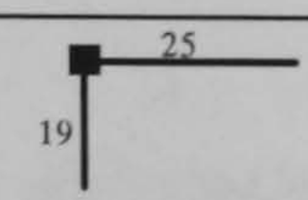
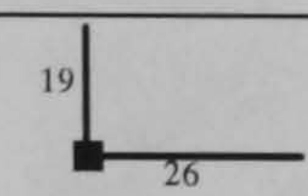
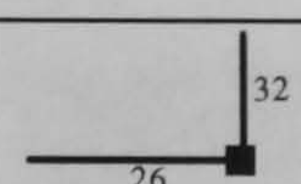
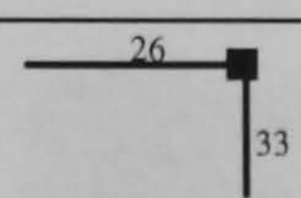
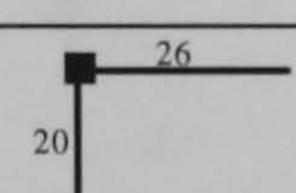
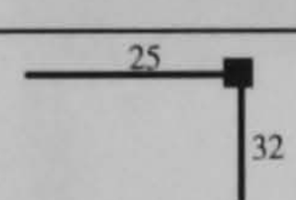
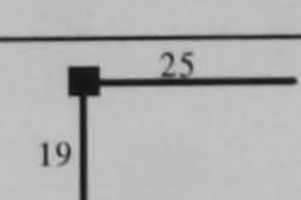
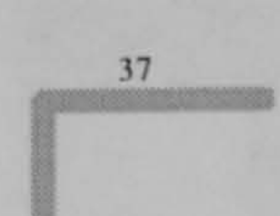
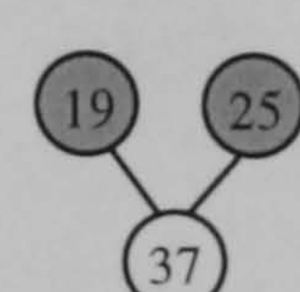
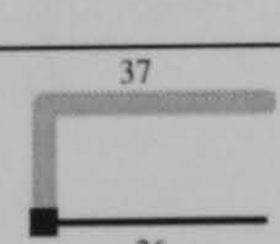
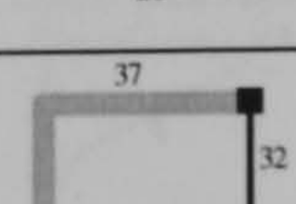
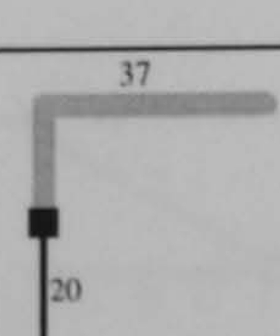
The process of hierarchy formation begin from the lowest level of description. At this level, all structural clusters are primitive clusters. Structural clusters with the best form according to the clustering criteria will be generated first and arranged in the hierarchy at next higher level of description. The process repeats until the structural system becomes a single structural cluster, i.e. the top level of the hierarchy is reached.

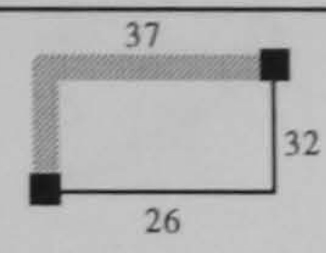
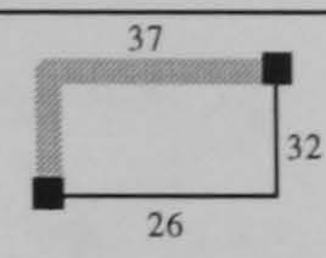
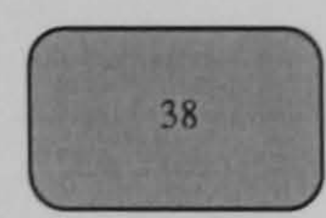
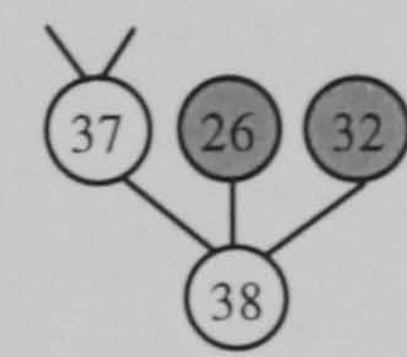
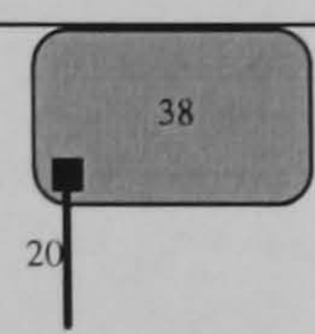
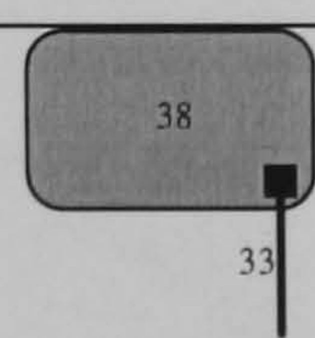
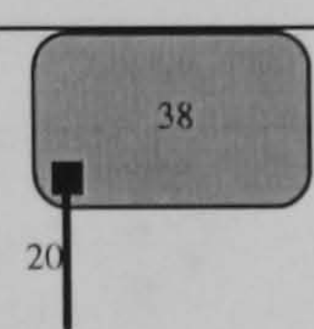
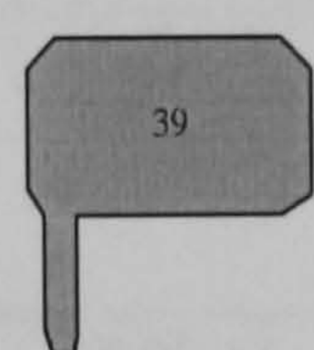
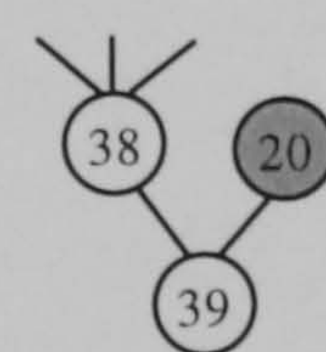
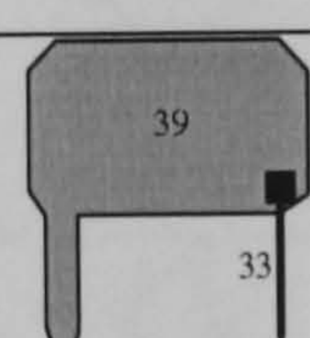
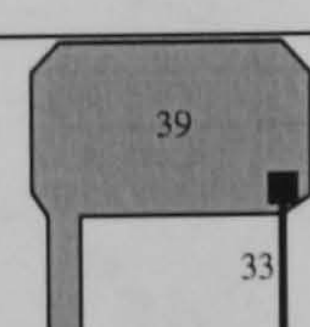
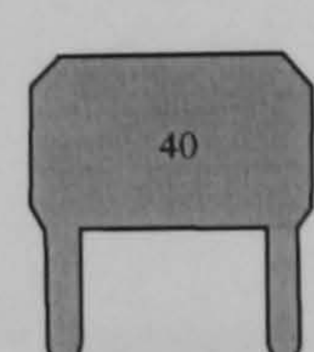
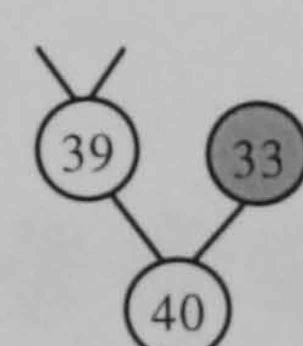
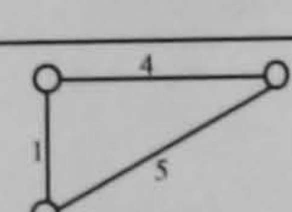
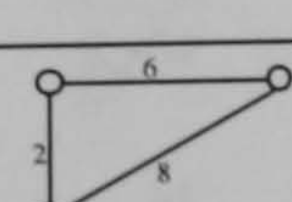
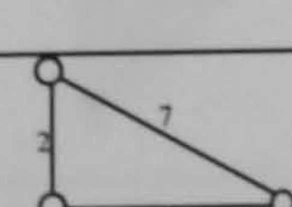
As discussed in section 4.5.2, there may be up to three major stages in the process of hierarchy formation: the initial, the secondary and the reference clustering stages. These stages are the levels in the hierarchy with distinct emergent properties. Between these stages, there is adjustment of clustering criteria corresponding to the emergent properties at that major level. The steps before reaching such stages are also levels of description, with different levels of detail. However, the clustering criteria for such a level is in consistent with its previous levels.

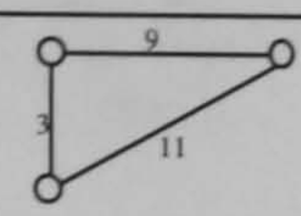
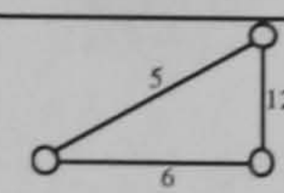
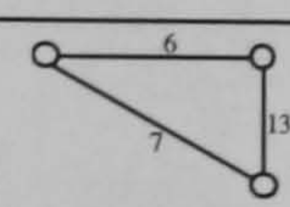
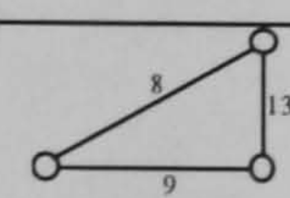
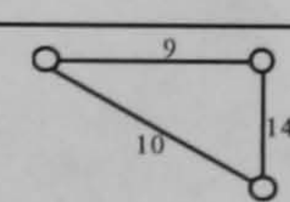
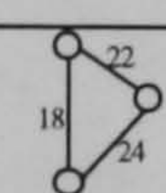
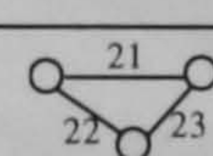
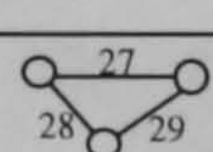
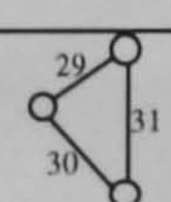
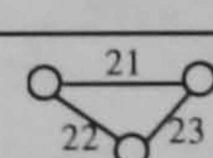
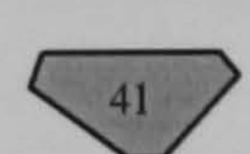
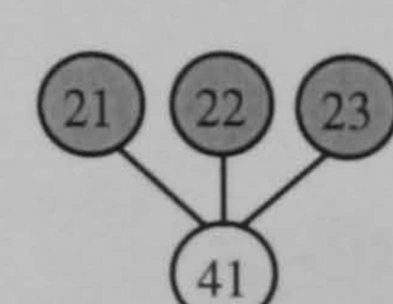
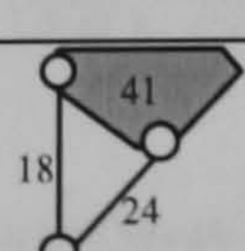
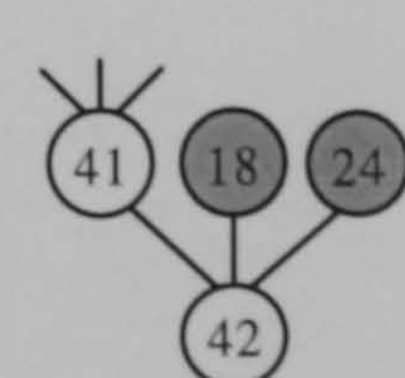
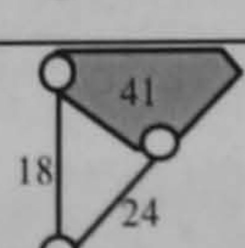
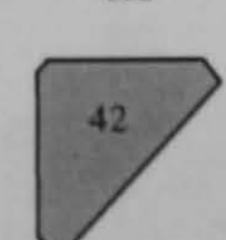
The example in Chapter 4 (see Section 4.6) will be used again to demonstrate the hierarchy formation.

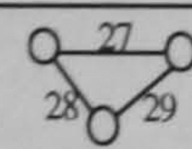
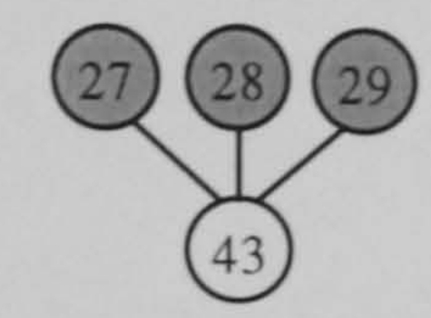
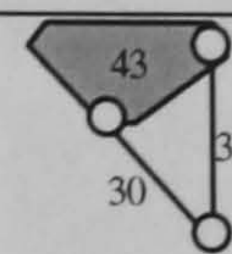
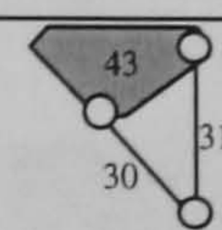
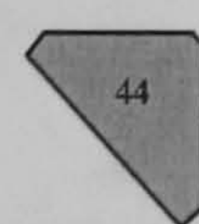
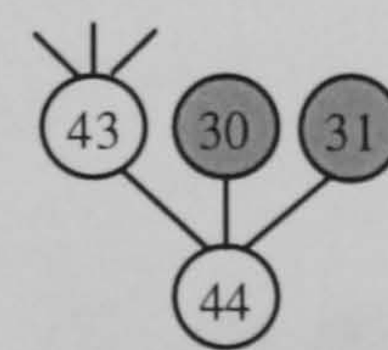
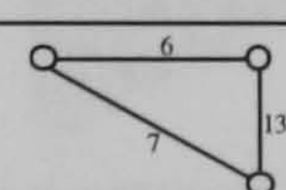
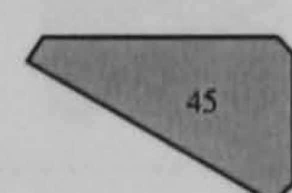
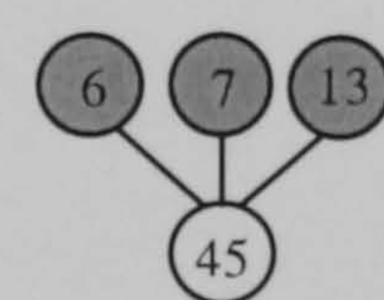
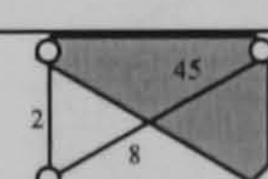
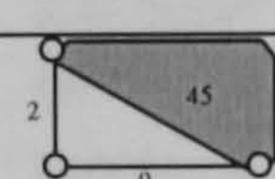
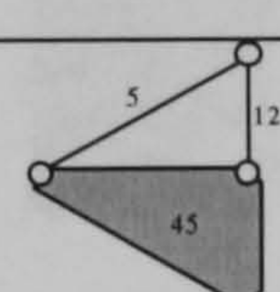
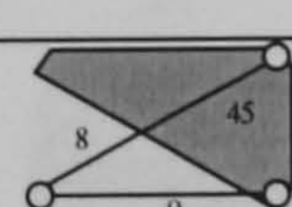
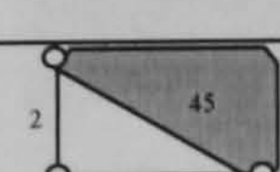
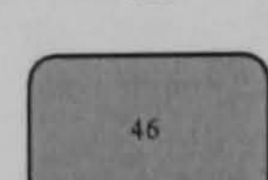
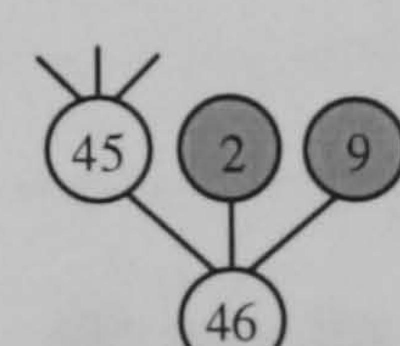
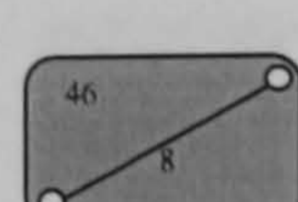
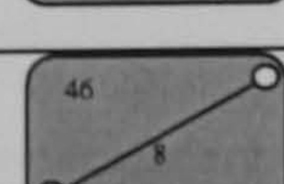
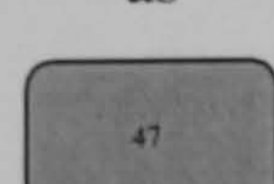
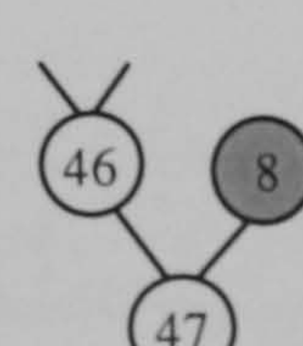
<u>Step-by-step Hierarchy Formation</u>

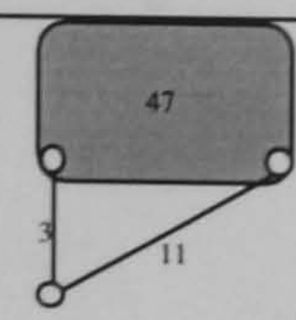
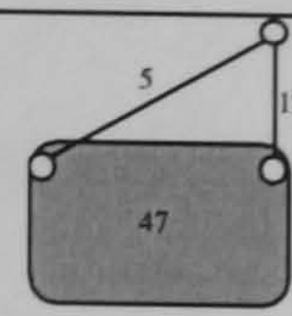
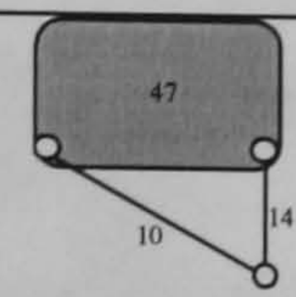
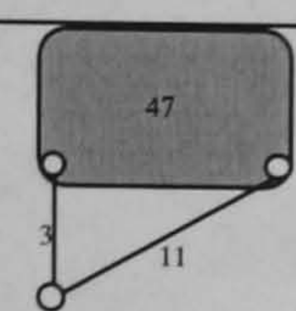
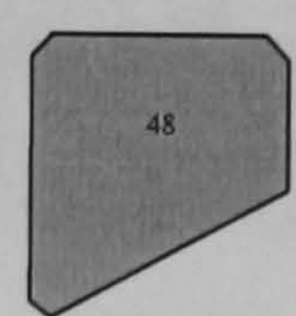
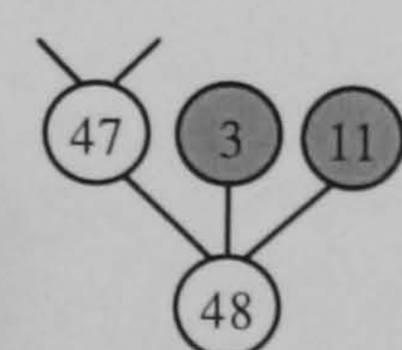
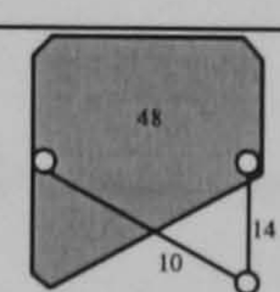
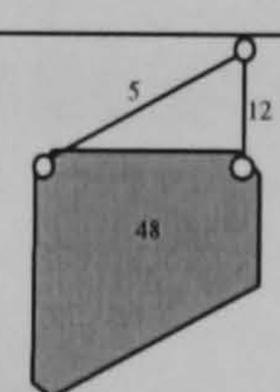
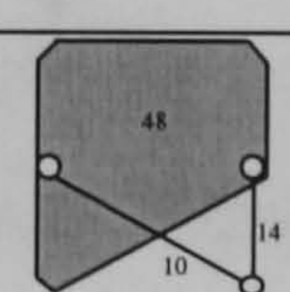
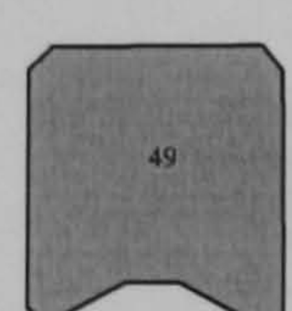
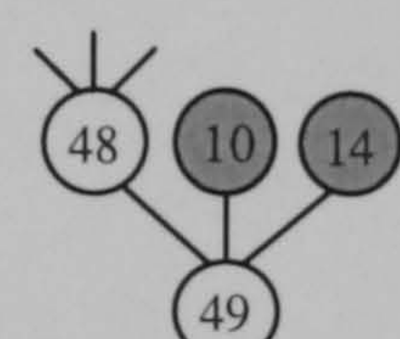
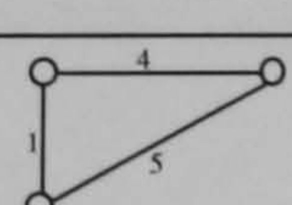
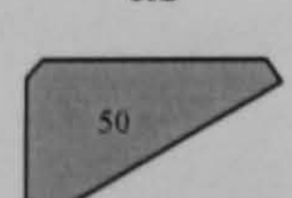
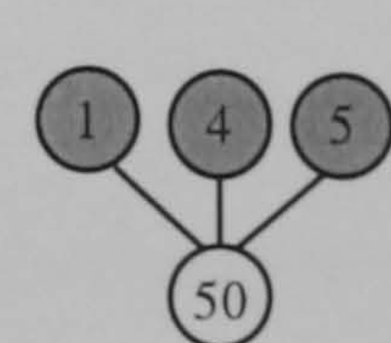
<table>
<tr><th>Levels</th><th>Component
s</th><th>Cluster formed</th><th>Hierarchy</th></tr>
<tr><td colspan="4"><u>The structure:</u>

</td></tr>
<tr><td colspan="4">----- Initial Clustering Stage -----</td></tr>
</table>

Level 1	19+20		
	19+25		
	19+26		
	26+32		
	26+33		
	20+26		
	25+32		
	32+33		
	Forming cluster 37	 as 	
Level 2	26+37		
	32+37		
	20+37		

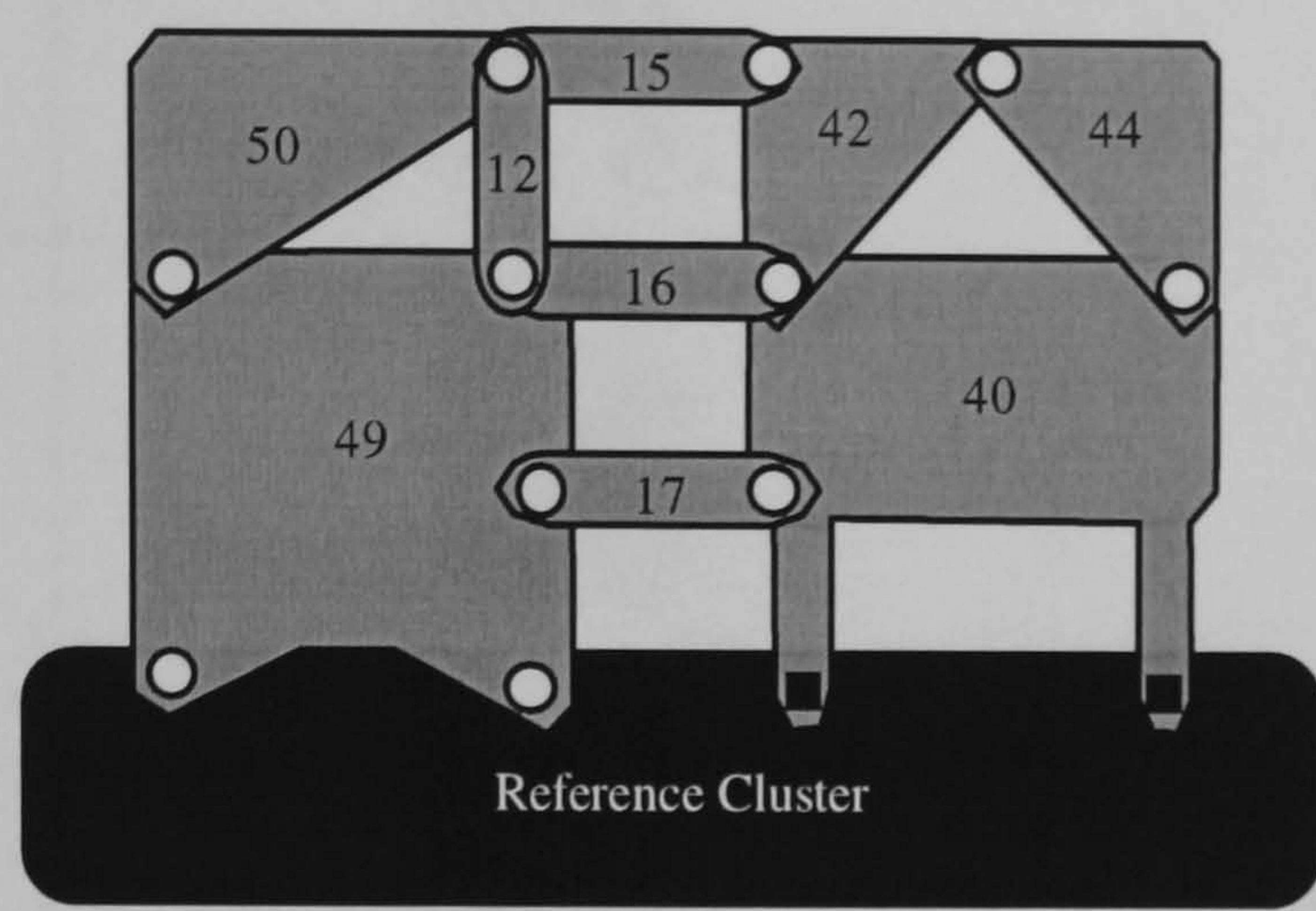
	20+26+37		
	Forming cluster 38	 as 	
Level 3	20+38		
	33+38		
	Forming cluster 39	 as 	
Level 4	33+39		
	Forming cluster 40	 as 	
	1+4+5 *		
	2+6+8 *		
	2+7+9 *		

Level 5	3+9+11 *		
	5+6+12 *		
	6+7+13 *		
	8+9+13 *		
	9+10+14 *		
	18+22+24 *		
	21+22+23 *		
	27+28+29 *		
	29+30+31 *		
	Forming cluster 41	 as 	
Level 6	18+24+41		
	Forming cluster 42	 as 	
Level 7	*	See Level 5	

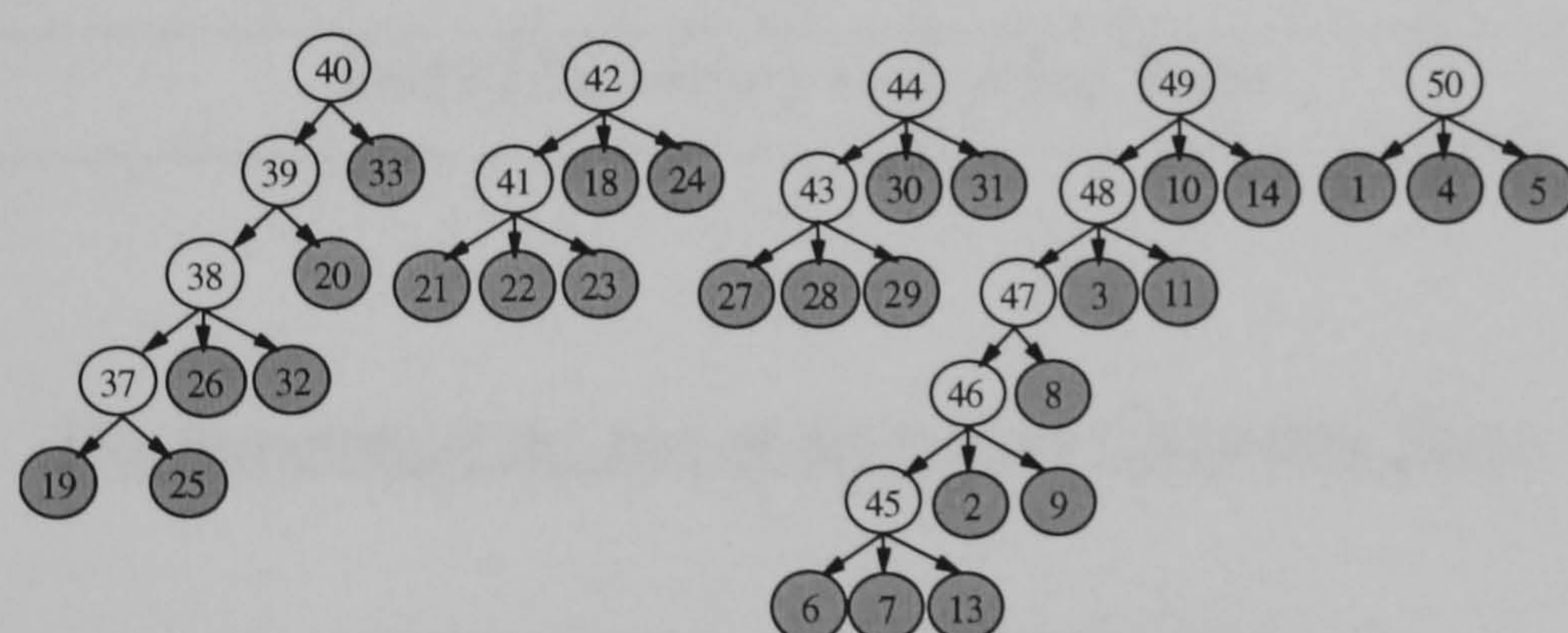
	Forming cluster 43	 as 	
Level 8	30+31+43		
	Forming cluster 44	 as 	
Level 9	*	See Level 5	
	Forming cluster 45	 as 	
Level 10	2+8+45		
	2+9+45		
	5+12+45		
	8+9+45		
	Forming cluster 46	 as 	
Level 11	8+46		
	Forming cluster 47	 as 	

Level 12	3+11+47		
	5+12+47		
	10+14+47		
	Forming cluster 48	 as 	
Level 13	10+14+48		
	5+12+48		
	Forming cluster 49	 as 	
Level 14	*	see step 5	
	Forming cluster 50	 as 	
End of Initial Clustering Stage			

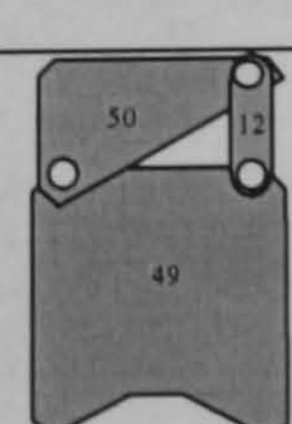
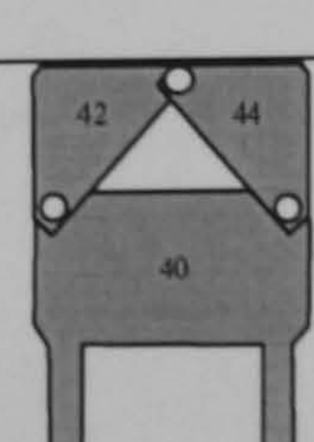
The structure at the end of ***Initial*** *Clustering Stage*:

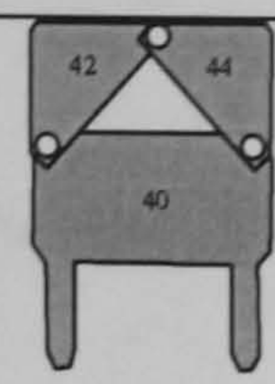
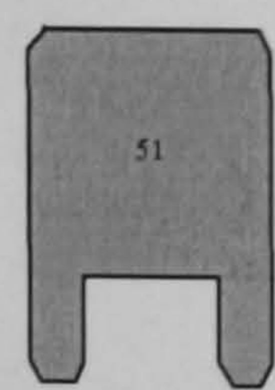
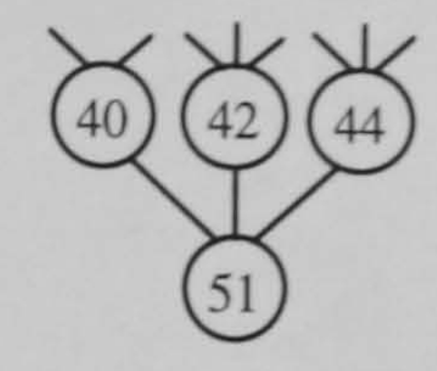
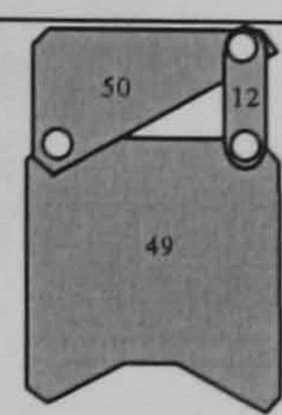
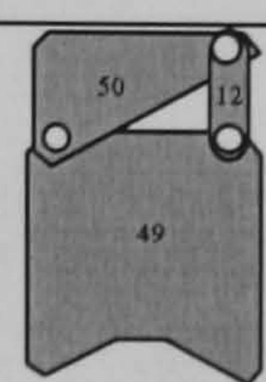
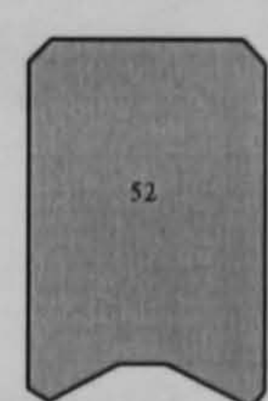
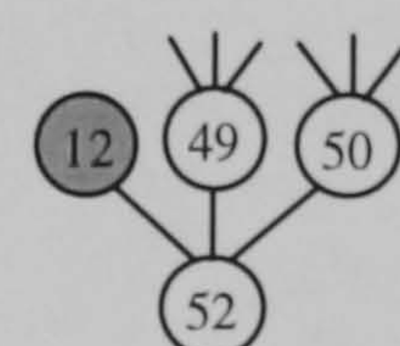


The hierarchy at the end of initial clustering stage:

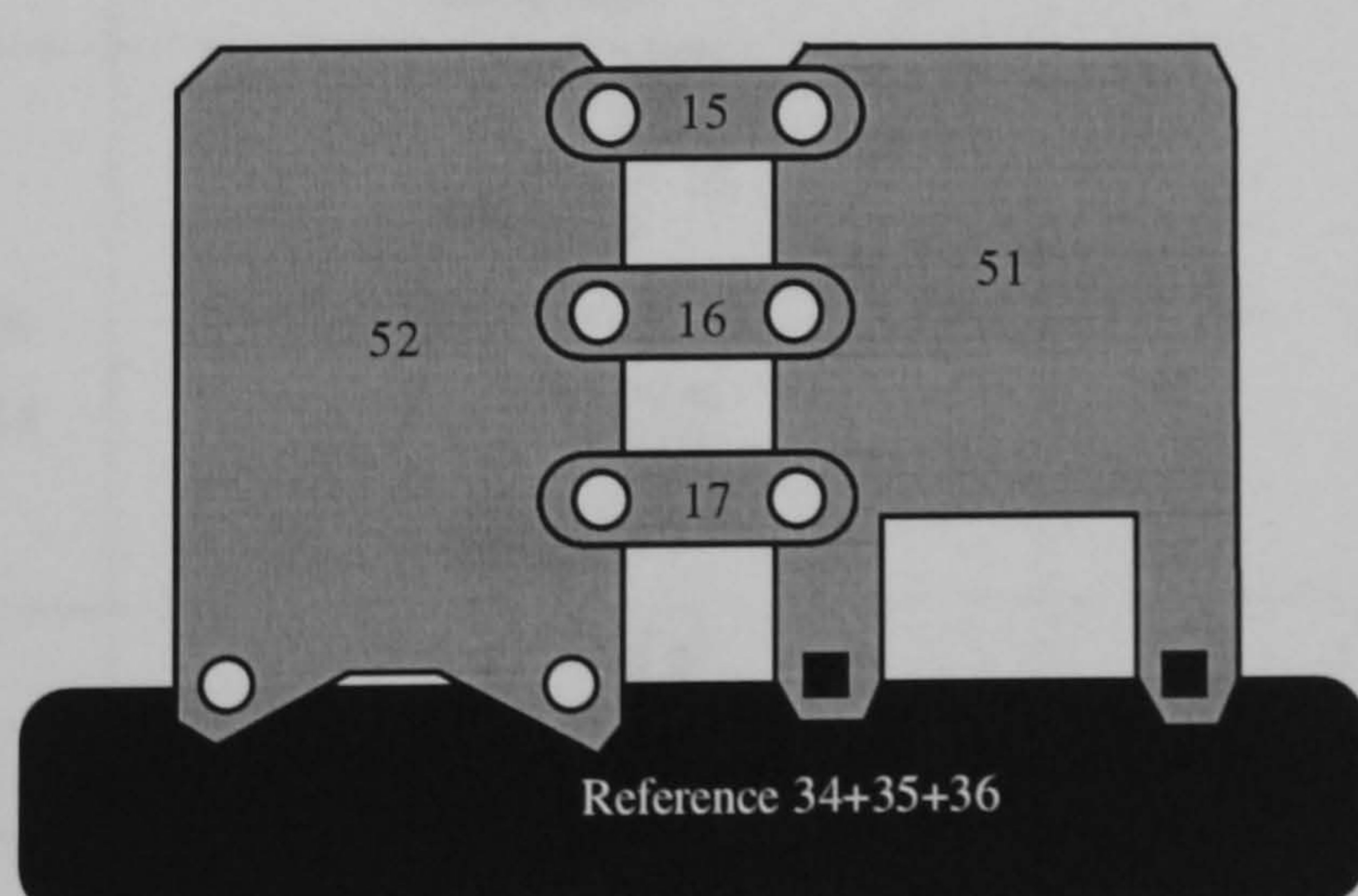


----- Secondary Clustering Stage -----

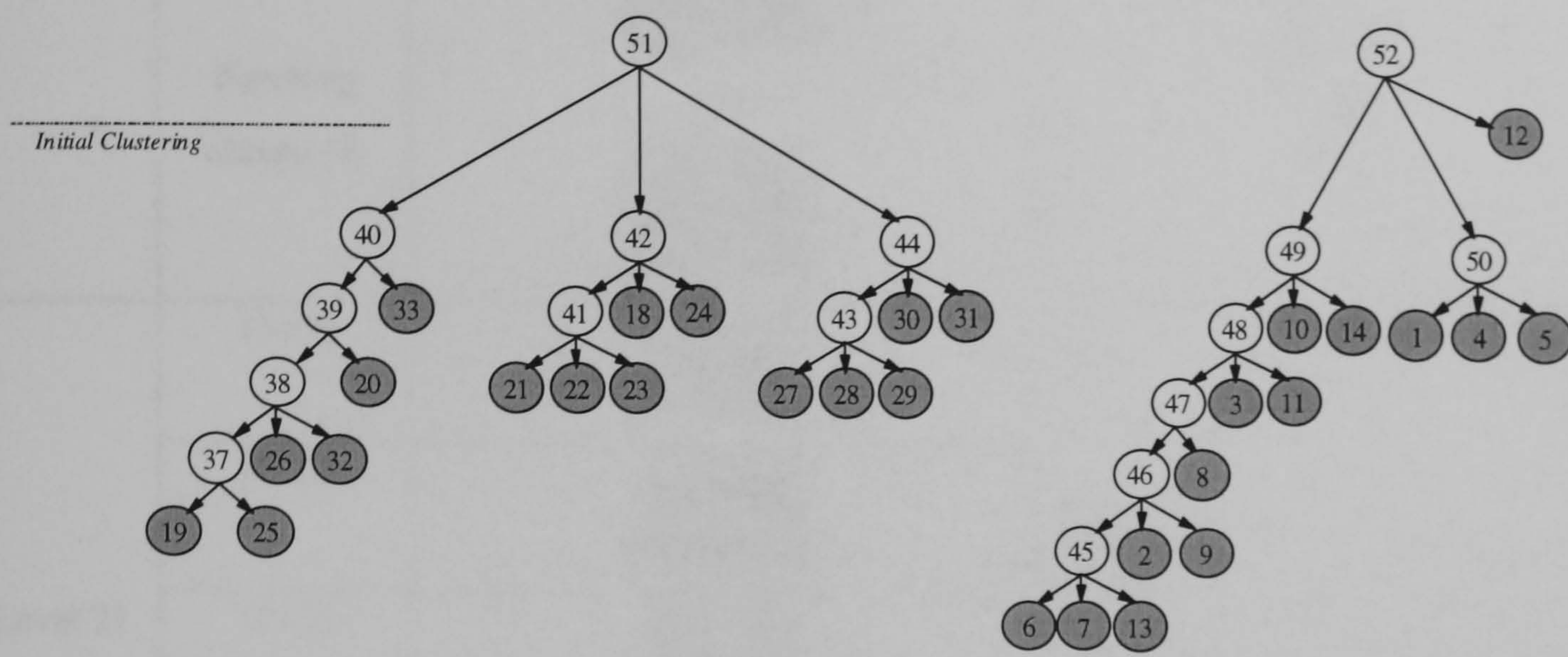
Level 15	12+49+50		
	40+42+44		

	Forming cluster 51	 as 	
Level 16	12+49+50		
	Forming cluster 52	 as 	
End of Secondary Clustering Stage			

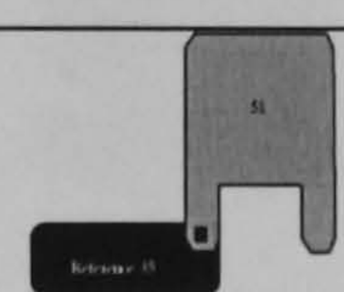
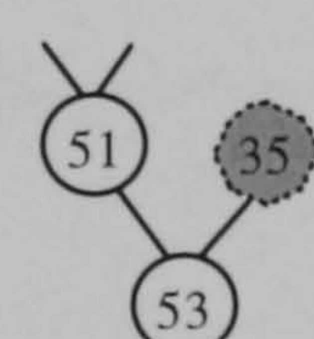
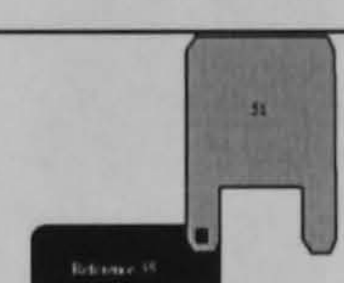
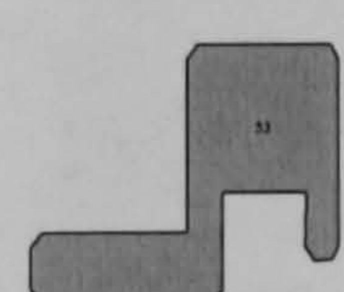
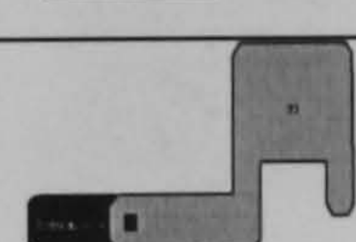
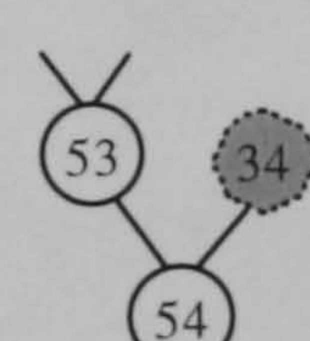
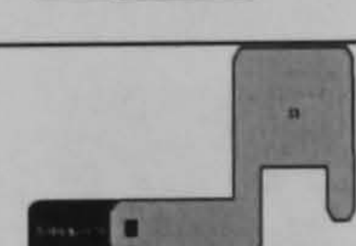
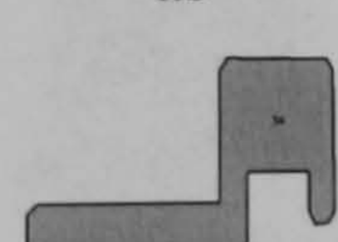
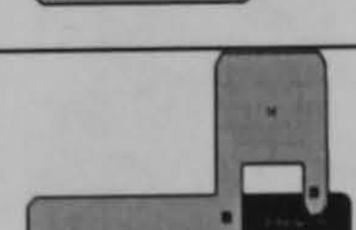
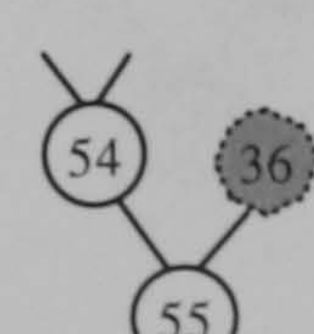
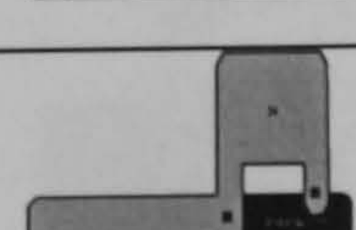
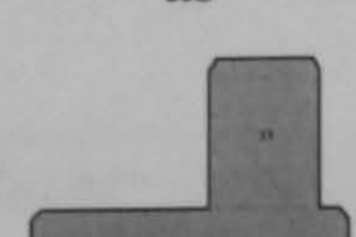
<u>The Structure at the end of ***Secondary** Clustering Stage*:</u>

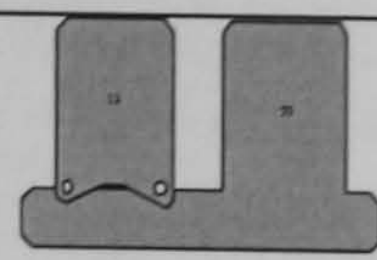
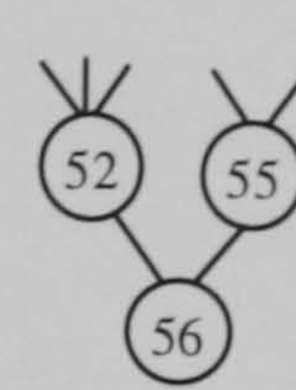
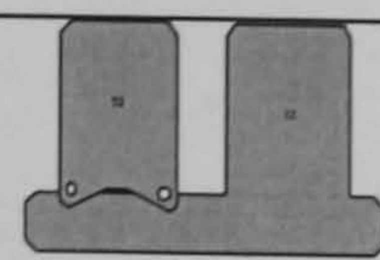
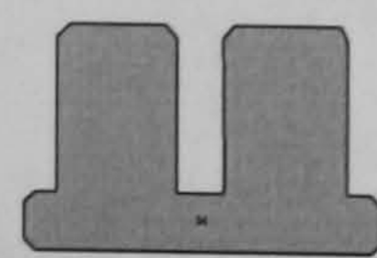
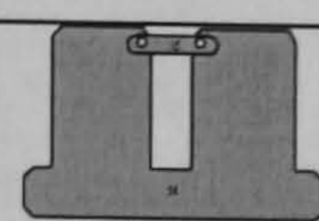
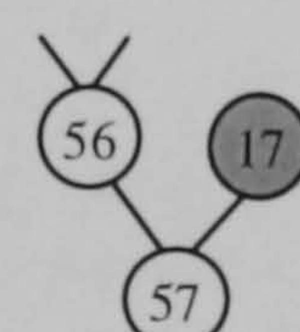
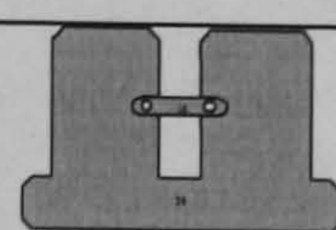
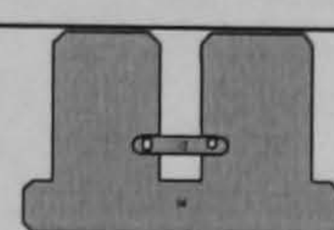
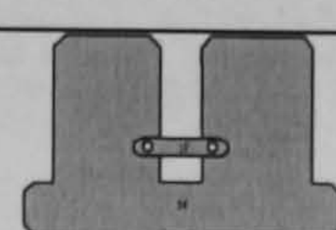
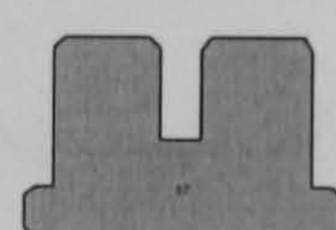
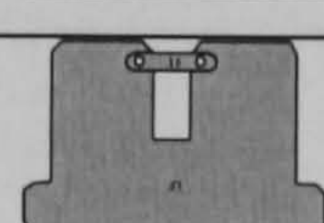
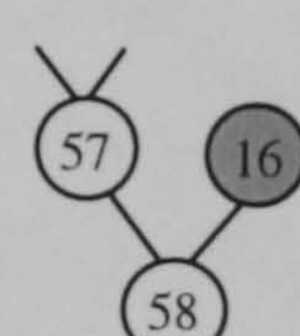
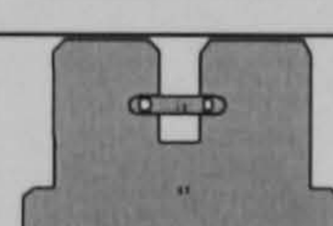
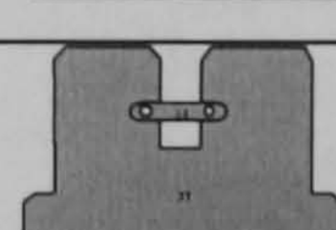
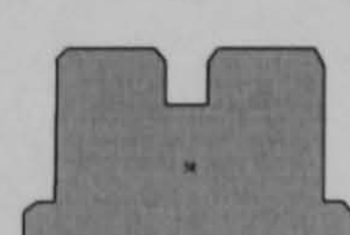
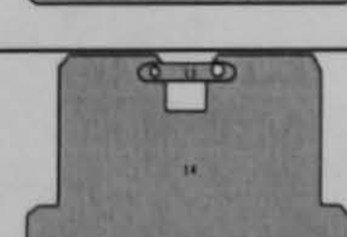
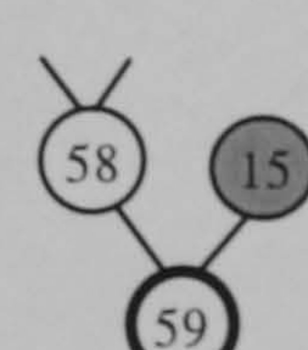
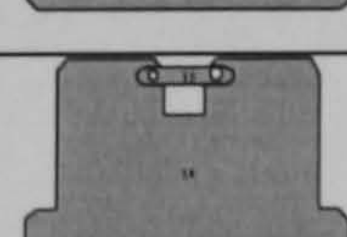
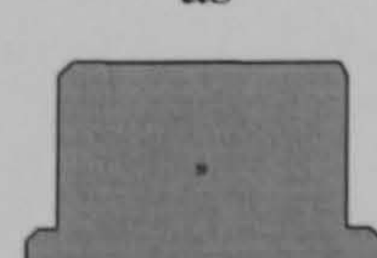


The hierarchy at the end of secondary clustering stage:

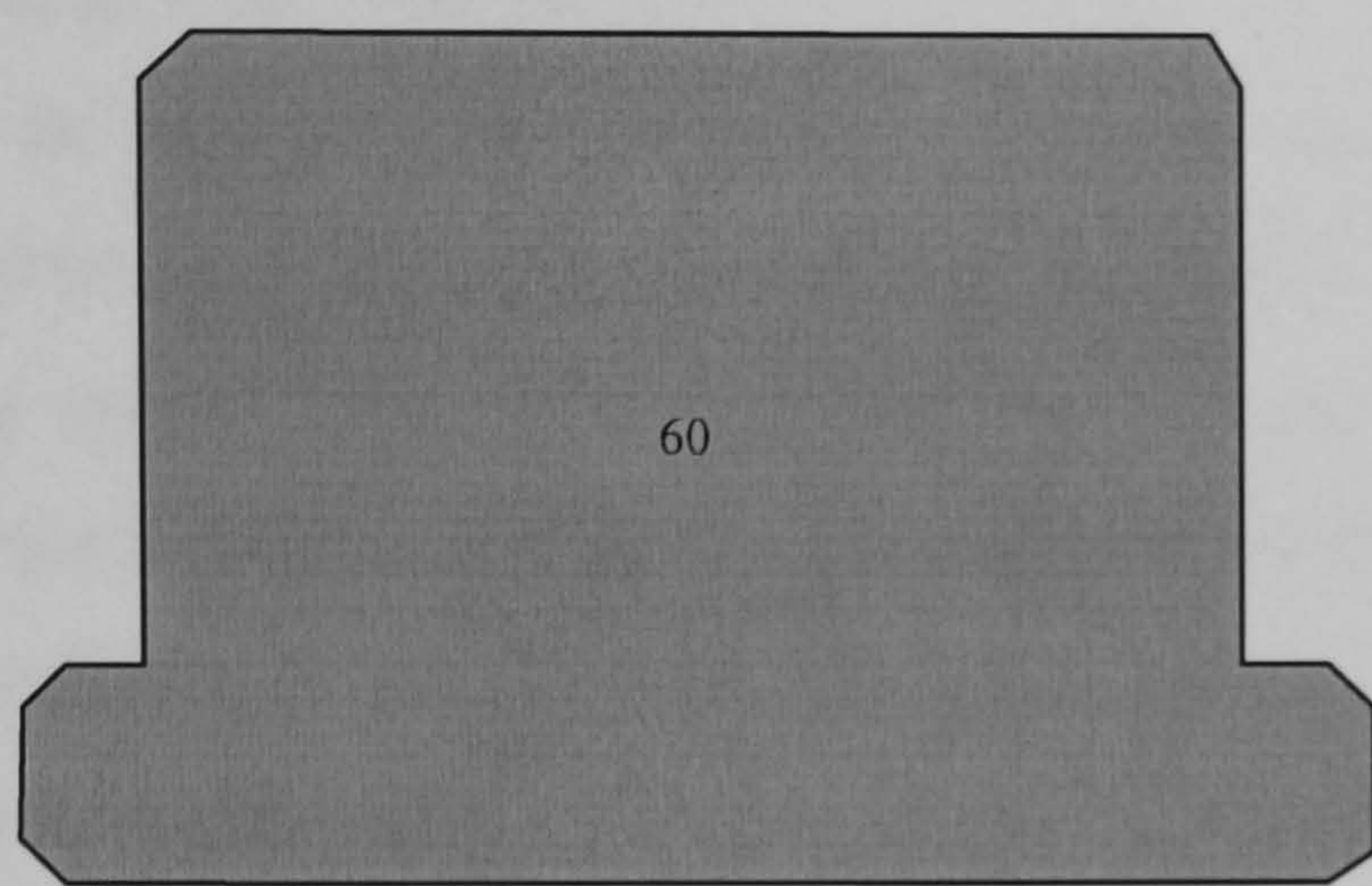


----- Reference Clustering Stage -----

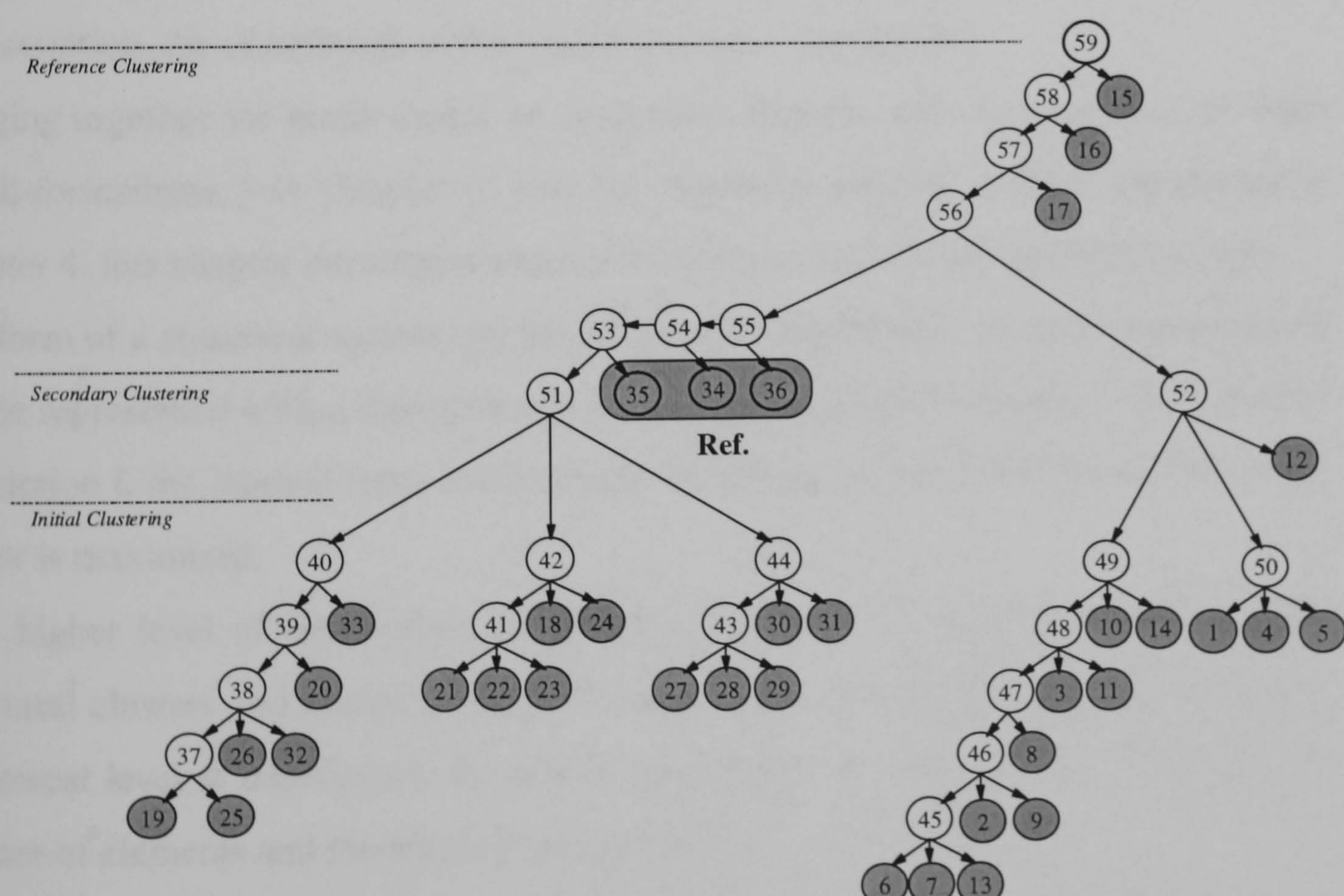
Level 17	35+51		
	Forming cluster 53	 as 	
Level 18	34+53		
	Forming cluster 54	 as 	
Level 19	36+54		
	Forming cluster 55	 as 	

Level 20	52+55		
	Forming cluster 56	 as 	
Level 21	15+56		
	16+56		
	17+56		
	Forming cluster 57	 as 	
Level 22	15+57		
	16+57		
	Forming cluster 58	 as 	
Level 23	15+58		
	Forming cluster 59	 as 	
Cluster formation complete ***Root reached***			

The Structure after Cluster Formation:



The complete hierarchy:



5.7 Conclusions

In this chapter, key concepts and methodology of systems approach were introduced. The systems paradigm is concerned with systems as *wholes*, their properties and the hierarchical arrangement of their organisations.

Complex systems can be structured hierarchically. Hierarchy discriminates entities, relations, processes and levels as the ingredients of the structure of a complex system. Hierarchy represents a system with multi-level description in the form of a graph. Each of the entities in the hierarchy is a holon. It is complete in itself and consists of parts from the levels below it in the hierarchy, yet it is also a constituent of some entities of higher levels in the hierarchy.

With hierarchy, a system can be described by a set of models each of which is concerned with the behaviour of the system from a different level of abstraction. By dealing with the peculiar problems and characteristics of the system at different levels of description, the complexity of the system becomes manageable.

Bringing together the graph model of structural systems and the measure of form (well-formedness) in Chapter 3, and the clustering analysis and its application in Chapter 4, this chapter introduced a hierarchical representation of structural systems.

The form of a structural system can be described hierarchically. A structural system S can be represented with a hierarchy of sub-structures/structural clusters. At a level of description l, the internal form and connectivity of each of the sub-structure/structural cluster is maximised.

At a higher level of description, the structural system is transformed into a set of structural clusters and complex joints. Comparing to the original structural system at the lowest level of description, the higher level system is simpler in terms of both the number of elements and the relation between them.

In the hierarchical representation of structural systems, each sub-structure/structural cluster is a holon. The structural clusters at lower levels in the hierarchy have more detailed information of the structure than those at higher levels of description.

At a higher level of description, a structural ring consists of a set of alternating structural clusters and complex joints and can be presented as a string pattern. The process of deterioration of a higher level structural ring can be identified using the deterioration hierarchy of structural rings (DHSR). Its degree of freedom and redundancy can be computed as discussed in Chapter 3.

The hierarchical representation of structural systems is the basis for the structural vulnerability analysis.

The example in Section 4.6 has been used again to illustrate detailed hierarchy formation process.

Chapter 6

Failure Scenarios & Vulnerability Analysis

6.1 Objectives

The objectives of this chapter are to:

- define what constitutes a failure scenario;
- discuss the consequences of a failure scenario;
- introduce a vulnerability index for failure scenarios;
- define five types of failure scenarios for vulnerability analysis of a structural systems;
- introduce methods for identifying the vulnerable failure scenarios;
- discuss practical aspects of vulnerability analysis.

6.2 Introduction

A system is vulnerable if it is susceptible to damage or failure which is disproportionate to the perturbation which triggers it. A system is vulnerable if it suffers from severe consequences at a given system level resulting from some damage or failure in the system defined at a much lower level in the hierarchy. A system is robust if it can withstand or tolerate damage in the system without having the consequence of significant loss in its function or in its form.

A structural system is vulnerable if some local damage or failure will cause serious loss in its form or integrity.

In this chapter, a method will be introduced to examine various failure scenarios through which a structural system may partially or completely fail. The vulnerability of a structural system can then be assessed by analysing the failure consequence and the damage demand of each of the failure scenarios.

6.3 Failure Scenarios

6.3.1 Structural rings and a structural system

In Chapter 3, the term structural ring (R) was introduced as a graphic description of a structure (S). A structure/sub-structure S^l/S_i^l at a level of description l can be described by a corresponding structural ring R^l/R_i^l.

The structural ring will be either a 2-link type or a 3-link type, consisting of two members and two joints or three members and three joints respectively.

At a level of description l, the structural system can be expressed by a set of structural rings either in a form of symbols:

$$R^l = \{ M_i^l , J_i^l \} \qquad (6.1)$$

where:

$i = (1, 2)$ for a 2-link-ring, and

$i = (1, 2, 3)$ for a 3-link-ring.

or in a form of string patterns:

$$R^l = \{ D_{M(i,j)}^l , D_{J(i,j)}^l \} \qquad (6.2)$$

where:

D_M is the string pattern for a member object, and

D_J is the string pattern for a joint object;

$i = (1, 2)$ for a 2-link-ring, and

$i = (1, 2, 3)$ for a 3-link-ring;

$j = (\mu, \nu, \theta)$, the degrees of freedom in principal direction

The following example illustrates this:

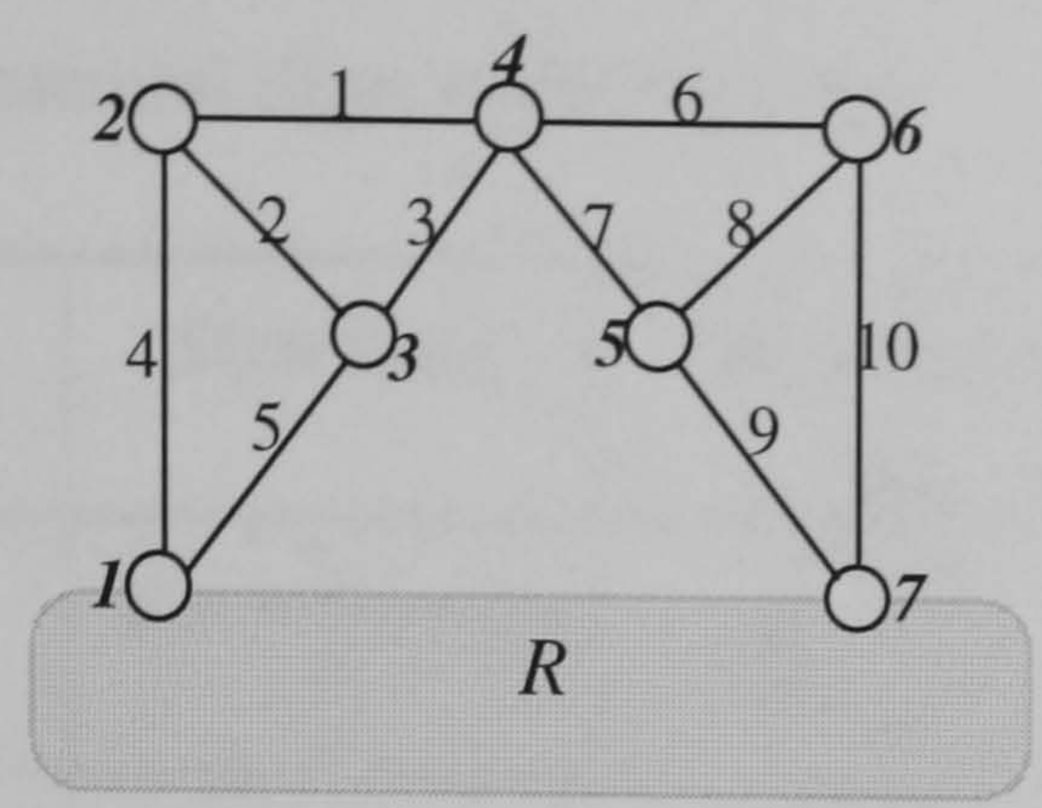


Figure 6.1 A structure

Assuming the properties of all member objects in the structure are identical, a hierarchy can be formed using the method introduced in Chapter 4. The detail of cluster formation is not shown in this case.

The following hierarchy represents the structure:

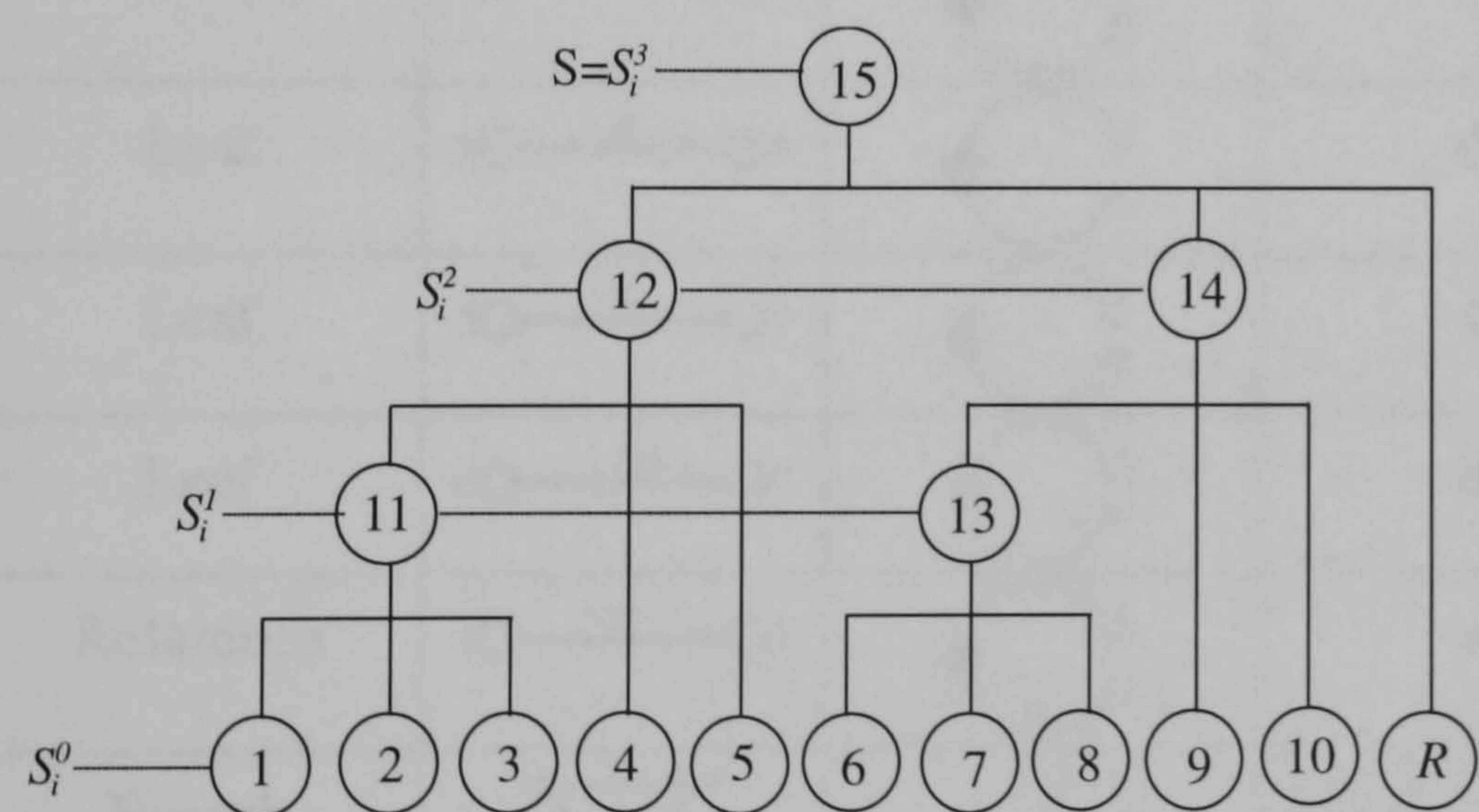
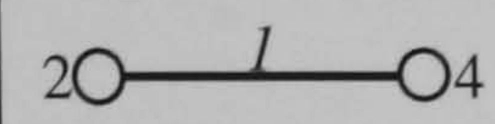
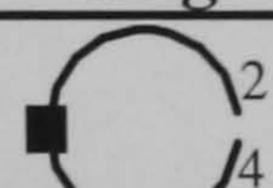
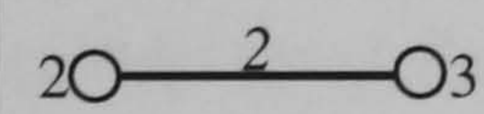
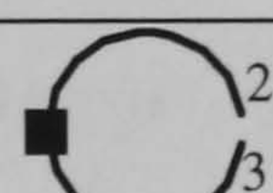
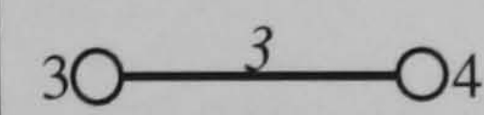
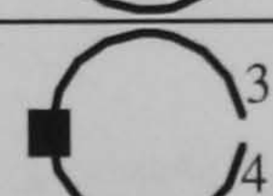
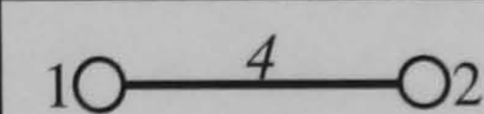
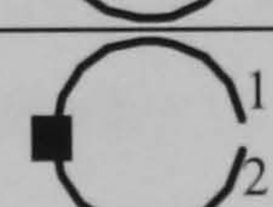
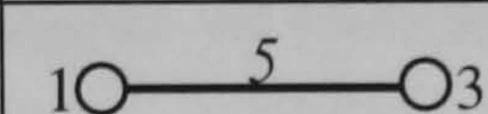
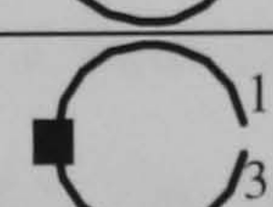
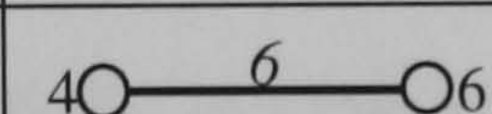
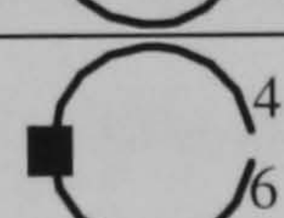
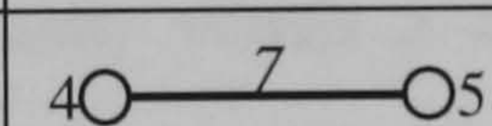
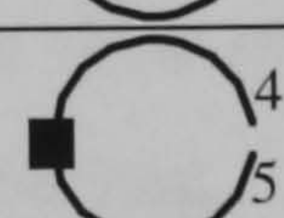
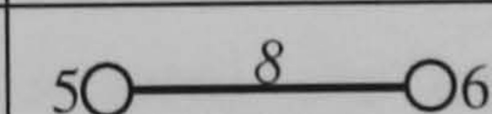
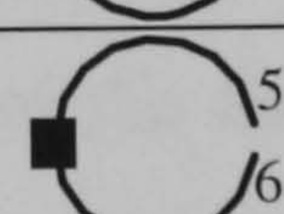
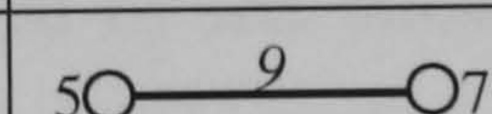
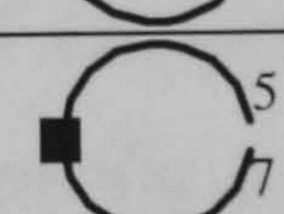
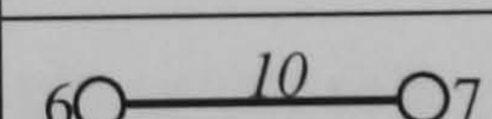
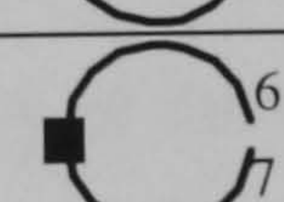
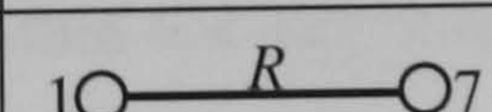
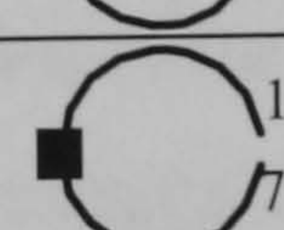
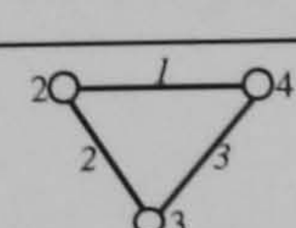
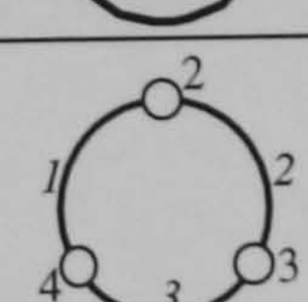
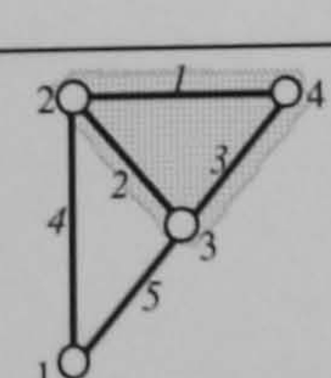
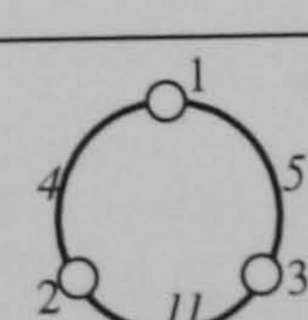
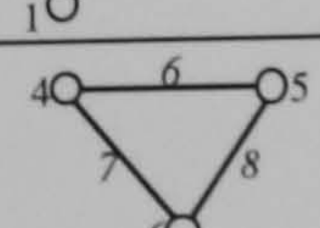
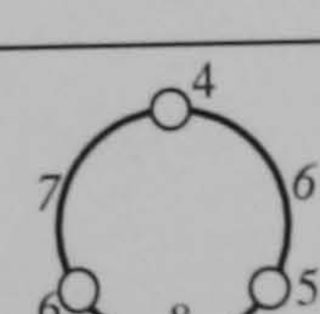
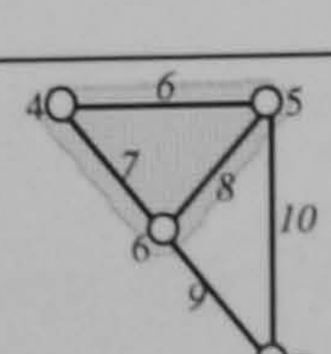
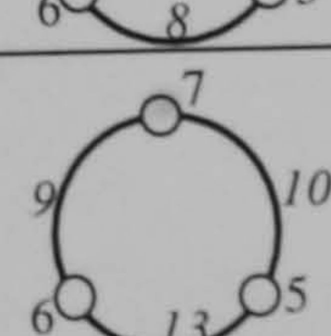


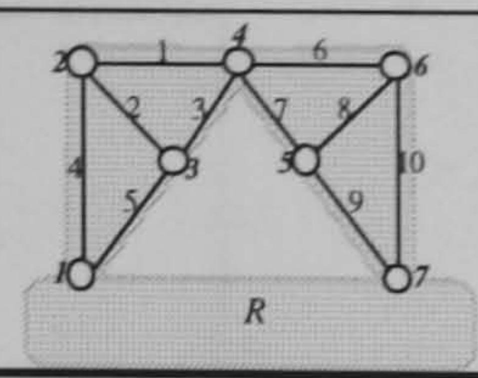
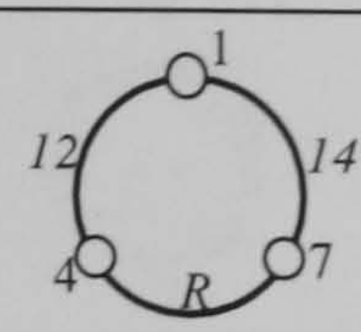
Figure 6.2 The hierarchical representation

Each of the structural clusters, or entities in the hierarchy, can be represented by a single corresponding structural ring, regardless of which level of description the entity is at. A member object in the structural ring can be one structural member or a sub-structure. Similarly, a joint object in the structural ring can be one structural joint or a complex joint which consists of several structural joints.

The structural rings are listed in the following table:

Table 6.1 Details of structural rings in the hierarchy

Cluster No.	Type of cluster	Structure	Structural ring	Description of the ring
1	Leaf			{{1,1,1},{1,1,1},{1,1,1},{0,0,0}}
2	Leaf			ditto
3	Leaf			ditto
4	Leaf			ditto
5	Leaf			ditto
6	Leaf			ditto
7	Leaf			ditto
8	Leaf			ditto
9	Leaf			ditto
10	Leaf			ditto
R	Reference			ditto
11	Branch			{{1,1,1},{1,1,0},{1,1,1},{1,1,0},{1,1,1},{1,1,0}}
12	Branch			ditto
13	Branch			ditto
14	Branch			ditto

15	Root			ditto

6.3.2 Damage process and failure state

Having represented a structural system in the form of structural rings, the damage process of the structural system can be studied through deterioration of the structural rings.

Any damage or fault is triggered by deteriorating events which occur in the structural ring. As defined in Chapter 3, a deteriorating event is the loss of the capacity to transmit force in one given degree of freedom in a structural ring, either adjacent to a joint or in a member object. A deteriorating event is the result of general actions. The detail and nature of the actions are not considered at this stage of the research.

The damage process will finally reach a failure state when there is not sufficient capacity in the structure to maintain static equilibrium, i.e. the structure becomes a mechanism. For a structural system, the length of the damage process before failure can be quantitatively described by the number of deteriorating events required. The number of deteriorating events required for a specific structural ring to achieve the failure state can be calculated using the redundancy of the structural ring.

The following example illustrates the representation of the damage process of a portal frame by the deterioration of the corresponding structural rings:

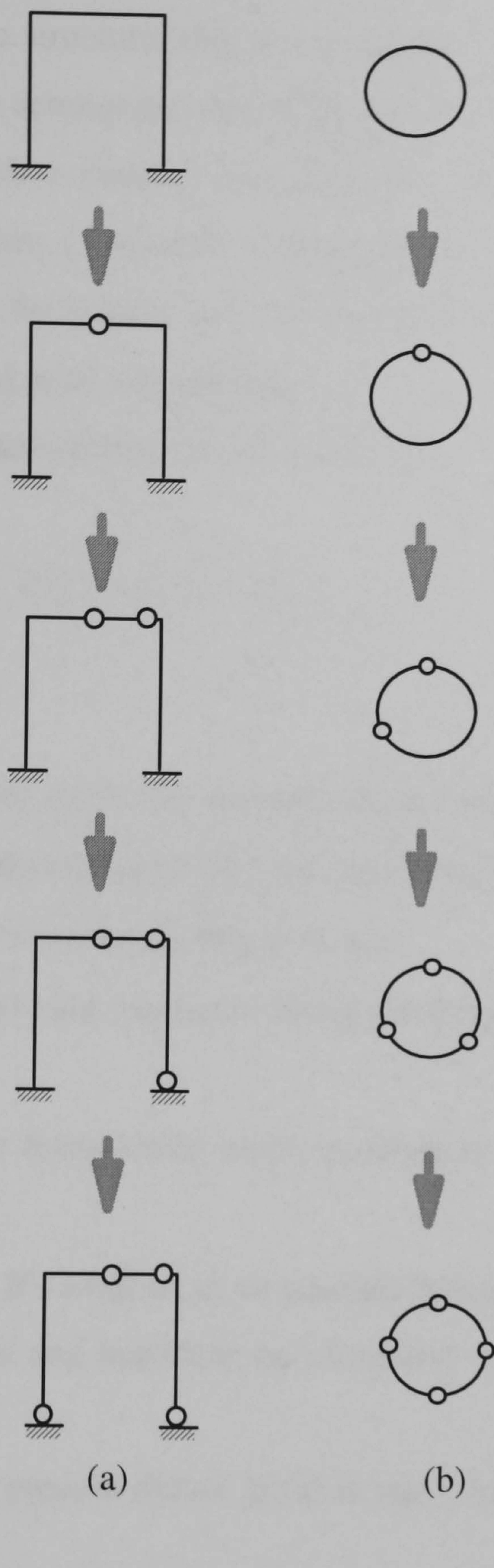


Figure 6.3 A damage process of a portal frame

6.3.3 Failure scenarios

In Chapter 3, the deterioration hierarchy of structural rings, DHSR was introduced. It is a hierarchy in which all possible ways of deterioration for a structural ring are listed. The structural ring at the top of the DHSR is a maximally redundant structural ring, and the elements at the bottom of DHSR are all mechanisms. At any intermediate levels in the hierarchy, the structural ring is *a deteriorated.* Thus, a structural ring at any intermediate level is a deteriorated ring of the one at the next higher level.

For a structural ring, a *failure scenario* is defined as a path in the DHSR such that the final element is a mechanism. (Wu, 1991). The path may not start from the top of the DHSR but always ends at the bottom of it. In another word, a failure scenario is a set of deteriorated rings ending with a mechanism.

A failure scenario of a structural ring can be written as:

$$F_h(R^l) = \{ \; R_k^l \mid k = 1, 2, \ldots, m_h \; \} \qquad (6.3)$$

where

$F_h(R^l)$ is the hth failure scenario of the structural ring R at the lth level of description in the total number of failure scenarios,

R_k^l is a deteriorated ring of R, and

m_h is the total number of deteriorated rings in the failure scenario.

From R_k to R_{k+1}, only one deteriorating event occurring in the structural ring, and it is the kth event in a scenario.

For a given structural ring R^l, a full set of all possible failure scenarios can be found by mapping the ring in DHSR and launching an exhaustive search down to the bottom level of the DHSR.

In Figure 6.3, the damage process shown in (a) is represented by the failure scenario pattern shown in (b).

For a structural ring, a failure scenario can be described as the record of a sequence of deteriorating events. (Wu, 1991) Using the same notion which have been used in Chapter 4, that is: $d_{i,j,k}^l$ as the capacity along a given degree of freedom of the joint and $s_{i,j,k}^l$ as the capacity along a degree of freedom of the member objects in a structural ring, Wu has described a failure scenario $F_h(R^l)$ for a structural ring R at the level of description l as:

$$F_h(R^l) = \{\ R_k^l \mid k = 1, 2, \ldots, m_h\ \}$$
$$= \{\ d_{i,j,k}^l\ ,\ s_{i,j,k}^l \mid k = 1, 2, \ldots, m_h\ ; i = 1, 2, \ldots, n; j = \mu,\nu,\theta;\ \} \quad (6.4)$$

The failure scenario in Figure 6.4 can be described as:

$$F_h(R^l) = \{\ R_k^l \mid k = 1, 2, \ldots, m_h\ \}$$
$$= \{\ d_{i,j,k}^l\ ,\ s_{i,j,k}^l \mid k = 1, 2, \ldots, m_h\ ; i = 1, 2, \ldots, n; j = \mu,\nu,\theta;\ \}$$
$$= \{\ [(1,1,0), (1,1,1), (1,1,0), (1,1,1)],$$
$$[(1,1,0), (1,1,0), (1,1,0), (1,1,1)],$$
$$[(1,1,0), (1,1,0), (1,1,0), (1,1,0)]\ \}$$

where $m_h = 3$ and $n = 2$.

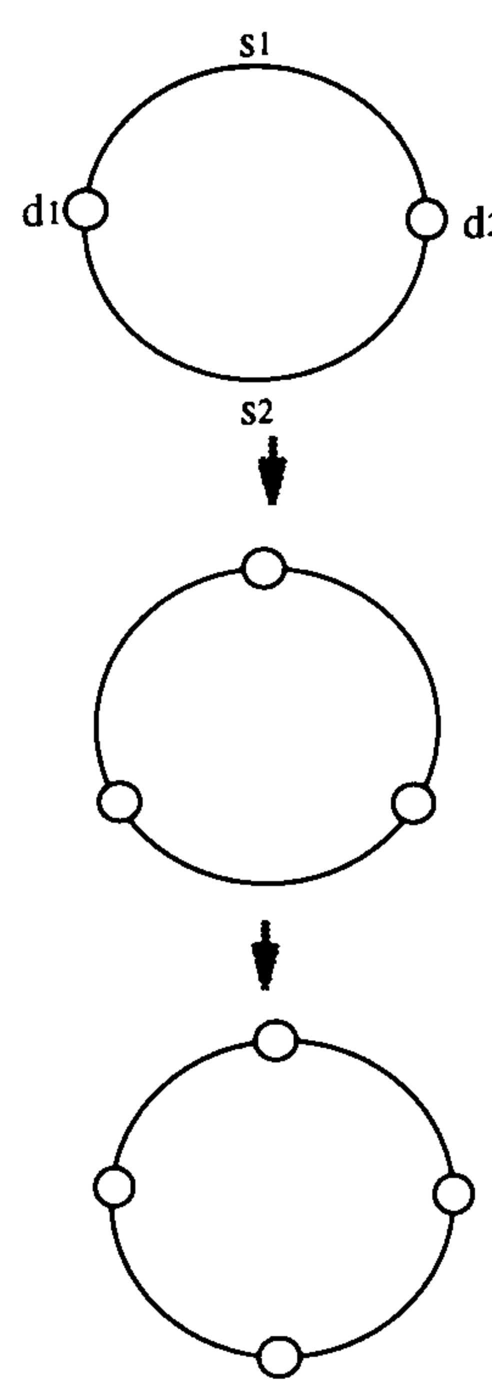


Figure 6.4 A failure scenario

It can also be described using the deteriorating events in the failure scenario as:

$$F_h(R^l) = \{\ f_{i,j,k}^l\ ,\ g_{i,j,k}^l \mid k = 1, 2, \ldots, m_h - 1; i = 1, 2, \ldots, n; j = \mu,\nu,\theta;\ \} \quad (6.5)$$

where $f_{i,j,k}^l$ is the deteriorating event that occurs adjacent to a joint, and

$g_{i,j,k}^l$ is the deteriorating event that occurs in a member.

Thus, the failure scenario in Figure 6.4 is:

$$F_h(R^l) = \{ f^l_{i,j,k} , g^l_{i,j,k} \mid k = 1, 2, ..., m_h -1; i = 1, 2, ..., n; j = \mu,\nu,\theta; \}$$
$$= \{ g^l_{1,\theta,1} , g^l_{2,\theta,2} \}$$

6.3.4 The damage demand of a failure scenario

A failure scenario for a structural ring consists of a sequence of deteriorating events. For each of the deteriorating events, there is a corresponding damage demand, i.e. the effort which is required in order to achieve the event.

The general definition and calculation of the damage demand has been given in Chapter 4. The damage demand for a deteriorating event $f^l_{i,j,k}$ or $g^l_{i,j,k}$ is defined as $e(f^l_{i,j,k})$ and $e(g^l_{i,j,k})$ respectively, where

$e(f^l_{i,j,k}) > 0$ when $f^l_{i,j,k} > 0$,

$e(f^l_{i,j,k}) = 0$ otherwise.

and

$e(g^l_{i,j,k}) > 0$ when $g^l_{i,j,k} > 0$,

$e(g^l_{i,j,k}) = 0$ otherwise.

Therefore, the damage demand for a failure scenario is

$$\boldsymbol{E}[F_h(R^l)] = \sum_k \sum_i \sum_j e(f^l_{i,j,k}) + \sum_k \sum_i \sum_j e(g^l_{i,j,k}) \qquad (6.6)$$

where $k = 1, 2, ..., m_h - 1$;

$i = 1, 2, ..., n$;

$j = \mu, \nu, \theta$.

6.4 Vulnerability Analysis

6.4.1 Failure scenarios in a structural system

In 6.3.3, failure scenarios for a structural ring were defined. A structural system can be represented by a hierarchy of interconnected structural clusters, each of which can be represented by a structural ring. A structural cluster in this hierarchy model is a holon, it may be part of another cluster, while it may contain other clusters at the same time. Because of this holistic property of the clusters, the identification of failure scenarios for a structural system is far more complicated than that for a structural ring. A failure scenario of a particular structural ring at a level of description will often lead to more failure scenarios of several other structural rings.

For instance, at a intermediate level of description, the structural ring which is to be deteriorated may represent a branch cluster, which may, again, consist of other branch clusters.

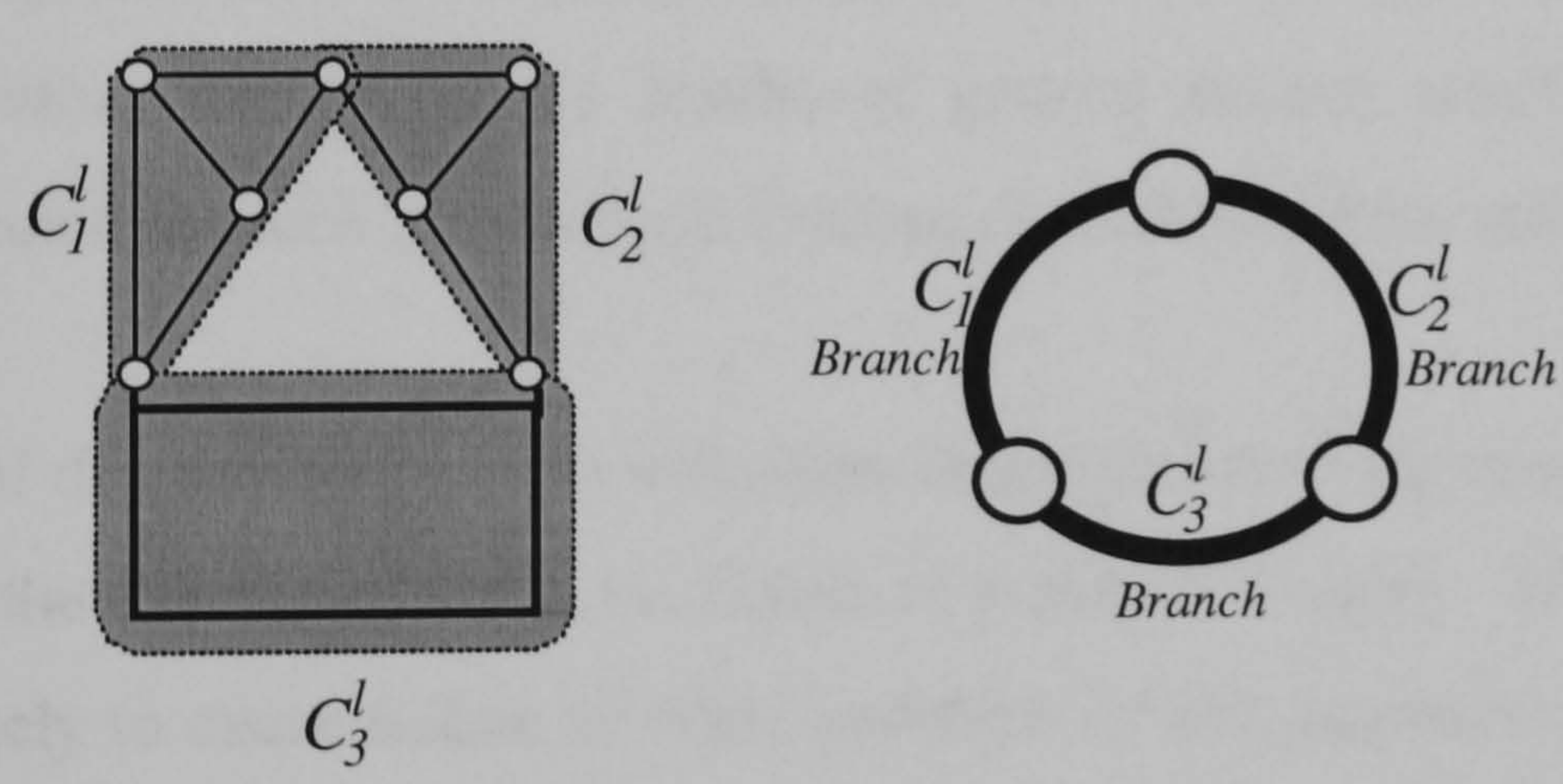


Figure 6.5 A branch cluster may contain other branch clusters

When dealing with such a ring in the analysis, "release one DOF" in the member object which is a branch cluster is not simply done by releasing one DOF in any one of its member or joint objects. The member object must be treated as a "whole", a structural sub-system in its own right.

Thus, a failure scenario in the structural system $F_h[S]$ consists of a sequence of deteriorating events which may occur at *different levels of description* in the hierarchical model of the structure and usually involves *more than one structural ring*. To simplify the presentation in the analysis, a failure scenario in a structural system can be written as a sequence of the member/joint objects together with the deteriorating event occurred to them:

$$F_h[S] = \{\ M_{i,k}(J_{i,k}),\ g^{l}_{i,j,k}(f^{l}_{i,j,k})\ \mid k = 1, 2, \ldots m_h,\quad i = 1, 2, \ldots n,\quad j = \mu, \nu, \theta\ \} \qquad (6.7)$$

6.4.2 Failure consequences

In a structural system, deteriorating events may occur in any of its component objects. These deteriorating events are the results of general actions which the structural system is subjected to, such as excessive loading, natural disasters, accidental force or human errors.

The presence of deteriorating events will cause deterioration in the structural form and even failure at the elementary level, i.e. failure of primitive clusters. When such failure occurs, it is likely to cause failure of other primitive or non-primitive clusters. In the extreme case, it can cause the complete breakdown of the structural system. The consequence of failure of different clusters varies and largely depends on how the cluster is configured in the structure.

The failure consequence is a very important part of the assessment of structural vulnerability. A structural system is vulnerable if failure of its element or elements will lead to large scale damage or failure consequence.

The simple example in Figure 6.6 shows different failure consequences of the failure of different clusters.

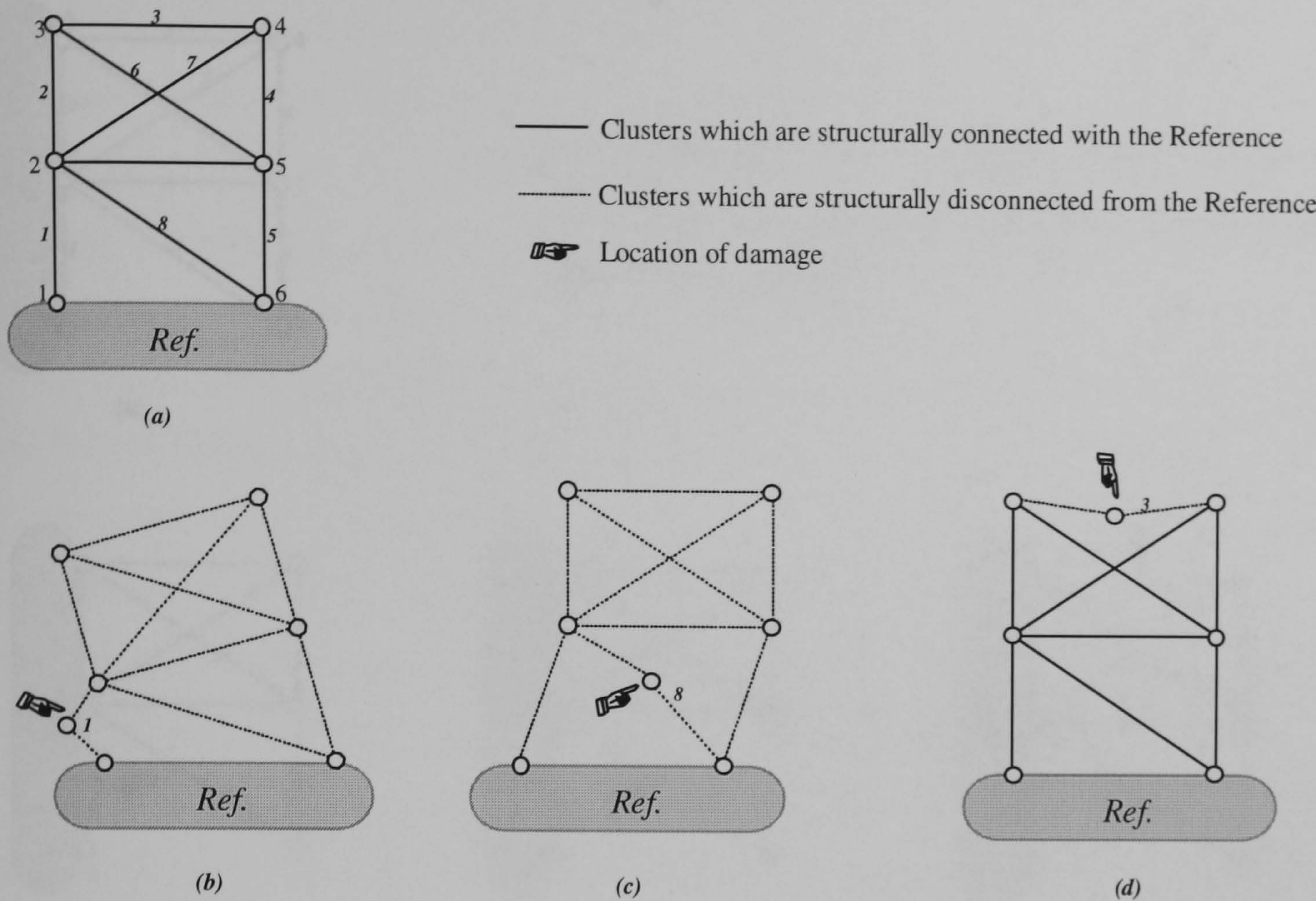


Figure 6.6 Failure consequences of various clusters in a structure

When discussing failure consequence, the ***reference*** plays a determinative role. For the same structure, had the reference cluster been a different one, the failure consequence could be very different (Figure 6.7). Therefore, the reference must be set clearly prior to any vulnerability analysis.

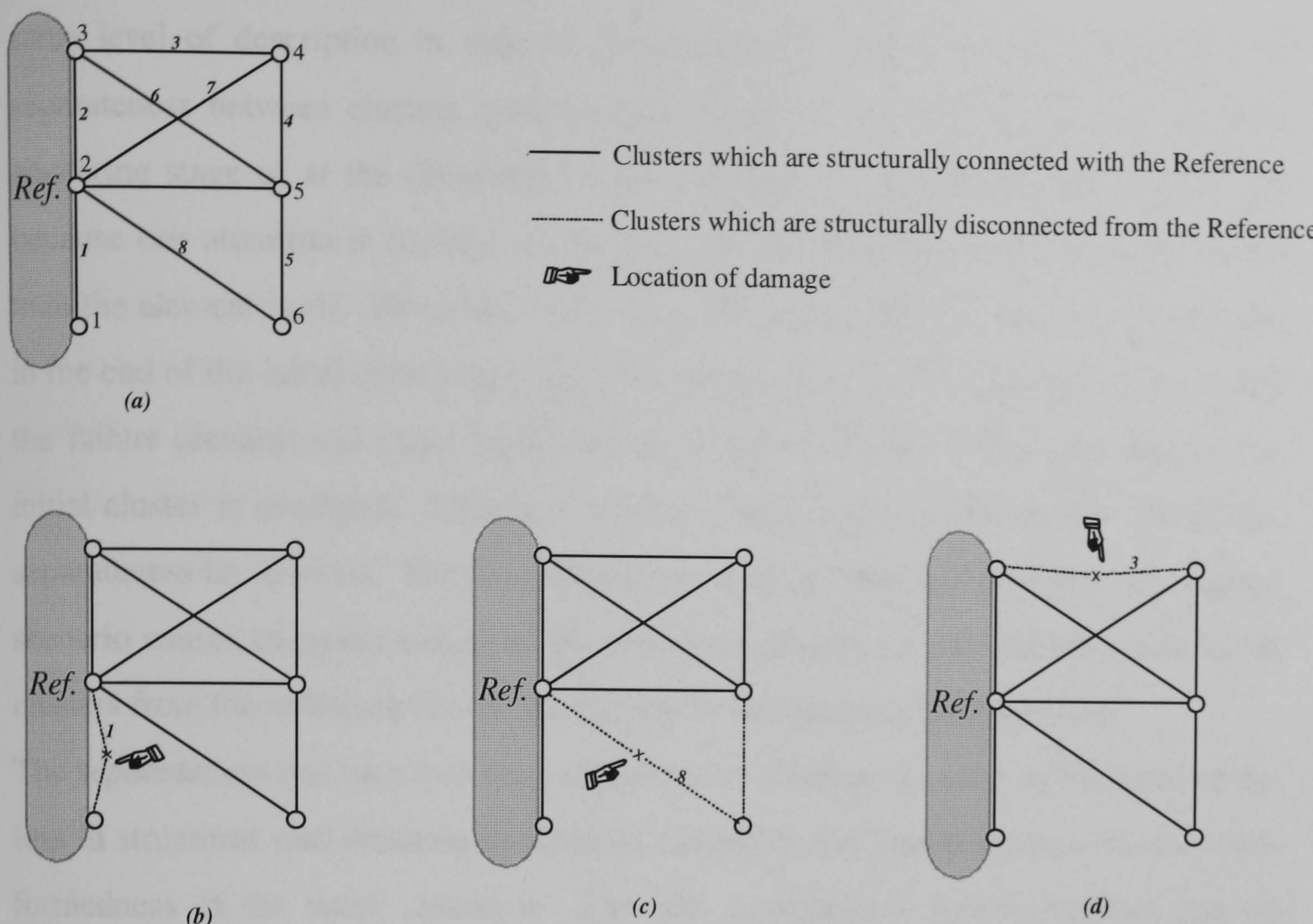


Figure 6.7 Failure consequence is dependent of the Reference

6.4.3 Damage scale & separateness

For the purpose of vulnerability analysis, we need to have a quantitative description of the failure consequences for various failure scenarios. The description or the damage scale must be able to represent the failure consequence in terms of damage in structural form, i.e. the well-formedness of the structure.

In previous work, Wu has defined the separateness as a measure of failure consequence. It was defined as *the number of clusters structurally disconnected from the reference cluster* and calculated by dividing the sum of structural well-formedness of all clusters which are structurally disconnected from the reference cluster with the sum of the structural well-formedness of all clusters which are still connected to the reference cluster.

This previous calculation of separateness has two problems. Firstly, the clusters, either connected to or disconnected from the reference cluster, must be defined at the same level of description in order to be compared. Therefore the comparison of separateness between clusters must be done either at the level at the end of initial clustering stage or at the elementary level. The latter option is obviously ruled out because our attention is focused on the form of the assembly of the elements rather than the element itself. However, when using the initial clusters (the clusters formed in the end of the initial clustering stage) for comparison, it is not always the case that the failure scenario will cause the failure of an initial cluster. Often only part of the initial cluster is damaged. Thus it is arbitrary which part of the cluster should the separateness be counted. The second problem is an arithmetical problem. If a failure scenario causes complete failure of the structural system, i.e. the disconnection of all clusters from the reference cluster, the divisor in the equation becomes zero.

The ***separateness*** has therefore been redefined for a failure scenario as the ratio of the loss in structural well-formedness which is caused by the failure scenario to the well-formedness of the intact structure. The loss in structural well-formedness can be calculated as the difference in the structural well-formedness between the intact structure and the deteriorated structure.

If S is the structure before the failure scenario F_h and S' is the deteriorated structure after F_h, then the separateness is:

$$\gamma[F_h] = \frac{Q[S]-Q[S']}{Q[S]} \qquad (6.7)$$

Apparently, the separateness γ is a bounded measure as $0 < \gamma \leq 1$. The separateness will be 1 if the failure scenario causes the complete failure in the structural system.

6.4.4 The vulnerability index of a failure scenario

The vulnerability index of a failure scenario is defined as the ratio of the separateness which is caused by the failure scenario to the relative damage demand of that failure scenario.

$$\xi[F_h] = \frac{\gamma[F_h]}{E_r[F_h]} \qquad (6.8)$$

The relative damage demand of a failure scenario, $E_r[F_h]$, is the ratio of the damage demand of the failure scenario to the maximum possible damage demand of a failure scenario in the structural system. The failure scenario which has the maximum possible damage demand is the one in which deteriorating events occur in every primitive cluster. If we choose the deteriorating event

$$g^0_{i,\theta,k} \big| i = 1,2,\ldots,n; k = 1,2,\ldots,n;$$

where

n is the total number of member object

the relative damage demand will be

$$E_r[F_h] = \frac{E[F_h]}{\sum_i \sum_k e(g^0_{i,\theta,k})} \qquad (6.9)$$

The maximum possible damage demand, for a given structural system, is a constant. Thus the vulnerability index is non-dimensional while comparisons are made among any failure scenarios in the structural system, and $0 < \xi < \infty$.

Here we choose three failure scenarios from the structure shown in Figure 6.6(a) to illustrate the calculation of the vulnerability index:

1) To form a hinge in member No. 1
2) To form a hinge in member No. 3
3) To form a hinge in member No. 8

The joint co-ordinates, the properties of members and the constraint conditions are listed in Table 6.2, Table 6.3 and Table 6.4.

Table 6.2 Joint co-ordinate detail of the structure in Figure 6.6(a)

Joint No.	X Co-od. (m)	Y Co-od. (m)
1	0.0	0.0
2	4.0	0.0
3	0.0	3.0
4	4.0	3.0
5	0.0	6.0
6	4.0	6.0

Table 6.3 Member detail of the structure in Figure 6.6(a)

Member	Node 1		Node 2		E	A	I
No.	Joint No.	Fixity	Joint No.	Fixity	(KN/m^2)	(m^2)	(m^4)
1	1	0	3	0	205×10^6	0.00474	0.000117
2	2	0	3	0	205×10^6	0.00474	0.000117
3	2	0	4	0	205×10^6	0.00474	0.000117
4	3	0	4	0	205×10^6	0.00474	0.000117
5	3	0	5	0	205×10^6	0.00474	0.000117
6	4	0	6	0	205×10^6	0.00474	0.000117
7	5	0	6	0	205×10^6	0.00474	0.000117
8	3	0	6	0	205×10^6	0.00474	0.000117
9	4	0	5	0	205×10^6	0.00474	0.000117

Note: Node fixity is the member end condition,
and is 1 if the end node is a fix joint, or 0 if the end node is a pinned joint.

Table 6.4 Constraint condition of the structure in Figure 6.6(a)

Constraint No.	**Joint No.**	x	y	θ
1	1	1	1	0
2	2	1	1	0

In this structure, $E_{max} = 9.72\times10^{-4}$, and $Q[S] = 1.591\times10^{11}$. Thus, the vulnerability indices for the three failure scenarios are:

Table 6.5 Calculation of the vulnerability index

Failure Scenario	E	E_r	$Q[S']$	γ	ξ
1	1.38×10^{-4}	0.137	0	1	7.3
2	1.33×10^{-4}	0.137	5.67×10^{9}	0.964	7.04
3	1.0×10^{-4}	0.103	1.27×10^{11}	0.251	2.4

In this example, it is apparent that how a structure being damaged and what the corresponding form of the structure effect the failure consequences. The vulnerability index is an indicator of the scale of potential damage with same effort.

Among the three cases, the failure scenario which forms a hinge in the member No. 1 is the most vulnerable scenario.

6.4.5 **Vulnerable scenarios**

In a structural system, the number of the possible failure scenarios at various levels of description can be enormous. However, only some of them are important in our structural vulnerability analysis. A vulnerable scenario might be the one which has a large value of vulnerability index, i.e. the one which will cause great loss in form out of relatively small amount of effort. Or it could be the one which shows the weakness in the form of the structural system. There are also other failure scenarios in which the engineer might be particularly interested and consider as vulnerable scenarios when carrying out structural vulnerability analysis. In this section, all the vulnerable scenarios will be discussed in detail.

Wu has specified the minimal and the maximal failure scenarios as the vulnerable scenarios which are of particular interest as far as the structural vulnerability is concerned. (Wu, 1991) The minimal failure scenario of a structural ring at a level of definition was defined as "the one in which the least damage demand is required to transform the structural ring into a mechanism". The maximal failure scenario was defined as "the one in which the least damage demand is required to cause the maximal number of clusters to structurally disconnect from a reference cluster at a given level of definition".

From this basis, those vulnerable scenarios which are associated with the weakness in the form of the structural system are categorised as the ***minimal failure scenarios***, and those ones which will have great failure consequence with minimum effort as the ***maximal failure scenarios***. Any other failure scenarios which might also have large failure consequences or which are of particular interest to the engineer fall into the third category as the ***interesting failure scenarios***.

6.4.5.1 The Minimal Failure Scenarios

Two scenarios which are associated with the weakness in the form of the structural system are as follows:

The minimum demand scenario is defined as the failure scenario which requires the least damage demand to cause the failure of a structural ring at a level of description. This is the scenarios in which the structural system is most vulnerable to *some* damage; the damage scale and the failure consequence is not considered. Thus, a minimum demand scenario may cause very little failure consequence in the structural system. However, it is the easiest possible way to cause damage to the structural system.

The least well-formed cluster scenario is the failure scenario where the configuration in the form is loose, hence poorly formed. Therefore, it shows where in the structural system the connection between clusters is the least.

The difference between these two scenarios derives from the fact that a member in a structural system may have low well-formedness itself but has high connection to a cluster. The former scenario looks for where the system may be damaged most easily, while the latter aims to locate the weak link in the structure of the form.

6.4.5.2 The Maximal Failure Scenarios

There are two scenarios associated with large failure consequence with least effort:

First, consider the situation of complete breakdown of the structural system, or total failure. There are many ways in which the structural system can be damaged leading to a complete breakdown. Among all the possible failure scenarios which lead to a total failure, the one which requires least damage demand is the most vulnerable scenario. It is ***the total failure scenario***. In this scenario, the scale of damage is set as total with least effort.

Secondly, consider the case where the scale of damage may not be a total failure. Some failure scenario will cause a partial failure in the structural system, but, the damage demand required for that can be very small, hence the vulnerability index very high. ***The maximum failure scenario*** is the one which has the highest vulnerability index, irrespective of the damage scale.

The total failure scenario may or may not be the same as the maximum failure scenario for a particular case.

6.4.5.3 **Other Interesting Failure Scenarios**

As previously stated, this category consists of those scenarios which (a) may have large failure consequence, or (b) are of particular interest to the engineer who carries out the vulnerability analysis.

The interesting failure scenarios in the first part can be those failure scenarios identified in the structural system which have high but not the highest value of vulnerability index. The threshold value needs to be determined according to the level of interest.

Those in the second part are rather dependent on the analyst. The detail of the failure scenario must be specified and further request of analysis must be made by the analyst.

6.4.6 **Identification of vulnerable scenarios**

In the previous chapter, the hierarchical representation of a structural system was developed. The various vulnerable scenarios in a structural system can be identified using the hierarchical modal which represents the structural system.

(1) *The minimal failure scenarios:*

The minimum demand scenario is defined as the failure scenario which requires the minimum damage demand to cause the failure of a structural ring at a level of description, irrespective of the scale of damage. Damage to the structural rings which represent primitive clusters requires least effort since it involves less number of (or at least not more) deteriorating events than those which represent branch or root clusters. Therefore, the minimum demand scenario is identified by finding failure scenarios which cause failure in the rings which represent primitive clusters and find the one or those ones which requires the minimum damage demand.

In the hierarchy formation, the first criterion is maximum well-formedness of structural clusters. Thus in the hierarchical model, those more tightly connected and well-formed clusters are in the lower levels of the hierarchy. The least well-formed cluster scenario is straightforward to identify using the hierarchical model.

(2) *The maximal failure scenarios:*

The process of identifying the total failure scenario and the maximum failure scenario is a pattern-matching search in the hierarchy in which the goal is to find a series of structural rings and damage them in various ways to achieve specific scales of damage. For the total failure scenario, the scale of damage is set as complete breakdown, i.e. the whole structure becomes a mechanism. In another word, all clusters should be structurally disconnected from the reference cluster. The relationship between the reference cluster and other clusters plays an important role in the identification.

For the maximum failure scenario, the scale of damage is not fixed. The identification involves finding failure scenarios of various scale of damage and identify the one or those ones with highest vulnerability index.

(3) *The interesting failure scenario:*

The above process in (2) can also be used to identify some interesting failure scenarios. A threshold value of vulnerability index can be set prior to the search, therefore any failure scenario with a vulnerability index higher than the threshold value is recorded as an scenarios of interest.

In this section, we only discussed the principle of identification of vulnerable scenarios. The details of the process will be given in Chapter 7.

6.4.7 Applications of vulnerability analysis

The purposes of vulnerability analysis are:

- to gain understanding on the quality of the form of a structural system,
- to identify the vulnerability of a structural system,
- to help engineers as a design aid to create robust structures.

In vulnerability analysis, the vulnerable failure scenarios provide information about the form of a structural system. When dealing with large-scale structural systems, it is not sufficient to rely on engineer's intuition or even good engineering experience to make a correct judgement. Vulnerability analysis may help to avoid those weaknesses in the form of the structural system being overlooked.

At this stage, the vulnerability analysis is focused on the form of a structural system. In practice, it is aimed for application which will be to achieve a robust design, to maintain an existing structure or to serve other specific purposes.

6.5 Conclusions

A structural system is vulnerable if certain damage processes in the system can cause severe and disproportionate consequences at the system level. A structural system is robust if it can withstand damage in the system without loss of its required functions.

In this chapter, the damage process of a structural system has been described in terms of failure scenarios of structural rings. For a structural ring, a failure scenario is defined as a path in the DHSR such that the final element is a mechanism. A failure scenario is a pattern in which the structural ring deteriorates to a failure state. A failure scenario can also be described using the deteriorating events in the failure scenario.

The damage demand of a failure scenario is the sum of the damage demands of all deteriorating events in the failure scenario.

As a measure of the failure consequence of a failure scenario, the separateness for a failure scenario is defined as the ratio of the loss in structural well-formedness which is caused by the failure scenario to the well-formedness of the intact structure.

The vulnerability index of a failure scenario has been defined as the ratio of the separateness to the relative damage demand of the failure scenario. For a failure scenario, the vulnerability index indicates its effectiveness in inflicting damage upon the structural system.

The purpose of vulnerability analysis is to find the vulnerability of the analysed structural system. The analysis is mainly concerned with the identification of various vulnerable failure scenarios. There are five vulnerable scenarios which fall into three categories: the minimal failure scenarios, the maximal failure scenarios and other interesting failure scenarios.

The minimal failure scenarios are associated with the weakness in the form of the structural system. This category includes the minimum demand scenario and the least well-formed cluster scenario. The former is where the system can be damaged most easily, whereas the latter is where inside the system the interconnectedness is the minimum.

The maximal failure scenarios are associated with the effective damage. The total failure scenario is the one which achieves total breakdown of the system with the minimum effort. The maximum failure scenario is the one which has the highest value of vulnerability index. The damage of the maximum failure scenario may be total or partial.

Apart from these four vulnerable scenarios, any other failure scenario which is of particular interest to the analyst is a interesting failure scenario. The details and the criteria for selection of interesting failure scenarios depend on the analyst and the purpose of specific application.

The various vulnerable failure scenarios can be identified using the hierarchical modal of the structural system. The principle of identification of various failure scenarios have been discussed in this chapter, however, the detailed algorithm will be given in chapter 7.

Vulnerability analysis can identify the vulnerability in the form of the structural system. Hence it is capable of aiding engineers either at the design stage or in carrying out monitoring or maintenance tasks.

Part III Implementation

Chapter 7

Algorithm

7.1 Objectives

The objectives of this chapter are to:

- introduce the computer program ***SAVE*** (Structural Analysis for Vulnerability Estimation);
- introduce the details of the implementation of *SAVE*.

7.2 Introduction

An algorithm is a procedure, which is suitable for computer implementation, for solving a problem. Often, there is more than one way of implementing an algorithm. The choice of algorithm and the efficiency of problem solving is largely dependent on understanding and defining of the problem to be solved. The complexity of the problem can be managed by decomposing the task into smaller subtasks which are easier to implement.

In this study of structural vulnerability, the problem was represented in the form of graphs. The graph-theoretic properties of the problem have been described in earlier chapters and therefore form the foundation of the algorithm.

In this chapter, the computer program *SAVE* (*S*tructural *A*nalysis for *V*ulnerability *E*stimation) will be introduced in detail. The main tasks of *SAVE* are to derive the following:

- a method for representing graph;
- a data structure to represent the interrelated structural clusters;
- a method for organising the data involved in the computations;
- a way of forming hierarchy;
- various search procedures to find and identify failure scenarios.

7.3 General Outline of *SAVE*

The program contains five modules, each of which performs a distinct task. They are:

- Data Input
- Data Preparation and Preliminary Calculation
- Hierarchy Formation
- Search for Minimal Failure Scenarios
- Search for Maximal Failure Scenarios

Before getting into the details of these modules, it is necessary to introduce the data structure.

7.4 Data Structure

The first task is to design a data structure to represent the problem. This data structure should contain of all the information which is considered to be important and relevant. Hence it is an abstraction of the problem model.

The selection of an appropriate data structure depends on many factors, such as the operations that are to be performed on the data, the degree of detail required, the computational facilities available (programming language and processor) and even different programming styles.

In general, a good data structure will enable:

- efficient abstraction of the problem to be solved;
- clear representation of the information;
- efficient in storing, processing and retrieving information.

In designing the data structure for the program Structural Analysis for Vulnerability Estimation (*SAVE*), the most important information can be captured in a *record structure.* This structure represents the graph object, "a structural cluster", and will be used throughout the whole process of forming the hierarchy to represent the structure and in the search for different failure scenarios.

A Cluster

In the graph model of the structural system, a structural cluster, at a level of description, is basically a *member object.* At various levels of description, it can be a primitive or non-primitive cluster. In the process of cluster formation which creates a hierarchy of clusters to represent the whole structural system, the primitive clusters are termed ***Leaf*** clusters and the cluster which represents the structure as a whole is termed ***Root*** cluster. All the clusters in the level between the *Leaf* and *Root* are termed ***Branch*** clusters. There is another type termed ***Reference-Leaf*** . This is the part of the structural system being specified from which other clusters will be disconnected during the failure process. Apart from their different type, they all share the same characteristics as a member object in the graph model of structural system. These characteristics are:

- Each cluster has two joint objects.
- Each member object has a string pattern to be used for the forming of structural rings.
- Each cluster has a value of structural tightness.

- Each cluster has a minimum damage demand to a defined action.

In the process of cluster formation to create the hierarchy, all the clusters formed in all levels of description will be stored in a single array. Therefore, in the data structure, the information of the position of the cluster in the hierarchy is required. This part of information is different for different type of clusters. According to the type of structural ring involved, in the hierarchy, any *branch/root* cluster consists up to three sub-clusters which form the structural ring, i.e., the *branch/root* cluster itself. The sub-clusters can be *Leaf* or *branch*. They are termed ***child*** clusters of the *branch/root* cluster they form in a higher level of description. Vice verse, the *branch/root* cluster is termed the ***parent*** cluster. For different type of clusters, this part of information in the data structure includes:

- The index of its parent cluster (if not a Root type),
- Numbers of its child clusters (if not a Leaf type),
- A list of the index of its child clusters (if not a Leaf type).

Since the formation of the hierarchy depends on the way the *reference* is defined in the structural system, the data structure also contains the information on what kind of relationship the cluster has with the *reference cluster*. There are three different types of such relationship, they are:

- whether the cluster contains a reference cluster (if not a Leaf type),
  this relationship will be either TRUE or FALSE;
- the relationship between the Leaf clusters in the cluster and the *reference cluster*,
  this relationship will be either FORM-OVERLAP, FORM-RING or FORM-NOTHING;
- the minimum distance of the cluster from the *reference cluster*,
  this is the minimum distance from any of the leaf clusters contained in the cluster to the reference cluster.

The type of cluster, i.e. Root/Branch/Leaf, is also recorded in the data structure of the cluster. The numbers of deterioration events required to bring the structural ring representing the cluster to failure is also recorded in the data structure. It is used in searching the hierarchy to identify failure scenarios.

A node in a cluster

It was mentioned in previous section that a cluster, as a member object, has two nodes. In the cluster data structure, the two node objects are represented by two record structures. Each of them contains the following information:

- the type of the node;
- the string pattern;
- the numbers of deterioration events required to bring the node to failure;
- the numbers of basic joint contained in a complex joint;
- the other clusters to which this cluster connects in order to form a structural ring at the next higher level of description.

With the information inside the node record structure, the clusters and joints which form the structural ring at any given level of description can be then traced.

A cluster object is abstracted as that shown below:

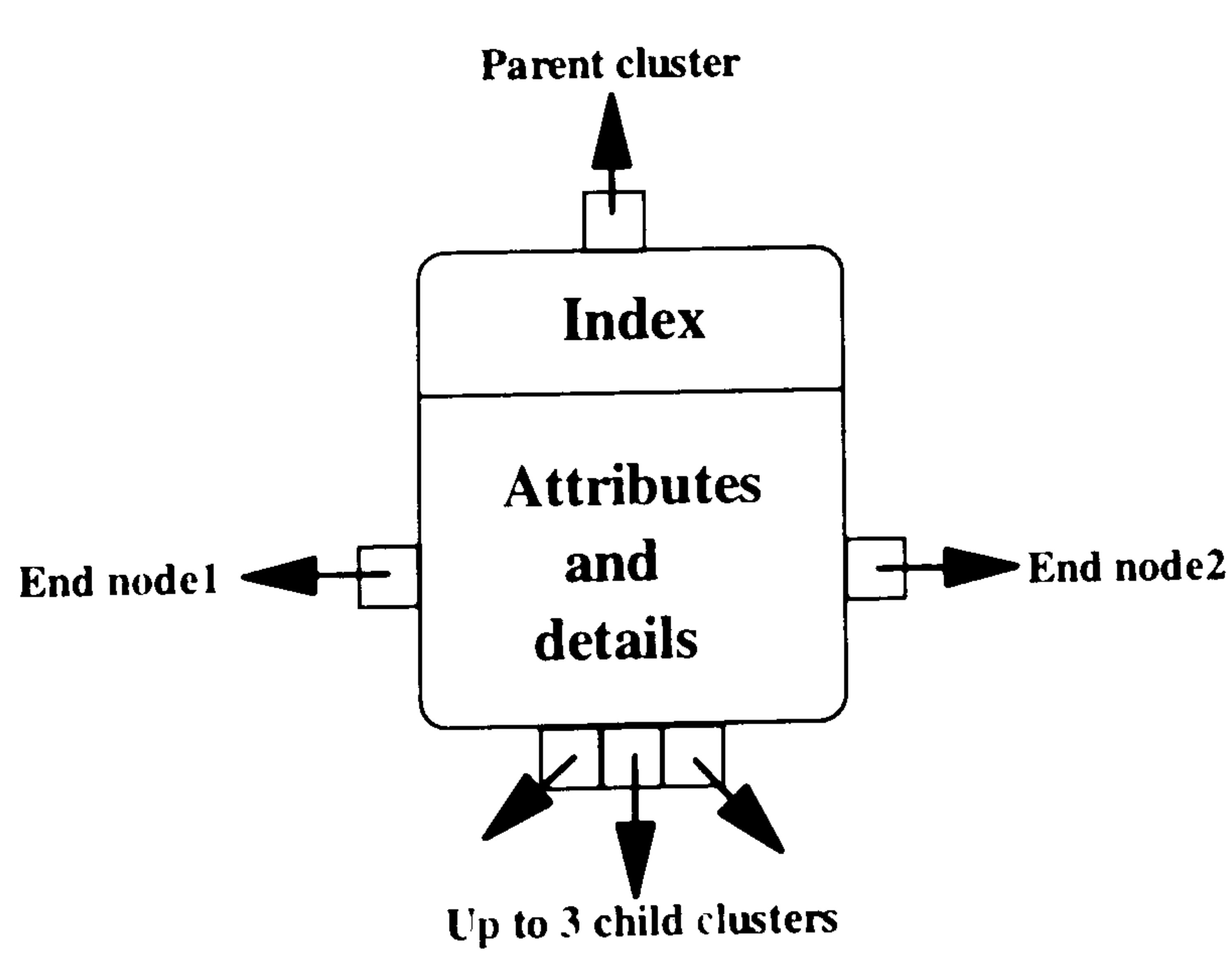


Figure 7.1 The data structure for a structural cluster

The complete data structure representing a cluster object is shown below:

- Type_of_cluster (*Root, Branch or Leaf*)
- Nos._of_deterioration_event_required
- String_pattern
- Structural_tightness
- Minimum_damage_demand
    - ⇒ Minimum_damage_demand_to_pin
    - ⇒ Minimum_damage_demand_to_cut
- Parent_cluster (*if not a Root*)
- Nos._children_clusters (*if not a Leaf*)
    - ⇒ List_of_children_clusters
- Node1
    - ⇒ Type_of_node
    - ⇒ String_pattern
    - ⇒ Nos._of_deterioration_event_required
    - ⇒ Nos._of_joints_contained (*if a complex joint*)
        - ◊ List_of_joints
    - ⇒ The other cluster it connects to in the next level of description
- Node2
    - ⇒ Type_of_node
    - ⇒ String_pattern
    - ⇒ Nos._of_deterioration_event_required
    - ⇒ Nos._of_joints_contained (*if a complex joint*)
        - ◊ List_of_joints
    - ⇒ The other cluster it connects to in the next level of description

Cluster_relationship_with_reference

⇒ Cluster_contatins_ref.

⇒ Leaf_relationship_with_ref.

⇒ Minimum_distance_from_ref.

7.5 Data Input

The data input is in the form of file input by writing the original data required to describe a structure into a data file. It can be created with any text editor provided it complies with the following format:

Total numbers of joint <return>

(For all the joints:)

Index of each joint <tab>

Co-ordinate---X of the joint <tab>

Co-ordinate---Y of the joint <tab> <return>

Total numbers of member <return>

(For all the members:)

Index of each member <tab>

Index of the joint as the member's first node <tab>

Index of the joint as the member's second node <tab>

The fixity at the first node <tab>

The fixity at the second node <tab> Modulus

The value of the member's Young's Modules <tab>

The area of the member's cross section <tab>

The member's second moment of area <tab> <return>

Total numbers of constraint <return>

(For all the constraint:)

Index of the constraint <tab>

Index of the joint at the constraint <tab>

Constraint in Horizontal direction (X) <tab>

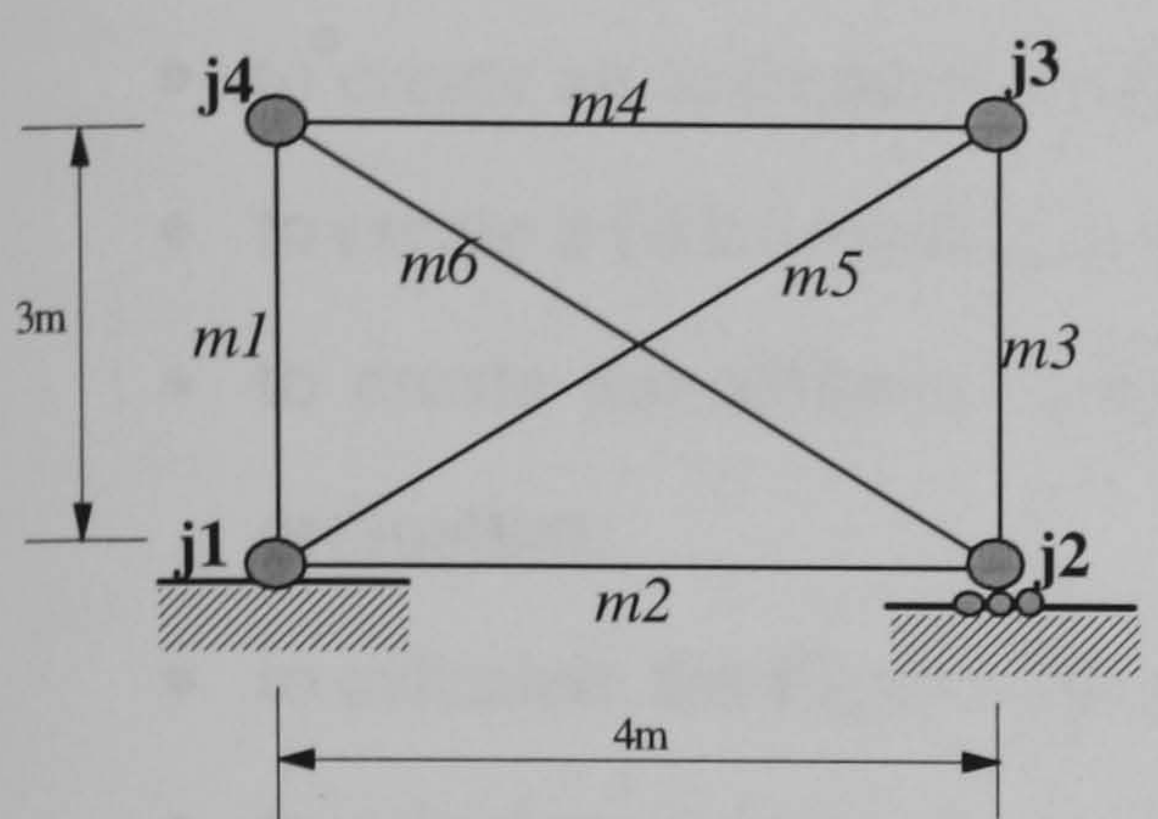
Constraint in Vertical direction (Y) <tab>

Constraint in Rotational direction (θ) <tab> <return>

In those required data listed above, note that:

- there is no spacing between the input data of joints, members and constraints;
- the fixity of a node is defined as the numbers of degrees of freedom maintained in it. This is:
  - *3* when a force can be transmitted along x, y and θ directions, i.e. fixed joint,
  - *2* when a force can be transmitted along x and y directions, i.e. pin joint,
  - *1* when a force can be transmitted along x or y directions only, i.e. roller joint,
  - *0* when no force can be transmitted through it, i.e. an open joint.
- the constraint in X, Y or θ direction will be:
  - 1 when there is a constraint along the specified direction, or
  - 0 when there is no such constraint along the specified direction.
- the units of all the dimensional values in the data file can be any of the user's choice provided they are consistent with the rest of the data.

Example 7.1:



Member	A (mm2)	I (mm4)
1	80	2000
2	90	2100
3	82.5	2050
4	92.5	2150
5	100	2500
6	120	3000

The data input file of the structure above will be as follows:

```
4
1    0     0
2    4000  0
3    4000  3000
4    0     3000
6
1    1    4    2    2    200   80    2000
2    1    2    2    2    200   90    2100
3    2    3    2    2    200   82.5  2050
4    3    4    2    2    200   92.5  2150
5    1    3    2    2    200   100   2500
6    2    4    2    2    200   120   3000
2
1    1    1    1    0
2    2    1    0    0
```

7.6 Data Preparation and Preliminary Calculation

In this stage, the original data is processed and calculated to generate the intermediate data to be used in later stages. Its purpose is:

- to create an association matrix of the graph model of the structure;
- to create a fixity matrix, in which the type of structural connection is specified;
- to create the stiffness matrices for all the structural components in global co-ordination;
- to calculate the Eigenvalues of the stiffness matrices;
- to calculate the minimum damage demand of each of the members;
- to generate the nodal connectivity information.

The Eigenvalues will be used in the cluster formation process to determine the well-formedness of structural clusters. The minimum damage demand will be written into the cluster data structure for further use in both cluster formation and searching stages. The nodal connectivity information will be used to help to determine the direction of cluster formation when having several clusters with identical other characteristics and conditions.

7.7 Hierarchy Formation

Cluster analysis is the process of classifying objects into subsets that have meaning in the context of a particular interest. In general, a cluster is defined as a subset of objects in a given set of objects which has some attributes more in common with each other than with those objects in other subsets.

If $O = \{ o_1, o_2, \ldots\ldots o_n \}$ is the given set of objects, a clustering is a type of classification imposed on O. The classification is based on the *likelihood* among the set of objects. Providing there is a suitable measure of this *likelihood*, cluster analysis technique can be applied widely in many fields.

Different clustering techniques are used according to different types of classification problem, as are the algorithms. It can be exclusive or non-exclusive, intrinsic or extrinsic and hierarchical or partitional.

As introduced in chapter 4, the method of cluster formation used in this particular research is the hierarchical clustering method.

A hierarchical clustering method is a procedure for transforming a set of entities into a sequence of nested partitions. The classification of the hierarchical clustering is both exclusive and intrinsic. At a level of description, one entity can only belong to one cluster. If { C_1, C_2, ... C_m } is the set of clusters at a level of description, then,

$$C_i \cap C_j = \Phi \qquad (\textit{for } i \textit{ and } j \textit{ from } 1 \textit{ to } m, \; i \neq j)$$

$$C_1 \cup C_2 \cup .... \cup C_m = O$$

The algorithm designed to generate the hierarchy using the graph model to represent the structural system is an agglomerative algorithm. It starts with the disjoint clustering, which places each of the objects in an individual *Leaf cluster.* This algorithm uses combination of measures firstly to evaluate the well-connectedness of the graph, i.e. the structural tightness of the structural ring and secondly to merge one or two other clusters into next partition. The process is then repeated to form a sequence of nested clusters, or *branch clusters.* Thus the number of clusters decreases as the sequence progresses until a single cluster containing all objects, called the conjoint clustering or *Root cluster*, is obtained.

The agglomerative algorithm includes the following steps in general:

(Assume: S is the set of all components in the structural system, k is the level of description in the hierarchy. S(1) contains same numbers of cluster as the total numbers of component, each of which is in a individual cluster. As k increment, the numbers of cluster in S(k) will decrease.)

Step 1: Begin with the disjoint clustering, which places each of the objects into a individual cluster, set k = 1.

Step 2: Form new S(k).

Starting from the most connected cluster.

If it is connected to other one or two other clusters and the

connectedness of the new formed cluster is greater than the previous one,

Refine S(k) by replacing those clusters with a cluster.

Step 3: If S(k) consists of every components in S,

Stop.

Else,

Set k = k + 1,

Go to **Step 2**.

The abstract algorithm for hierarchy formation in SAVE is:

Step 1: Place each of the members in the structural system in a Leaf cluster. Mark all Leaf clusters as "UNUSED".

Set level of description = 1.

Step 2: Create Reference Leaf clusters by placing a dummy member, which has both end joints as constraint joints, in a Reference Leaf cluster.

Step 3: Construct structural rings with the UNUSED Leaf clusters and find the one with highest measure of structural tightness.

Step 4: Create a new Branch cluster as the parent cluster of those chosen Leaf clusters and mark them as "USED". Mark the new cluster as "UNUSED".

Set level of description = level + 1.

Step 5: Use the new Branch cluster to construct new structural rings with other UNUSED Leaf clusters.

Step 6: If there are structural rings that can be formed,

Find the highest increase in the measure of the structural tightness.

If it is greater than previous maximum value:

Go to **Step4.**

Step 7: If no structural ring can be formed,

and If any Leaf cluster is still "UNUSED",

Go to **step3**.

Otherwise,

Move on to **step 8**.

Step 8: Construct structural rings with UNUSED Branch clusters.

If there are structural rings formed,

Find the one with the highest measure of structural tightness.

Create a new Branch cluster to replace those chosen Branch clusters and mark them as "USED".

Set level of description = level + 1.

Repeat **Step 8**.

Otherwise,

Move on to next step.

Step 9: Construct structural rings with all UNUSED and Reference Leaf clusters. Find the one with the highest measure of structural tightness.

Create a new Branch cluster to replace those chosen ones and mark them as "USED".

Set level of description = level + 1.

Step 10: If every Leaf and Reference Leaf clusters are not yet contained in one cluster,

Repeat **Step 9**.

Otherwise,

Define the last cluster as Root cluster.

Cluster formation completed.

The structure of this module is:

{

Initialise Leaf Clusters & Reference_Leaf Clusters,

(If all Leaf and Reference Leaf clusters are not in one single cluster)

{

Form clusters without participation of Reference_Leaf clusters

{

(If there exist 2-link-rings)

{

Find the best 2-link-ring

Write into a new branch cluster

Increase the level of description by 1

}

(If the 2-link-rings are exhausted)

{

Find the best 3-link-ring

Write into a new branch cluster

Increase the level of description by 1

}

}

(Otherwise)

Form clusters with participation of Reference_Leaf clusters

{

Repeat the same procedures as above.

}

}

(Otherwise)

{

Mark Root cluster

Stop

}

}

For example 7.1 (see Section 7.5), the result of hierarchy formation is, (the process of hierarchy formation is omitted)

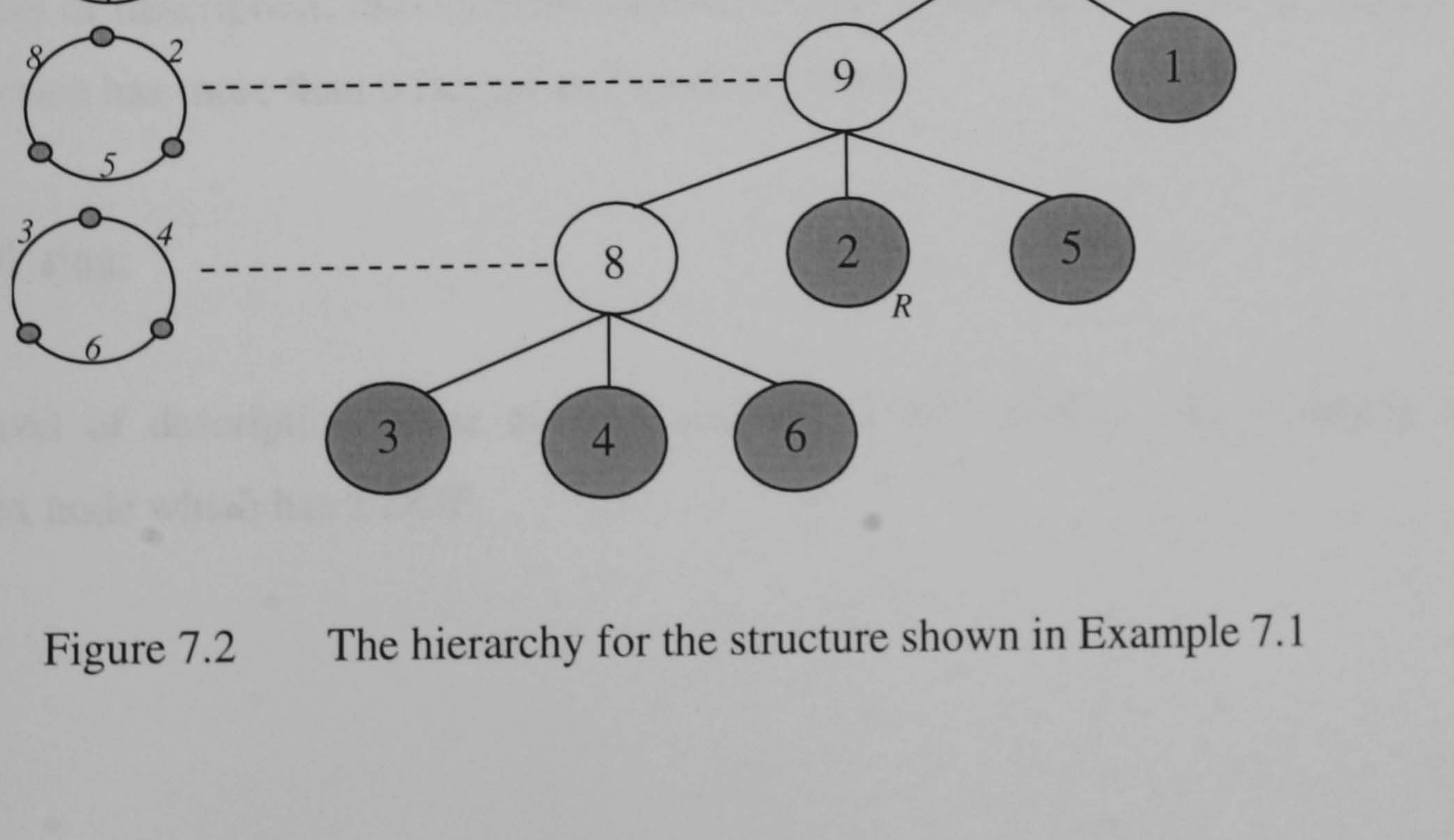


Figure 7.2 The hierarchy for the structure shown in Example 7.1

In the algorithms shown above, there are several points need to be explained in further detail. From the general algorithm to the specific ones used in SAVE, the expansions are due to the complexity in the process of formation of the hierarchy. The complexity is induced by the measure of the structural well-formedness and connectedness, which is a combination of several measures, and the further analysis which is to be performed on the hierarchy in later stages.

In the SAVE algorithm, the clustering is performed without the participation of Reference Leaf clusters first. Only after all possibilities for cluster formation without those Reference Leaf clusters are exhausted, are the same procedures then performed including Reference leaf clusters. This is a requirement for the future use of the hierarchy (see Section 7.8). In the future process of identification of failure scenarios, the Reference cluster is the one from which other clusters are disconnected to define total collapse. For earth bound structures, this will normally be the ground. For other type of structures it may be selected otherwise, such as in a space structure, it may be the living accommodation. Therefore, it is logical to form the clusters first without including the Reference. The Reference cluster is treated as a special part of the structure.

In the algorithm, the *2-link-ring* is given priority to the *3-link-ring*. As defined ealier in Section 3.4.2:

<u>A 2-link-ring:</u>

at a level of description, two clusters connect to each other with a simple or complex node which has more than 3 Degree of Freedom (DOF).

<u>A 3-link-ring:</u>

at a level of description, three clusters connect to one another with a simple or complex node which has 2 DOF.

If two structural rings, one 2-link-ring and one 3-link-ring, are of identical value of well-formedness, the 2-link-ring has higher structural tightness because it has a smaller number of nodes. Therefore, when in the presence of both types of structural ring, the 2-link-ring will be given priority.

The core part of the algorithm is the measure of the connectedness of the structural ring. The measure is implemented as a combination of rules concerning the structural tightness, toughness and the potential consequence. It can represent the quality of the structural cluster in terms of structural form (well-formedness and node connectivity), strength (damage demand) and effective damage (distance from Reference).

In finding the best structural ring of both types, the rules imposed are:

Find the cluster with highest Well-formedness;

(If Well-formedness identical)

Find the cluster with highest Damage Demand;

(If Damage Demand identical)

Find the cluster with highest node connectivity;

(If Node Connectivity identical)

Find the cluster with highest Distance from Reference.

7.8 Identification of Failure Scenarios

In section 7.7, the process of hierarchy formation (combining a set of primitive structural clusters into a single one cluster) has been introduced in detail. Using the hierarchy, the process of identification of various failure scenarios is mainly concerned with unzipping and searching the hierarchy for desired paths.

7.8.1 Minimal failure scenarios

As defined in chapter 6, the minimal failure scenarios include (1) the minimum demand scenario, and (2) the least well-formed cluster scenario. Both of the scenarios are associated with weakness in the form of the structural system.

As defined in section 6.4.5.1, the minimum demand scenario is the failure scenario which requires the least damage demand to cause the failure of a structural ring at a level of description. Since in a structural ring, the minimum damage demand of a "member" object at any level of description is the same as the minimum value of damage demand of its child clusters, at the end of the search, it will always lead to the primitive cluster/clusters which has the minimum damage demand. Hence the scenario can be identified either using the hierarchy or using the intermediate data which has been generated in the hierarchy formation stage.

If using the hierarchy, an exhaustive search from the top level is required. The primitive cluster/clusters with an overall minimum damage demand, together with an specified action which requires lower damage demand, will be identified as the minimum demand scenario. Or, simply, the same primitive cluster/clusters can be identified by using the array which stores the damage demand value for all the LEAF clusters, i.e. primitive clusters, and find the LEAF with minimum damage demand.

The least well-formed cluster scenario is the failure scenario which will cause least loss in the form of the structural system. On the other hand, it is the part of the structural system which is most loosely formed and connected. It is easy to spot using the hierarchy since the hierarchy has been built in the way that the more tightly connected or more well-formed clusters which comes in first, or in lower levels. Hence the clusters which are combined into the hierarchy at the end of the process are those which have least contribution to the overall well-formedness of the whole structure.

The searching process is as shown below:

===

{

from top level,
Search down the hierarchy;
Find the first structural ring with a LEAF or LEAVES in it;
if (*There is only one LEAF in it*)

```
        {
          Record the LEAF;
          for this LEAF
          {
               Compare all possible actions;
               Find the one with the minimum damage demand;
               Record the action;
          }
        }
        else if ( There are more than one LEAF )
        {
          Compare the damage demand of these LEAF clusters;
          Find the one with minimum damage demand;
          Record this LEAF.
          for this LEAF
          {
               Compare all possible actions;
               Find the one with the minimum damage demand;
               Record the action;
          }
        }
    }
```
==

7.8.2 Maximal failure scenarios

The maximal failure scenarios include (1) the total failure scenario, and (2) the maximum failure scenario. (Section 6.4.5.2)
The searching processes for both scenarios are similar except the scale of search may be different. In both processes, the target is to break up or unzip a single cluster (root or branch). The result of the search will be a series of actions which cause deteriorating events in a corresponding series of structural clusters which are capable of causing the break up of that single cluster with minimum required effort.

The hierarchy is used to find the maximal failure scenarios. In the searching process, several factors are important in the decision-making. They are:

- whether the failure state has been reached, and includes:
    - whether sufficient number of actions have been carried out. This involves the constant checking and modifying of the number of

deteriorating events required in each cluster when any of the action has been committed in the process. The failure state is not reached until all required number of deteriorating events has been used up;

    - whether any of the child clusters are independent clusters according to the cluster's relationship with the Reference cluster. If so, this particular child cluster must be failed before total failure can be achieved.

- whether the appropriate clusters which has minimum damage demand has been found for breaking, this includes:
    - whether any of the child clusters are dependent-clusters according to the cluster's relationship with the Reference cluster. If so, it will be ignored and the number of deteriorating events required will be modified accordingly when searching down to the next lower level;
    - whether the kinds of relationships between the child clusters and the Reference cluster are appropriate. Those child clusters, which have lower demand and higher effect on damaging the connection between the Reference and the structure, should be broken;
    - whether the child clusters which have the minimum damage demand have been identified. When other conditions are satisfied, the child cluster with the minimum damage demand is the one to be broken;
- whether the appropriate actions have been selected, and is related to:
    - the number of deteriorating events required to achieve failure, and
    - the damage demand of different actions.

The detail of the searching process for maximal failure scenarios is shown in the flow-charts in Figure 7.3 ~ Figure 7.6.

<u>Notions in Figure 7.4 ~ 7.6:</u>

n_p --- The number of deterioration events required of the parent cluster at a level of description.

n_c --- The number of deterioration events required of a child cluster at a level of description.

$n_{c(alloc)}$ --- The number of deterioration events required which is allocated to a child cluster from its parent cluster at a level of description.

A *Leaf* cluster is

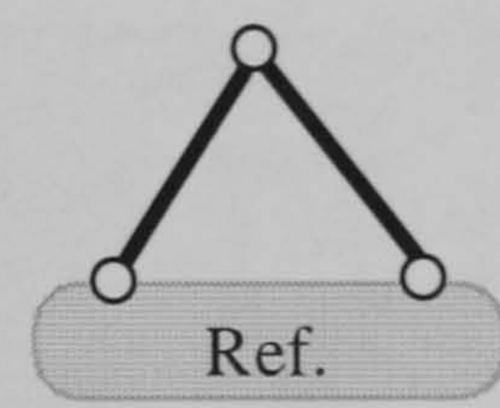
a primitive cluster which consists of a single structural member.
A Leaf cluster has no child cluster.
All structural members are Leaf clusters at the first level of description.

A *Ref. Leaf* cluster is part of the reference cluster.
There can be one or more Ref. Leaves in a structural system to form the one and only reference.

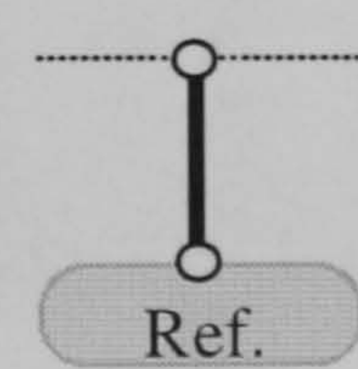
A cluster is said to have:

X-relationship with the reference, if
any Leaf cluster in this cluster shares the same node with the reference, and the node has 3 degrees of freedom, i.e. a fixed joint with the reference.

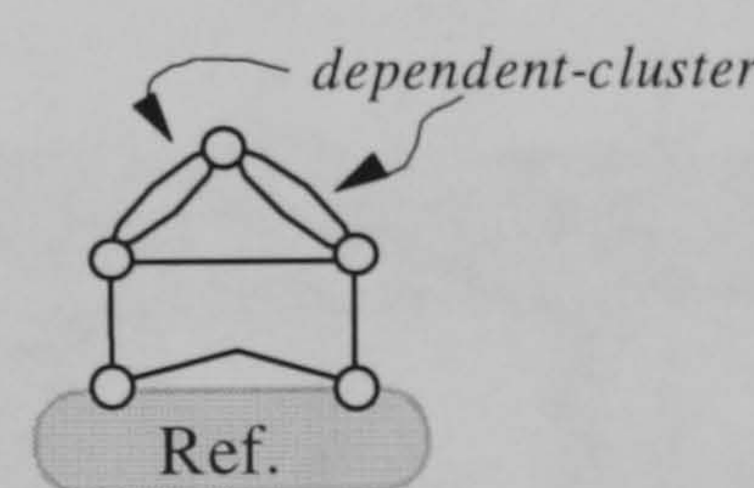
Y-relationship with the reference, if
any Leaf cluster in this cluster can, in presence of another Leaf cluster, form a structural ring with the reference.



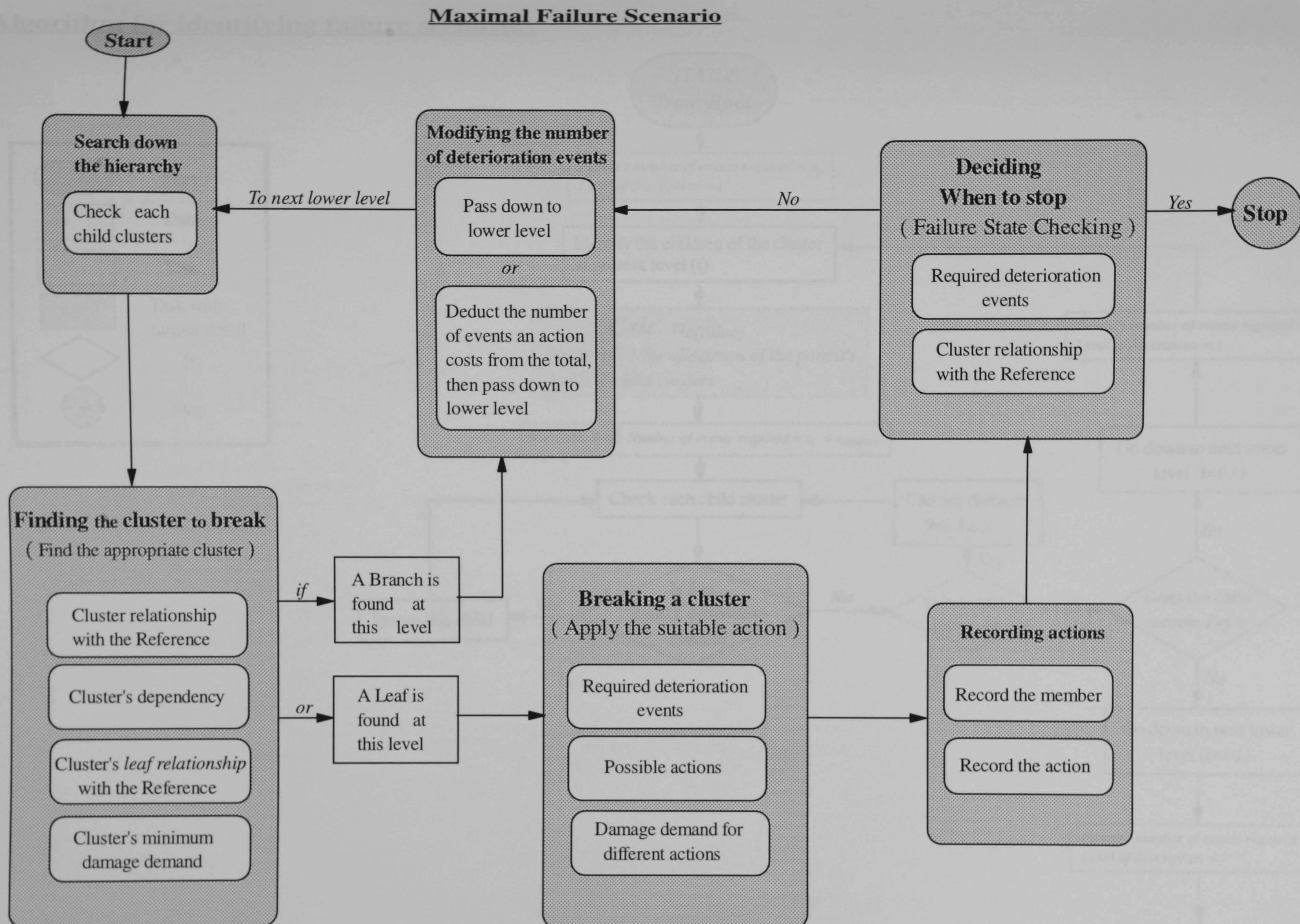
Null-relationship with the reference, if
none of the Leaf cluster in this cluster has any of the above relationships with the reference.



A child cluster is said to be
a dependent-cluster, if
this cluster only connects to the reference via its siblings.



A cluster is said to
contain the reference, if
any of its sub-clusters is a Ref. Leaf.


Maximal Failure Scenario
Start
Search down the hierarchy
Check each child clusters
To next lower level
Modifying the number of deterioration events
Pass down to lower level
or
Deduct the number of events an action costs from the total, then pass down to lower level
No
Deciding When to stop
(Failure State Checking)
Required deterioration events
Cluster relationship with the Reference
Yes
Stop
Finding the cluster to break
(Find the appropriate cluster)
Cluster relationship with the Reference
Cluster's dependency
Cluster's *leaf relationship* with the Reference
Cluster's minimum damage demand
if
A Branch is found at this level
or
A Leaf is found at this level
Breaking a cluster
(Apply the suitable action)
Required deterioration events
Possible actions
Damage demand for different actions
Recording actions
Record the member
Record the action

Algorithm for identifying failure scenarios

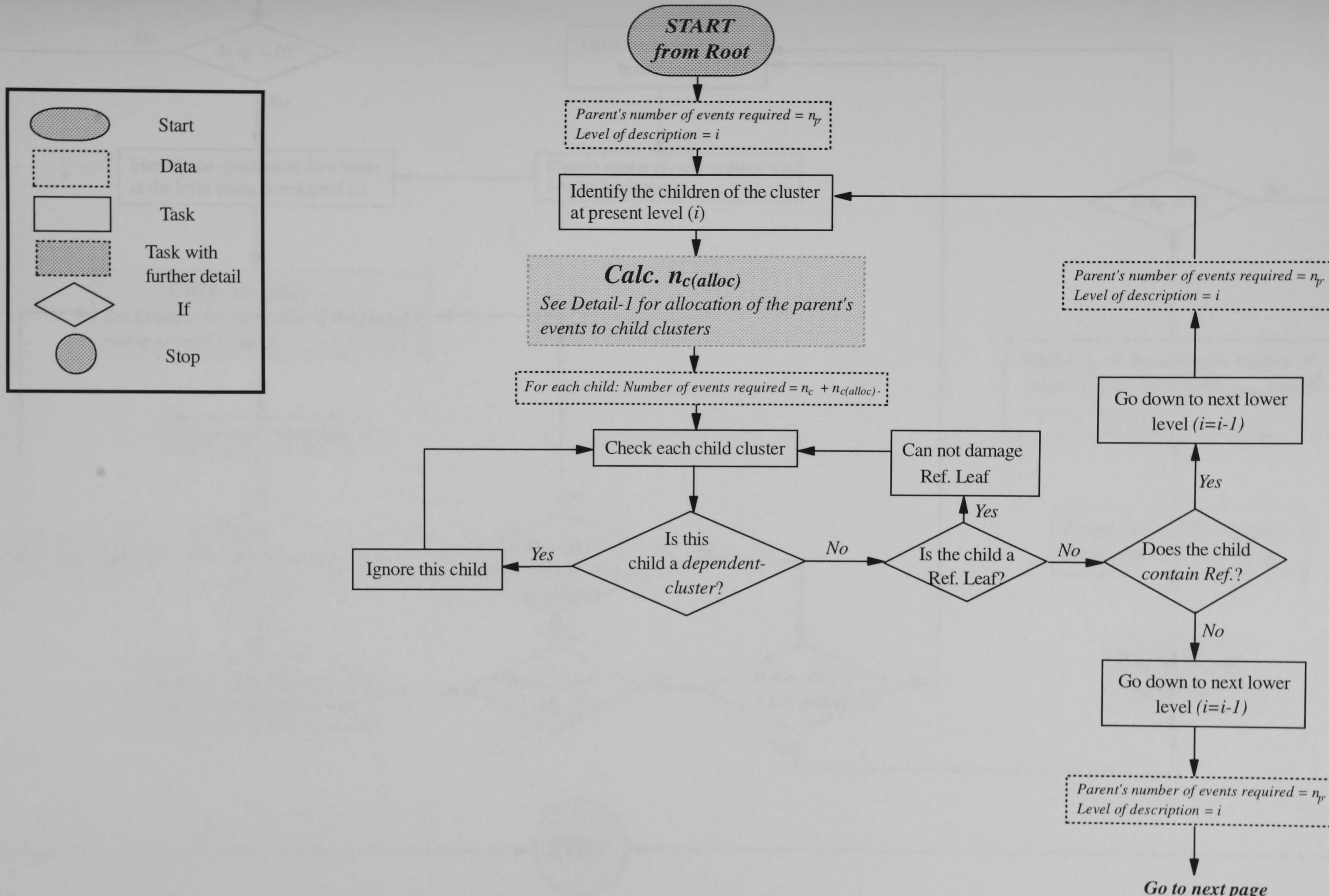


Figure 7.4 Detail of the unzipping process


From last page
Is $n_p = 0$?
Yes
No
Identify the children of the cluster at the level being considered (i)
Calc. $n_{c(alloc)}$
See Detail-1 for allocation of the parent's events to child clusters
Check each child cluster
Is this child a dependent-cluster?
Yes
Ignore this child
No
Find the child C^i_{min} at level i with min. damage demand.
Is this child C^i_{min}?
No
Yes
Has this child got X-relationship with the reference?
No
Yes
Is this child a Leaf cluster?
No
Yes
Go down to next lower level ($i=i-1$)
Cluster's number of events required = n_p.
Level of description = i
Break Leaf
See Detail-2
Record the Leaf cluster and the action which has been taken.
Modify n_p by deducting the number of events used by the action n_{action} from it.
$n_p = n_p - n_{action}$.
Is $n_p = 0$?
No
Yes
STOP

Detail-1:

Entry
Check the number of child clusters, (number-of-child)
Is number-of-child =
3 (a,b&c)
Is any one child (x) a Ref.-Leaf?
Yes
No
Find the child with min. damage demand. C_{min} of the other two children (y & z)
Is this child =?
z
y
(n_p -1) to z, and 0 to y.
2 (a&b)
Is any one child (x) a dependent-cluster?
Yes
0 to both a & b.
Exit
No
Does one child (x) contain Reference?
Yes
0 to x, and (n_p -1) to the other.
Exit
No
Is any child (x) a Ref-Leaf?
Yes
No
Do either of the children have Y-relationship with the Reference?
Yes
Find the child with min. damage demand. C_{min}
Is n_p>= 3?
Yes
3 to C_{min} and (n_p-3) to the other
No
1 to C_{min} and (n_p-1) to the other
Exit
No
Do any one child (x) have Y-relationship with the Reference?
Yes
Find the child with min. damage demand. C_{min}
No
Is this child =x?
Yes
(n_p -1) to x, and 0 to the other.
Exit
No
1 to x, and

Detail-1:

Figure 7.5 Detail-1 in Fiugure 7.4

Figure 7.5 Detail-1 in Fiugure 7.4

Detail-2:

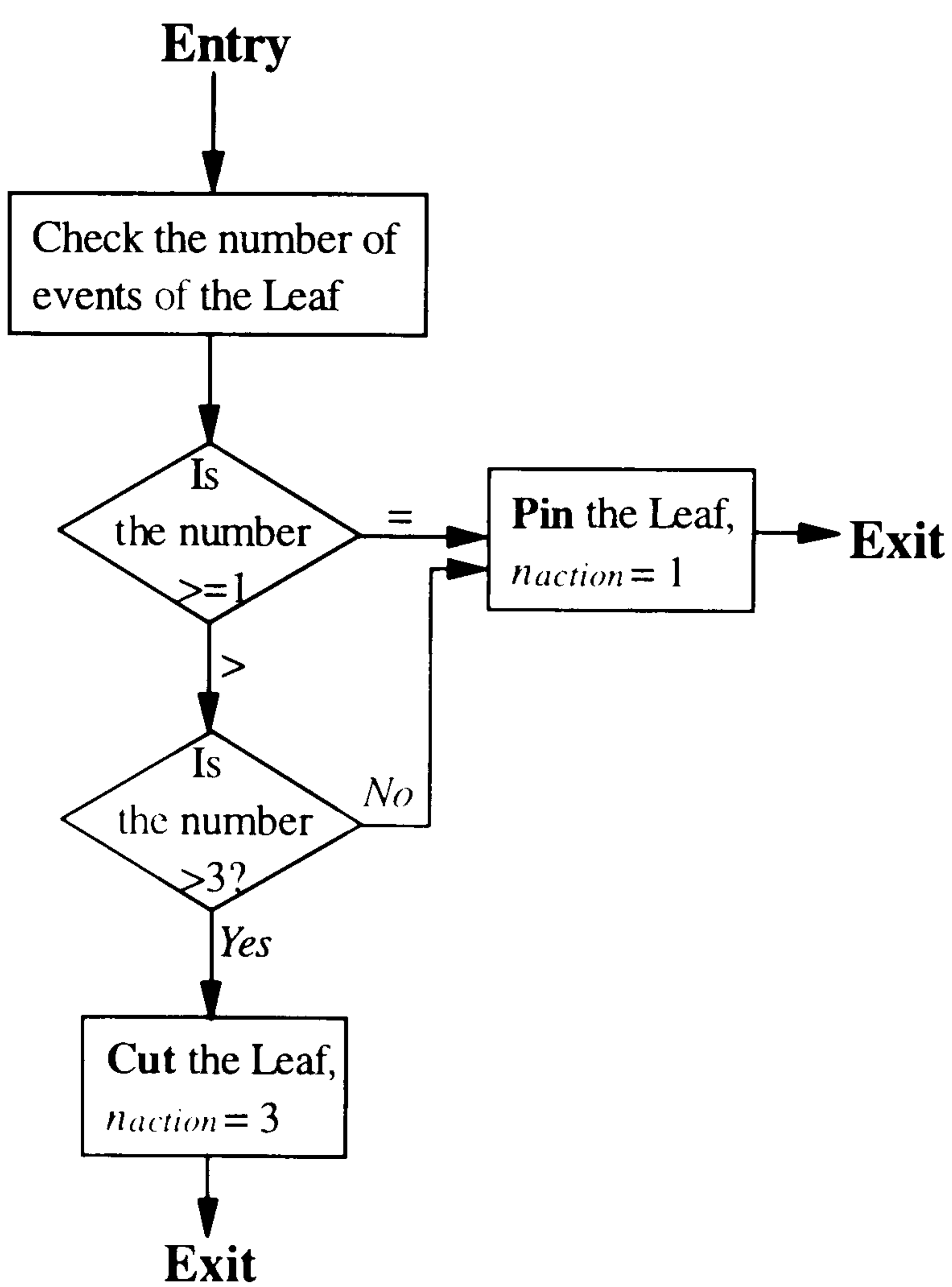


Figure 7.6 Detail-2 in Fiugure 7.4

Using the example in chapter 4 (see Section 4.6), the searching process is shown with the printout as part of the result file:

****

search_hierarchy(cluster 59, inherited_number_of_action 0)
search_down_overlap(cluster 59, total_number_of_action 2)
Cluster 15 is a dependent_cluster of cluster 58.
(Ignore Cluster 15, carry on search down the hierarchy.)

search_hierarchy(cluster 58, inherited_number_of_action 0)
search_down_overlap(cluster 58, total_number_of_action 2)
Cluster 16 is a dependent_cluster of cluster 57.
(Ignore Cluster 16, carry on search down the hierarchy.)

search_hierarchy(cluster 57, inherited_number_of_action 0)
search_down_overlap(cluster 57, total_number_of_action 2)
Cluster 17 is a dependent_cluster of cluster 56.
(Ignore Cluster 17, carry on search down the hierarchy.)

search_hierarchy(cluster 56, inherited_number_of_action 0)
search_down_overlap(cluster 56, total_number_of_action 2)
Cluster 55 is an independent structure & Cluster 52 has OVERLAP Leaf relationship
with the Reference, therefore can be a stand alone sub-structure.
Search down both clusters
No inherited actions for cluster 55 (because it contains Reference), inherit actions
2-1 = 1 are passed to cluster 52.
(This is done to make sure that the redundant number of deterioration events required
introduced by the participation of the Reference cluster can be taken into account
for Cluster 52, which does not contains Reference in the hierarchy.)

search_hierarchy(cluster 52, inherited_number_of_action 1)
search_down_ring(Cluster 52, total_number_of_actions 2)
no_reference_in_ring(12, 49, 50, 2)
Cluster 12 & 50 are dependant_clusters of cluster 49
(Ignore Cluster 12 & 50, carry on search down the hierarchy. Redundent actions 2-1
= 1 are passed down to Cluster 49.)

search_hierarchy(cluster 49, inherited_number_of_action 1)
search_down_ring(Cluster 49, total_number_of_actions 2)
no_reference_in_ring(10, 14, 48, 2)
Cluster 10 & 14 & 48 can all Form_Ring with the Reference Cluster
search_ring(10, 14, 48, 2)
Cluster 10 has the lowest damage demand
search_hierarchy(cluster 10, inherited_number_of_action 1)
pin_leaf(10)
Pin - member 10.
Modify event: 2-1=1. There is 1 event left, therefore, carry on searching.

search_ring(14, 48, 1)
Both 14 & 48 can FORM_RING with the Reference cluster
Cluster 48 has lower minimum damage demand
search_hierarchy(cluster 48, inherited_number_of_action 0)
search_down_ring(Cluster 48, total_number_of_actions 1)
no_reference_in_ring(3, 11, 47, 1)
Cluster 47 is a dependent_cluster of clusters 3 & 11
(Ignore Cluster 47, try Cluster 3 & 11)

Cluster 11 has lower minimum damage demand
Search down Cluster 11
search_hierarchy(cluster 11, total_number_of_action 1)
pin_leaf(11)
Pin - member 11.

search_hierarchy(cluster 55, inherited_number_of_action 0)
search_down_overlap(cluster 55, total_number_of_action 4)
Cluster 36 is a Ref_LEAF!
(Can't damage Reference, ignore Cluster 36, redundent actions 4-1 = 3 are passed down to Cluster 54.)

search_hierarchy(cluster 54, inherited_number_of_action 3)
search_down_overlap(cluster 54, total_number_of_action 4)
Cluster 34 is a Ref_LEAF!
(Can't damage Reference, ignore Cluster 34, redundent actions 4-1 = 3 are passed down to Cluster 53.)

search_hierarchy(cluster 53, inherited_number_of_action 3)
search_down_overlap(cluster 53, total_number_of_action 4)
Cluster 35 is a Ref_LEAF!
(Can't damage Reference, ignore Cluster 35, redundent actions 4-1 = 3 are passed down to Cluster 51.)

search_hierarchy(cluster 51, inherited_number_of_action 3)
search_down_ring(Cluster 51, total_number_of_actions 4)
no_reference_in_ring(40, 42, 44, 4)
Cluster 42 & 44 are dependent_clusters of cluster 40
(Ignore Cluster 42 & 44, redundent actions 4-1 = 3 are passed down to Cluster 40)

search_hierarchy(cluster 40, inherited_number_of_action 3)
search_down_overlap(cluster 40, total_number_of_action 4)
Both clusters can Form Overlap with the Reference cluster.

search_overlap(33, 39, 4)
Both 33 & 39 can FORM_OVERLAP with the Reference cluster
Need to damage both clusters, however,
Cluster 33 has lower minimum damage demand
Allocate 3 redundent actions to it,
search_hierarchy(cluster 33, inherited_number_of_action 3)
cut_leaf(33)
Cut - member 33.
Modify event: event left is 4-3 = 1.

search_hierarchy(cluster 39, inherited_number_of_action 0)
search_down_overlap(cluster 39, total_number_of_action 1)
Cluster 38 is a dependent_cluster of cluster 20.
(Ignore Cluster 38, damage cluster 20)

search_hierarchy(cluster 20, inherited_number_of_action 0)
pin_leaf(20)
Pin - member 20.
Event left is 1-1 = 0.
Searching completed.
****

7.9 Conclusions

In this chapter, the structure and algorithm of the computer program *SAVE* has been introduced. *SAVE* is a computer program which is designed to demonstrate Structural Vulnerability Theory. It comprises five modules, which are:

- Data Input,
- Data Preparation and Preliminary Calculation,
- Hierarchy Formation,
- Search for Minimal Failure Scenarios,
- Search for Maximal Failure Scenarios.

The graph property of the structural system is represented with a commonly used representation --- the association matrix (or otherwise called adjacency matrix). The structural properties are abstracted in the following form:

- a fixity matrix,
- an array of stiffness matrices of structural components,
- an array of Eigenvalues of the stiffness matrices,
- an array of damage demand of the members, and
- an array of nodal connectivity of the joints.

A data structure has been designed to allow effective representation of the problem model and efficient computation and information manipulation.

A structural cluster is represented as a record structure which contains information about its index, type, attributes and details, relationship with other structural clusters and detail of its connection with other clusters.

The algorithm used in hierarchy formation is agglomeritive. In this process, individual *Leaf* clusters are organised into *Branch* clusters step by step, according to the clustering criteria (see chapter 4), until a *Root* cluster is found.

An example has been given to illustrate the data input and the hierarchy formation.

The process of identification of various failure scenarios is mainly concerned with unzipping and searching the hierarchy for desired paths.

The algorithm for unzipping the hierarchy has been shown in a set of detailed flow-charts. To demonstrate its functions as fully as possible, an example from chapter 4 was used and the search was conducted. From the results, the process of searching was clearly traced.

Chapter 8

*S*tructural *A*nalysis for *V*ulnerability *E*stimation

8.1 Objectives

The objectives of this chapter are to:

- test structural vulnerability theory and its implementation against examples;
- examine closely the operation of key parts of the algorithm;
- use a variety of different type of structures to illustrate how the program *SAVE* works and to test its application.

8.2 Introduction

In previous chapters, the theoretical background of structural vulnerability analysis was presented (Chapter 3-6). An algorithm was described that was designed to carry out the analysis effectively (Chapter 7). A computer program *SAVE* (*S*tructural *A*nalysis for *V*ulnerability *E*stimation) has been developed in this research.

In this chapter, we will use a variety of different types of structure to illustrate the operation of the program. Meanwhile, the operation and implementation of the key parts of structural vulnerability theory will be closely examined.

The two dimensional structures used as examples include the following types:

- Truss structures,
- Frame structures,
- Mixture of truss and frame structures.

The program is able to distinguish a structure from a mechanism or a structure which inherently relies on the ground, or in other words, has insufficient internal stiffness to stand on its own. In the latter case, the vulnerability analysis does not apply. Such examples will be shown in Appendix-3.

8.3 Truss Structures

Two truss structures are chosen in this section. The structures Truss-1 and Truss-2 are illustrated in Figure 8.2 and Figure 8.4 correspondingly. The details of the two structures are given in Table 8.1 - 8.3 and Table 8.7 - 8.9 respectively.

The processes of cluster formation for both structures are shown in the step-by-step form (see Table 8.4 and Table 8.10). The step-by-step illustration shows the details of clustering criteria and precisely which one governs a selection at each step.

Following the cluster formation process, the hierarchical representation of the two trusses are shown in Figure 8.3 and Figure 8.5. With the hierarchy, the structural rings associated with each of the structural clusters are shown. Also shown is the number of deterioration events required for each structural ring at a particular level of description. The information contained in the hierarchical representation are shown in Figure 8.1.

The result of vulnerability analysis for Truss-1 and Truss-2 are shown in Table 8.5 - 8.6 and Table 8.11 - 8.13 correspondingly.

For Truss-2, the process of searching for total failure scenario is demonstrated, the purpose of which is to illustrate the algorithm shown in Chapter 7 (see Figure 7.3-7.5) at work.

Finally, some of the interesting failure scenarios of Truss-2 are listed in Table 8.13.

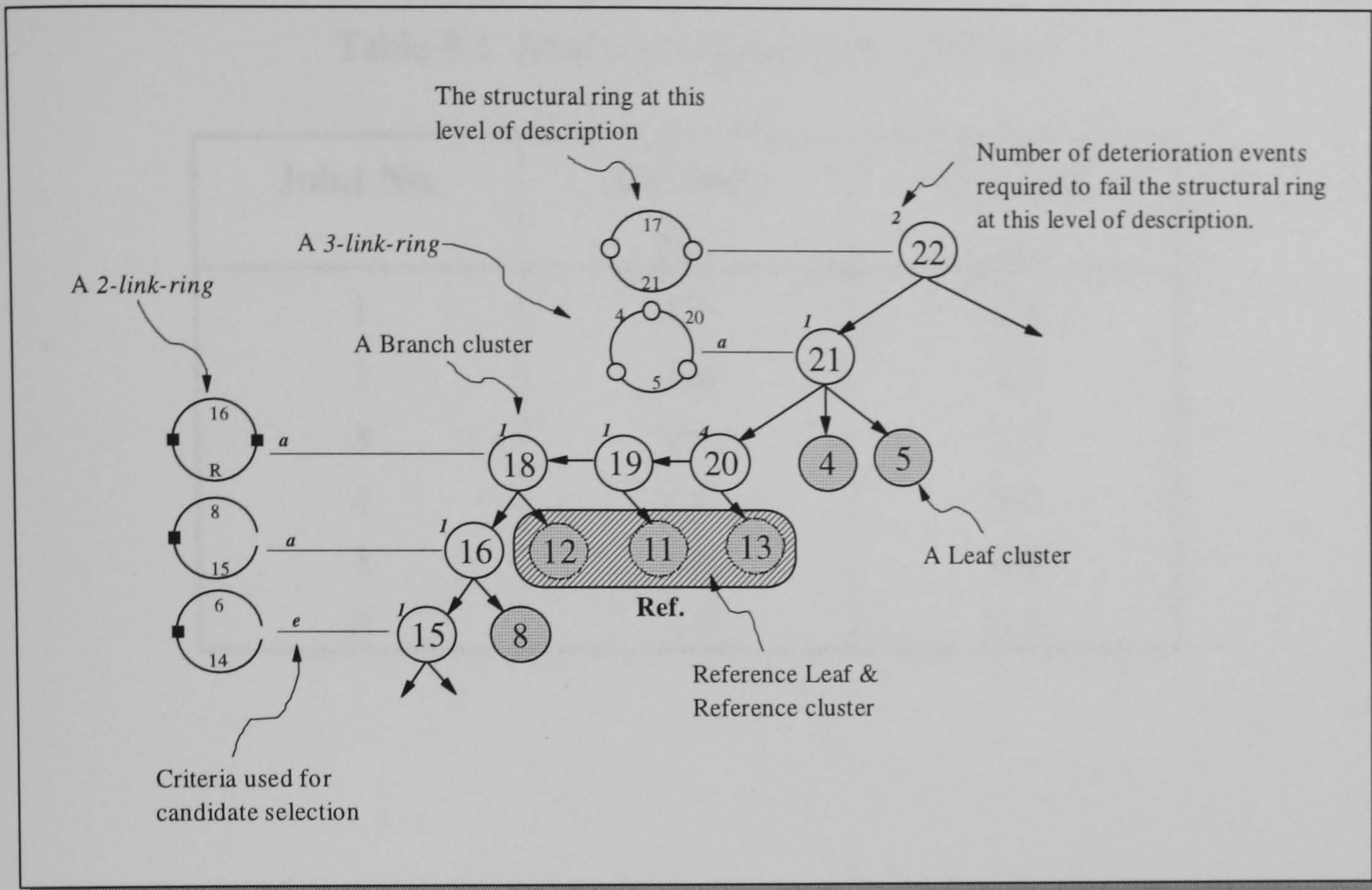


Figure 8.1 Information in the hierarchical representation

Truss-1:

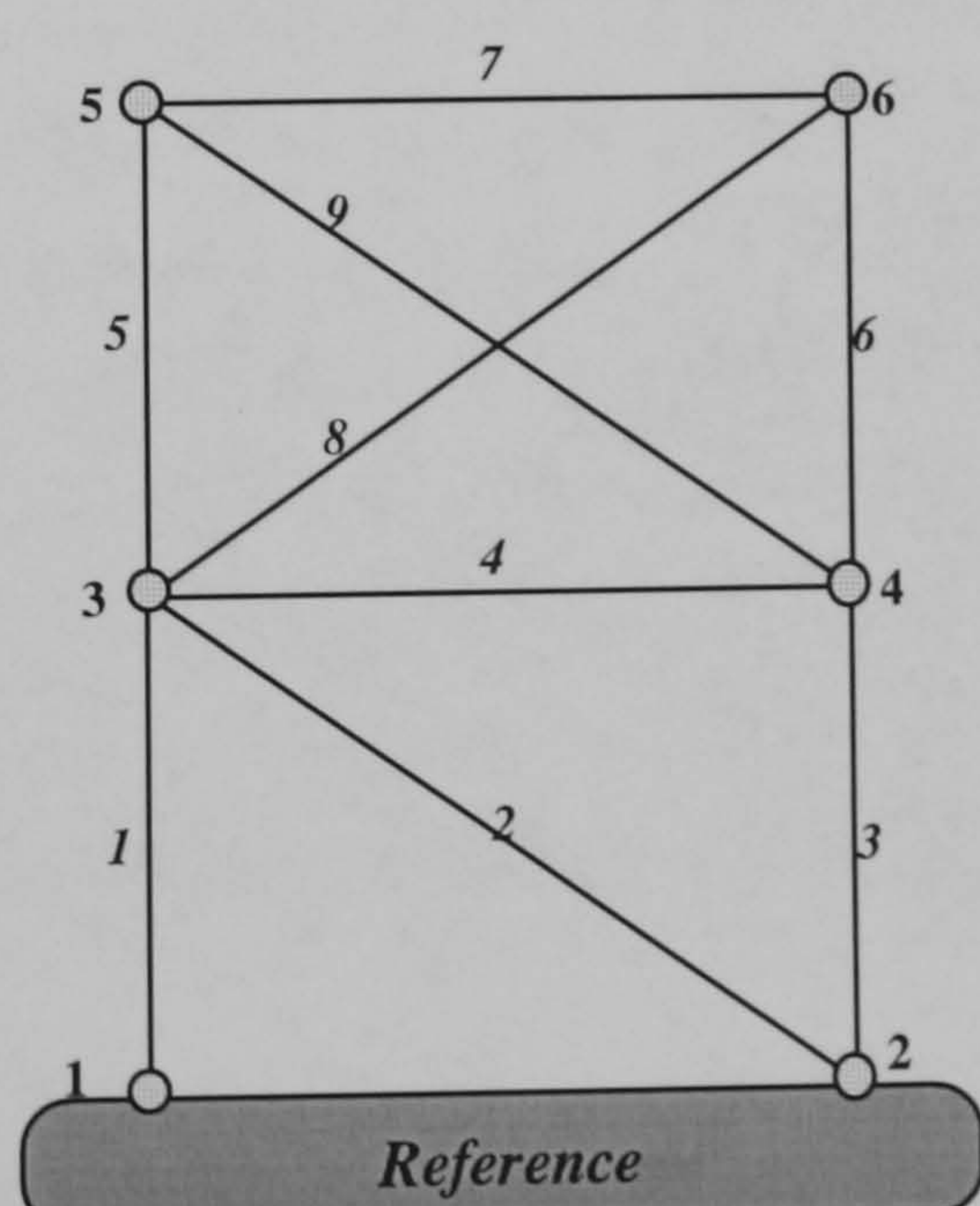


Figure 8.2 The structure --- Truss-1

Table 8.1 Joint co-ordinate table of Truss-1

Joint No.	X Co-od. (m)	Y Co-od. (m)
1	0.0	0.0
2	4.0	0.0
3	0.0	3.0
4	4.0	3.0
5	0.0	6.0
6	4.0	6.0

Table 8.2 Member properties table of Truss-1

Member No.	End Joints		End Fixity		E	A	I
	Joint-1	Joint-2	Joint-1	Joint-2	(n/m^2)	(m^2)	(m^4)
1	1	3	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}
2	3	2	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}
3	2	4	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}
4	3	4	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}
5	3	5	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}
6	4	6	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}
7	5	6	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}
8	3	6	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}
9	5	4	0	0	205×10^6	4.74×10^{-3}	1.17×10^{-4}

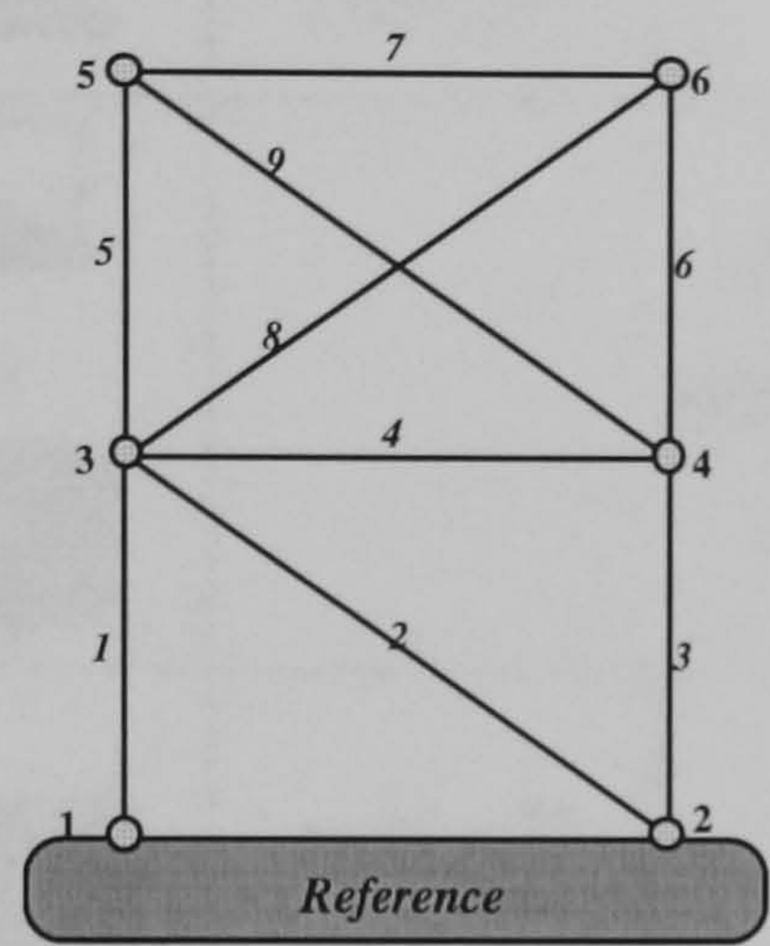
Table 8.3 Constraint condition of Truss-1

Constraint No.	Joint No.	x	y	θ
1	1	1	1	0
2	2	1	1	0

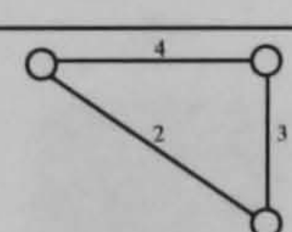
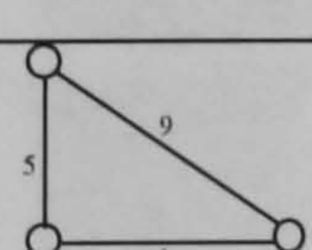
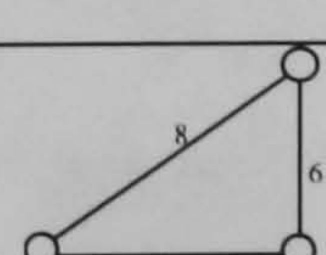
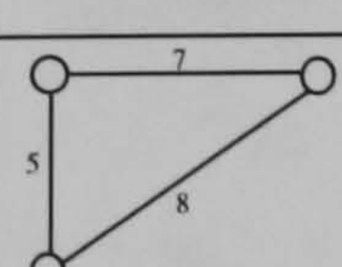
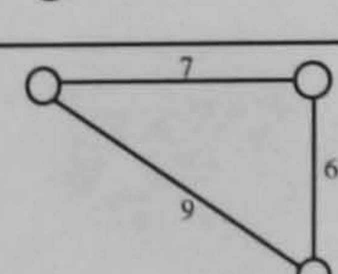
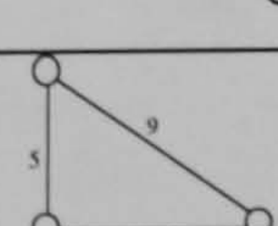
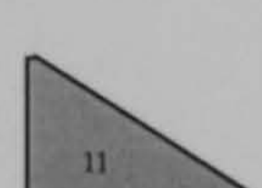
Table 8.4 Step-by-step cluster formation --- Truss-1

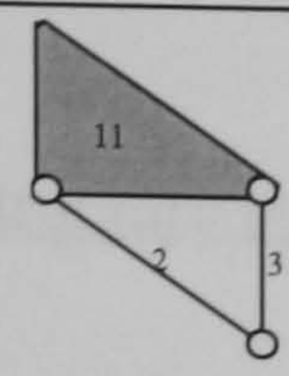
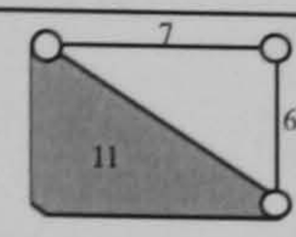
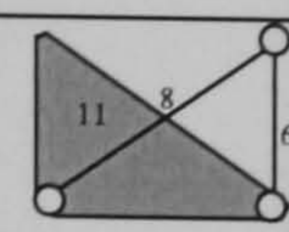
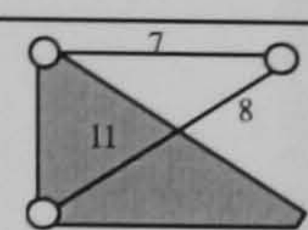
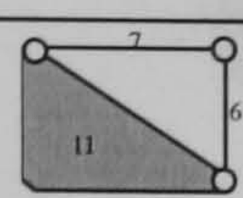
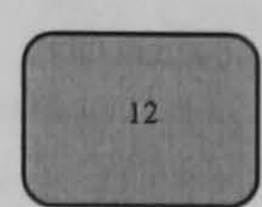
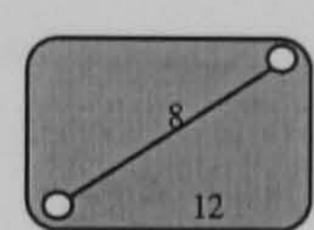
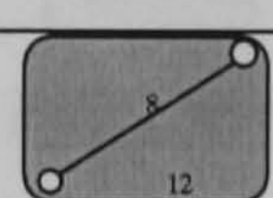
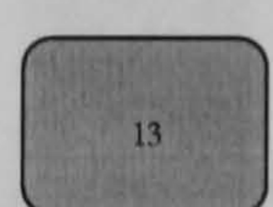
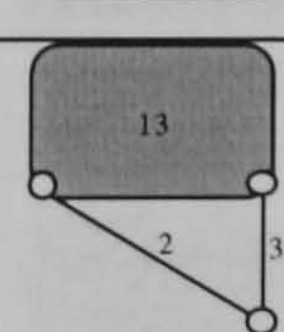
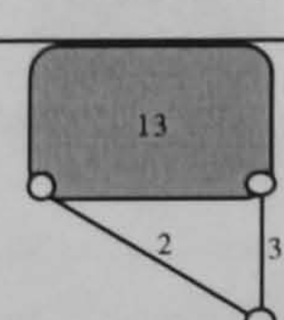
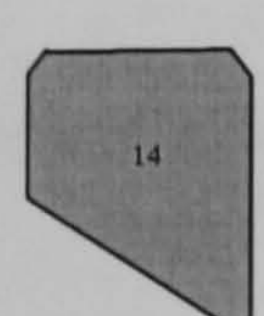
Steps	Components	Cluster Formed	Well-formedness	Damage demand	Nodal Degree	Distance

<u>The structure:</u>

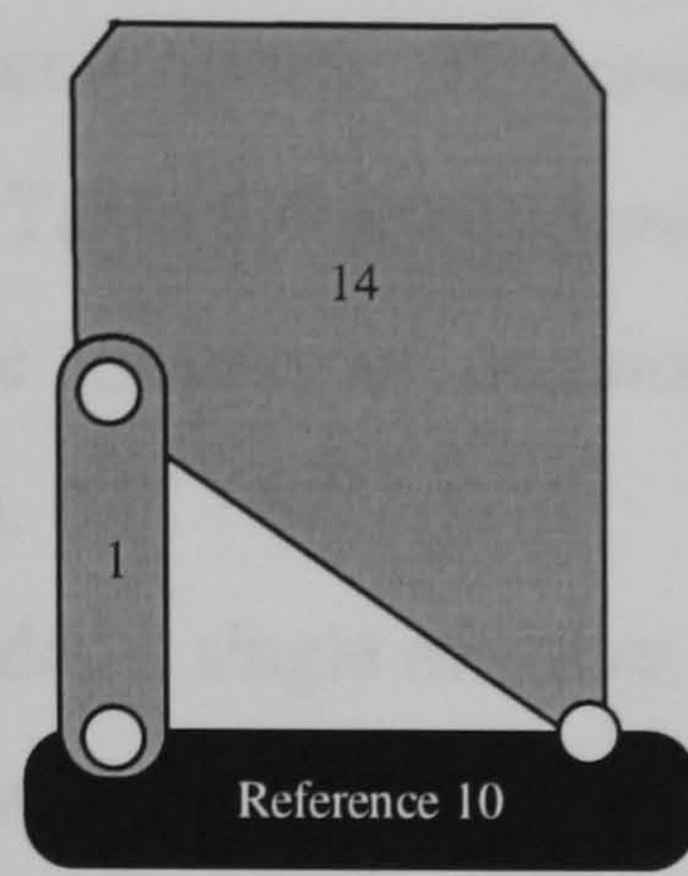


----- Initial Clustering Stage -----

Steps	Components	Cluster Formed	Well-formedness	Damage demand	Nodal Degree	Distance
	2+3+4		4.53×10^{10}	19.7	11	3.0
	4+5+9		4.53×10^{10}	19.7	**12**	9.0
	4+6+8		4.53×10^{10}	19.7	12	9.0
Step 1	5+7+8		4.53×10^{10}	19.7	10	-
	6+7+9		4.53×10^{10}	19.7	11	3.0
	Forming Cluster 11	 as 	*Selection Criteria*: Higher nodal connectivity			

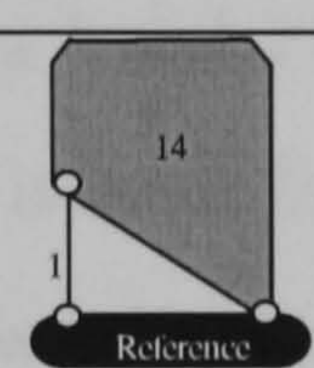
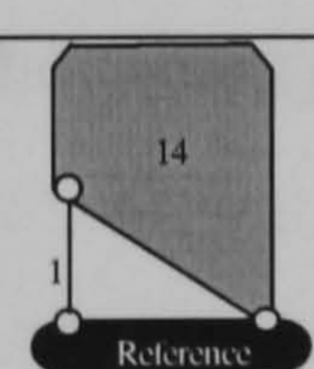
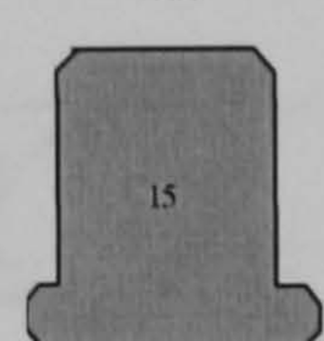
Step 2	2+3+11		8.8×10^{10}	19.7	14	3.0
	6+7+11		$\mathbf{10.7\times10^{10}}$	19.7	15	12.0
	6+8+11		8.8×10^{10}	-	-	-
	7+8+11		7.65×10^{10}	-	-	-
	Forming Cluster 12	 as 	*Selection Criteria*: Higher well-formedness			
Step 3	8+12		$\mathbf{13.4\times10^{10}}$	19.7	15	6.0
	Forming Cluster 13	 as 	Selection Criteria: One choice. Increased well-formedness.			
Step 4	2+3+13		$\mathbf{15.9\times10^{10}}$	19.7	17	3.0
	Forming Cluster 14	 as 	Selection Criteria: One choice. Increased well-formedness.			
End of Initial Clustering Stage						

The structure at the end of Initial Clustering Stage:



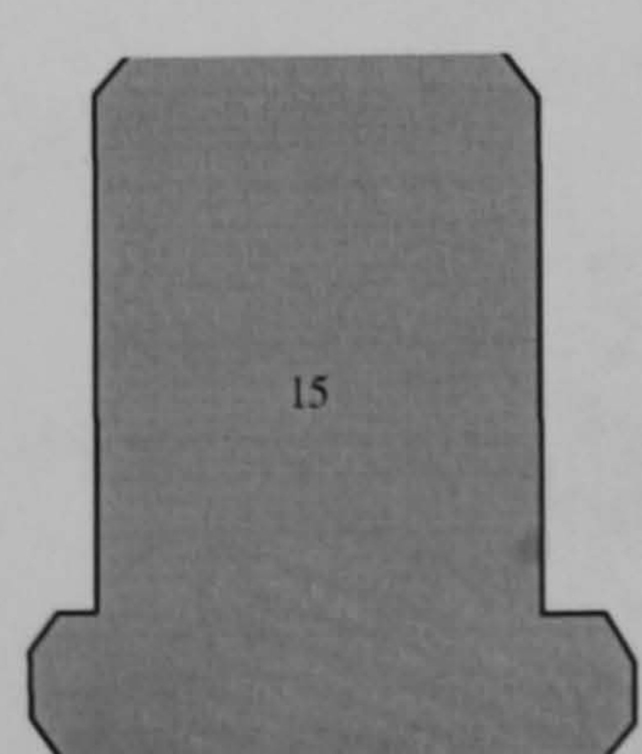
No secondary cluster identified.

----- Reference Clustering Stage -----

	1+10+14		15.9×10^{10}	19.7	18	0.0
Step 5	Forming Cluster 15	 as 	*Selection Criteria*: One choice.			

End of Reference Clustering Stage

The structure at the end of Reference Clustering Stage:



Cluster Formation Completed.

At the centre of vulnerability analysis is the hierarchical model to represent the structure of the form of a structural system. The process of cluster formation is a very important part of the program. Table 8.4 shows how the cluster formation module of SAVE works and sets out the process of decision making governed by the new ordered set of clustering criteria.

The old clustering criteria included a single measure, that of well-formedness. In step 1, the five candidates for cluster formation all have the identical well-formedness. With the old criteria, one candidate would be selected randomly from these five choices since it was unable to discriminate between them any further. Using the new criteria, further measures are used when a single measure fails to discriminate a candidate. In this particular case, two candidates are selected from the original set of five using the new criteria (nodal connectivity) and they are identical in all accounts. Finally, a random choice is made to select one.

The new criteria, which is an ordered set of criteria related to the structural form, has significantly improved the quality and resolution of candidate selection from the previous algorithm.

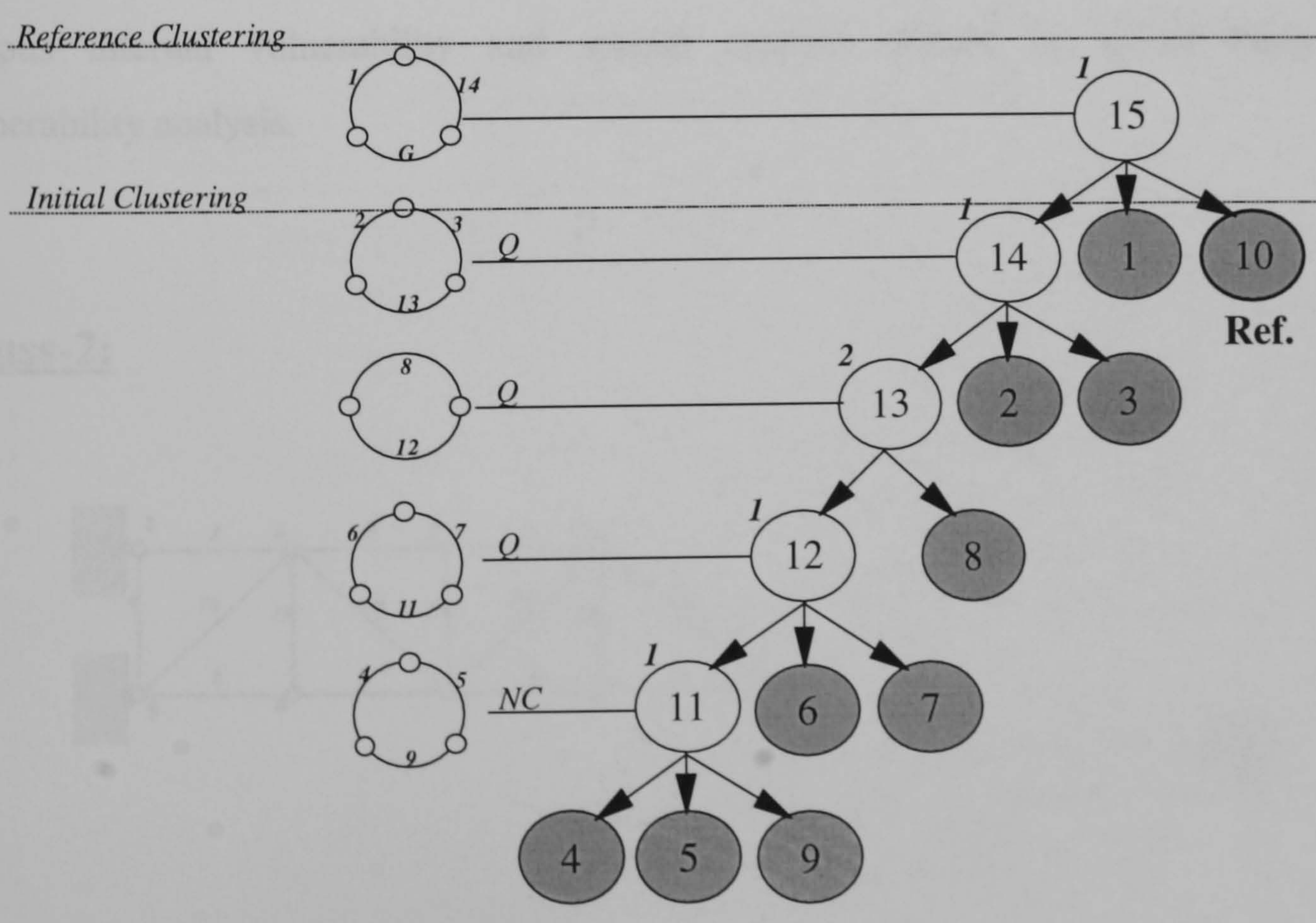


Figure 8.3 Hierarchical representation of Truss-1

Table 8.5 Minimal failure scenarios for Truss-1

Min. demand failure scenario	The least well-formed cluster scenario
To damage: Cluster 2 or Cluster 8 or Cluster 9	Cluster 1

The Minimum demand failure scenario identified that Cluster 2, 8 and 9 can be damaged most easily. Cluster 1 which has the least connection with other part of the structure is identified as the least well-formed cluster.

Table 8.6 Maximal failure scenarios for Truss-1

Total failure scenario	The Maximum failure scenario
To form a pin in Cluster 2.	Same as Total failure scenario.

In this case, the maximum failure scenario is the same as the total failure scenario. Comparing Table 8.6 and Table 8.5, the total/maximum failure scenario happens to be one of the minimum demand failure scenario. This is a case where the structure has a serious internal vulnerability and special concern should be raised from the vulnerability analysis.

Truss-2:

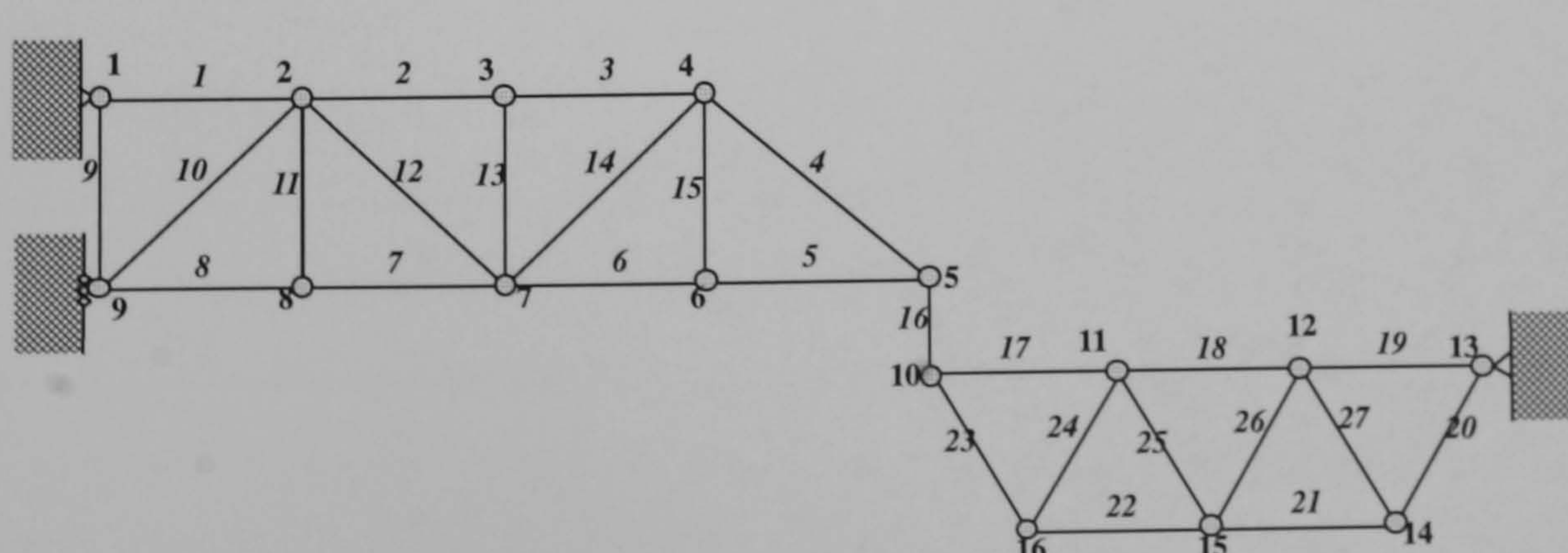


Figure 8.4 The structure --- Truss-2

Table 8.7 Joint co-ordinate table of Truss-2

Joint No.	X Co-od. (m)	Y Co-od. (m)
1	0.0	9.0
2	3.0	9.0
3	6.0	9.0
4	9.0	9.0
5	12.0	4.0
6	9.0	4.0
7	6.0	4.0
8	3.0	4.0
9	0.0	4.0
10	12.0	3.0
11	16.0	3.0
12	20.0	3.0
13	24.0	3.0
14	22.0	0.0
15	18.0	0.0
16	14.0	0.0

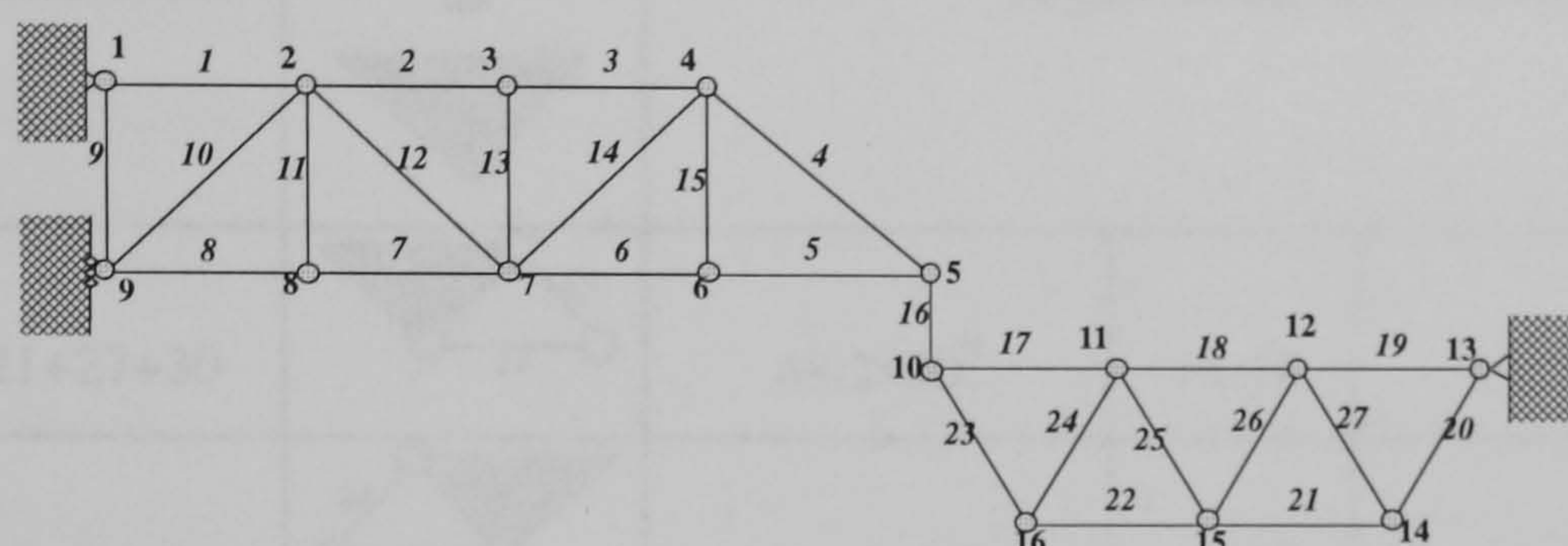
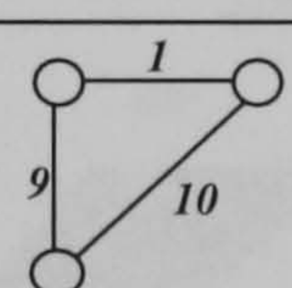
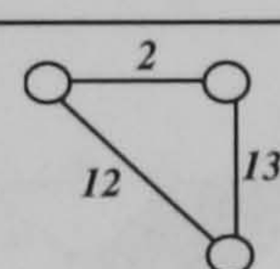
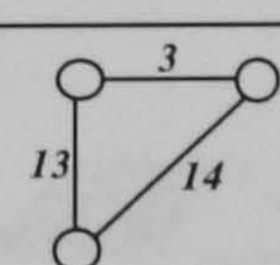
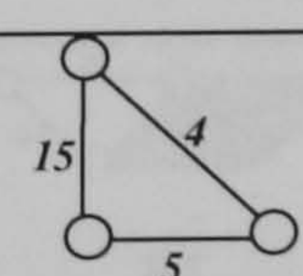
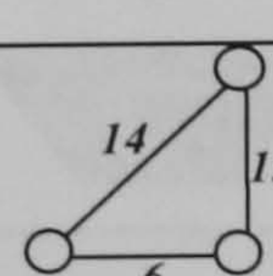
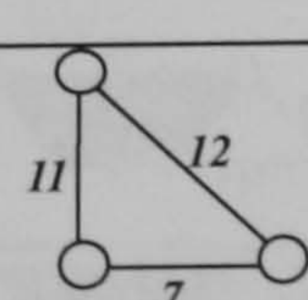
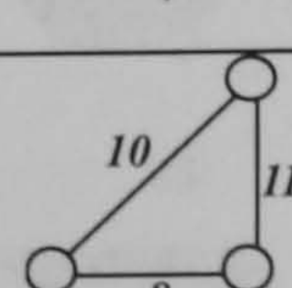
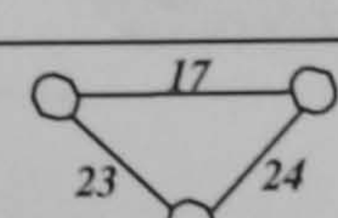
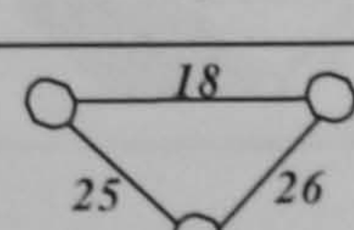
Table 8.9 Constraint condition of Truss-2

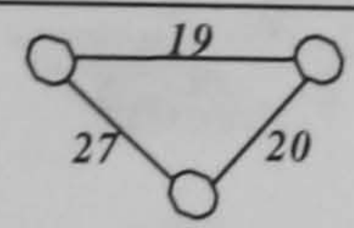
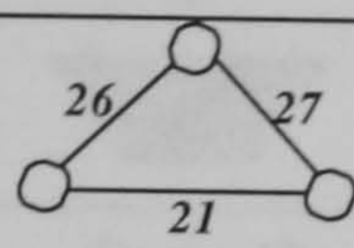
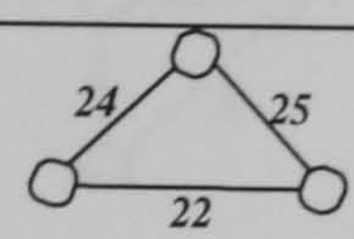
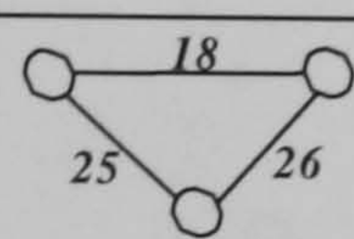
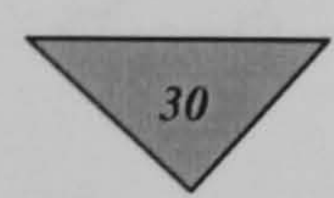
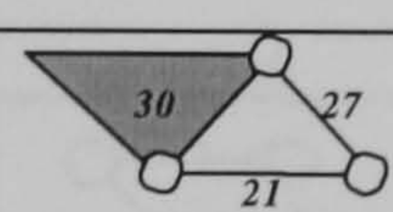
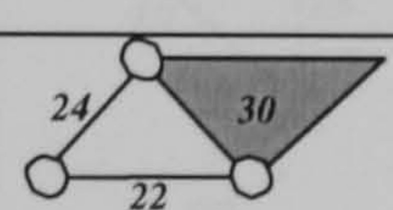
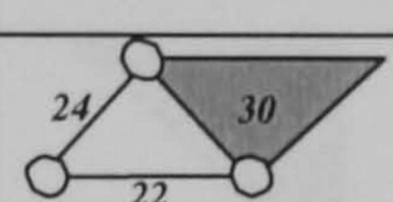
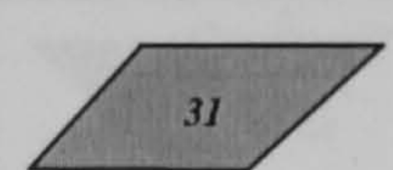
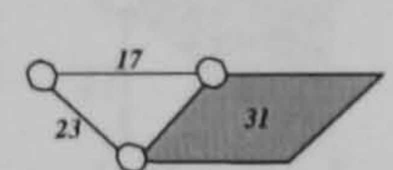
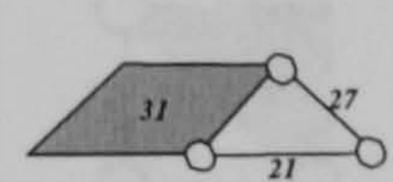
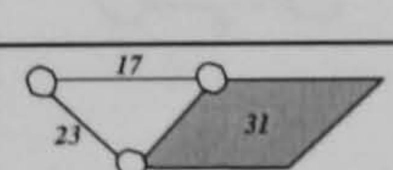
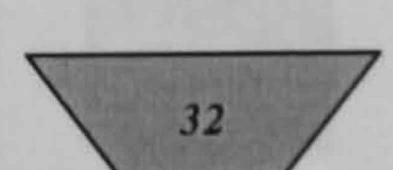
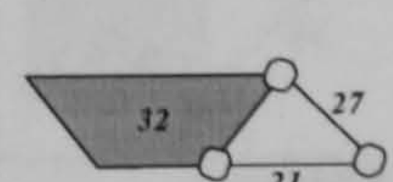
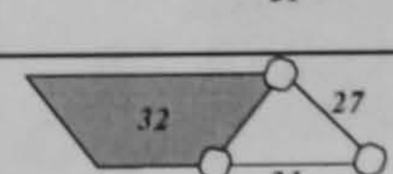
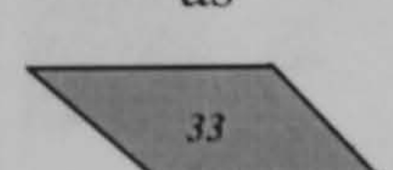
Constraint No.	Joint No.	x	y	θ
1	1	1	1	0
2	9	1	0	0
3	13	1	1	0

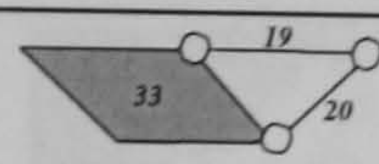
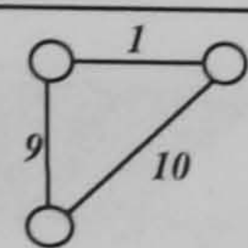
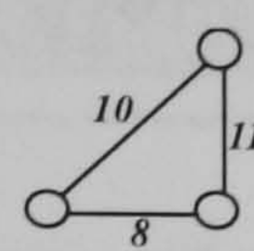
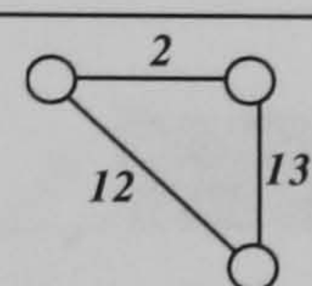
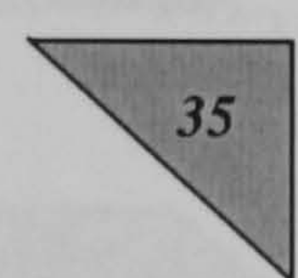
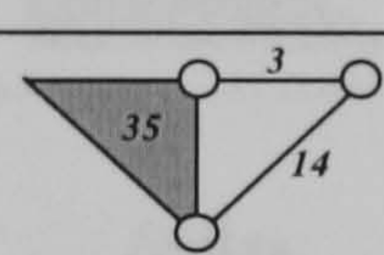
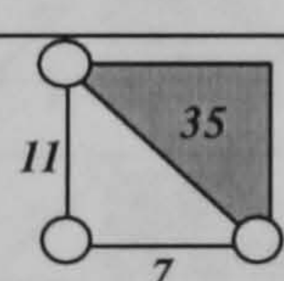
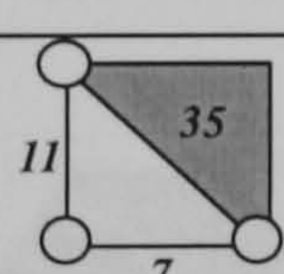
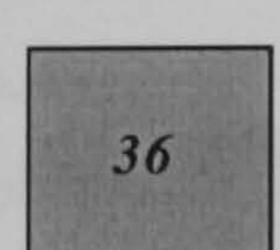
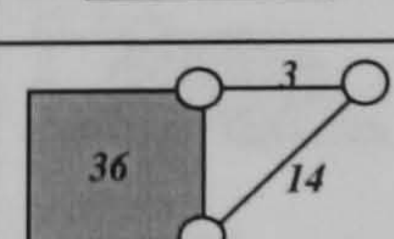
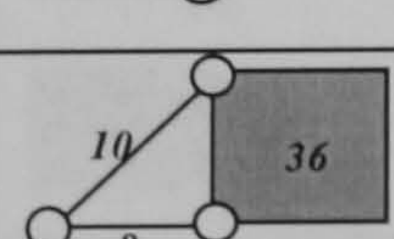
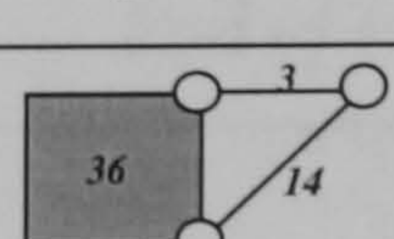
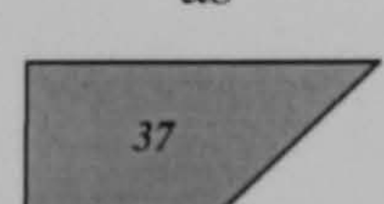
Table 8.8 Member properties table of Truss-2

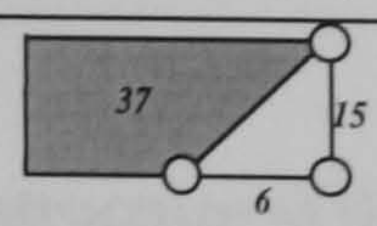
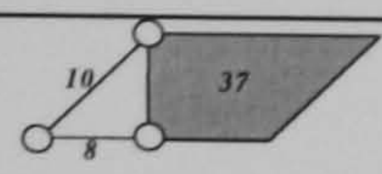
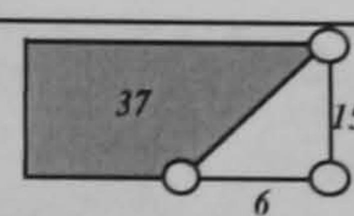
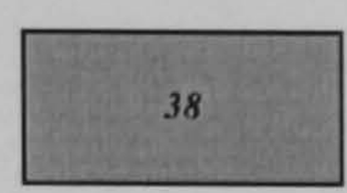
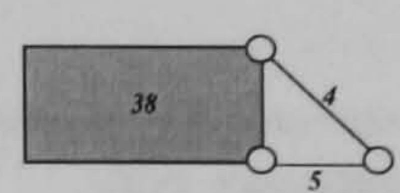
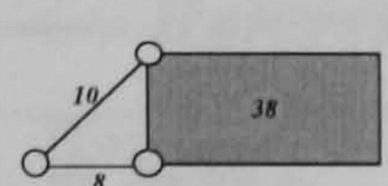
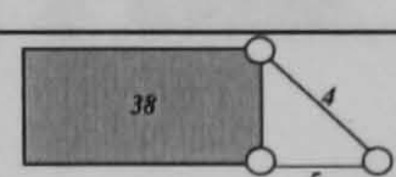
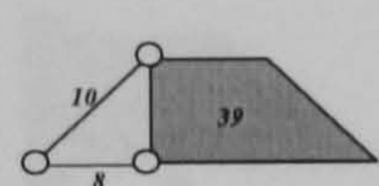
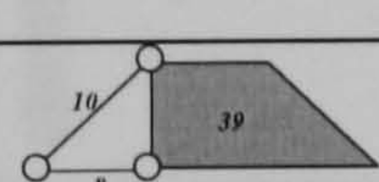
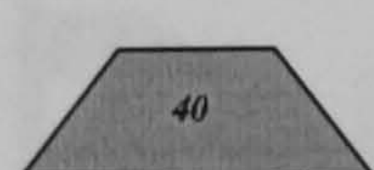
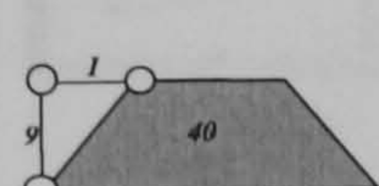
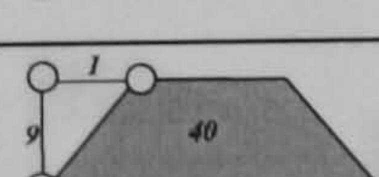
Member No.	End Joints		End Fixity		E	A	I
	Joint-1	Joint-2	Joint-1	Joint-2	(n/m^2)	(m^2)	(m^4)
1	1	2	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
2	2	3	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
3	3	4	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
4	4	5	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
5	5	6	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
6	6	7	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
7	7	8	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
8	8	9	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
9	1	9	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
10	2	9	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
11	2	8	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
12	2	7	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
13	3	7	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
14	4	7	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
15	4	6	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
16	5	10	0	0	70×10^6	20×10^{-3}	6.7×10^{-5}
17	10	11	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
18	11	12	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
19	12	13	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
20	13	14	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
21	14	15	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
22	15	16	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
23	10	16	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
24	11	16	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
25	11	15	0	0	205×10^6	3.23×10^{-3}	3.7×10^{-5}
26	12	15	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
27	12	14	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}

Table 8.10 Step-by-step cluster formation --- Truss-2

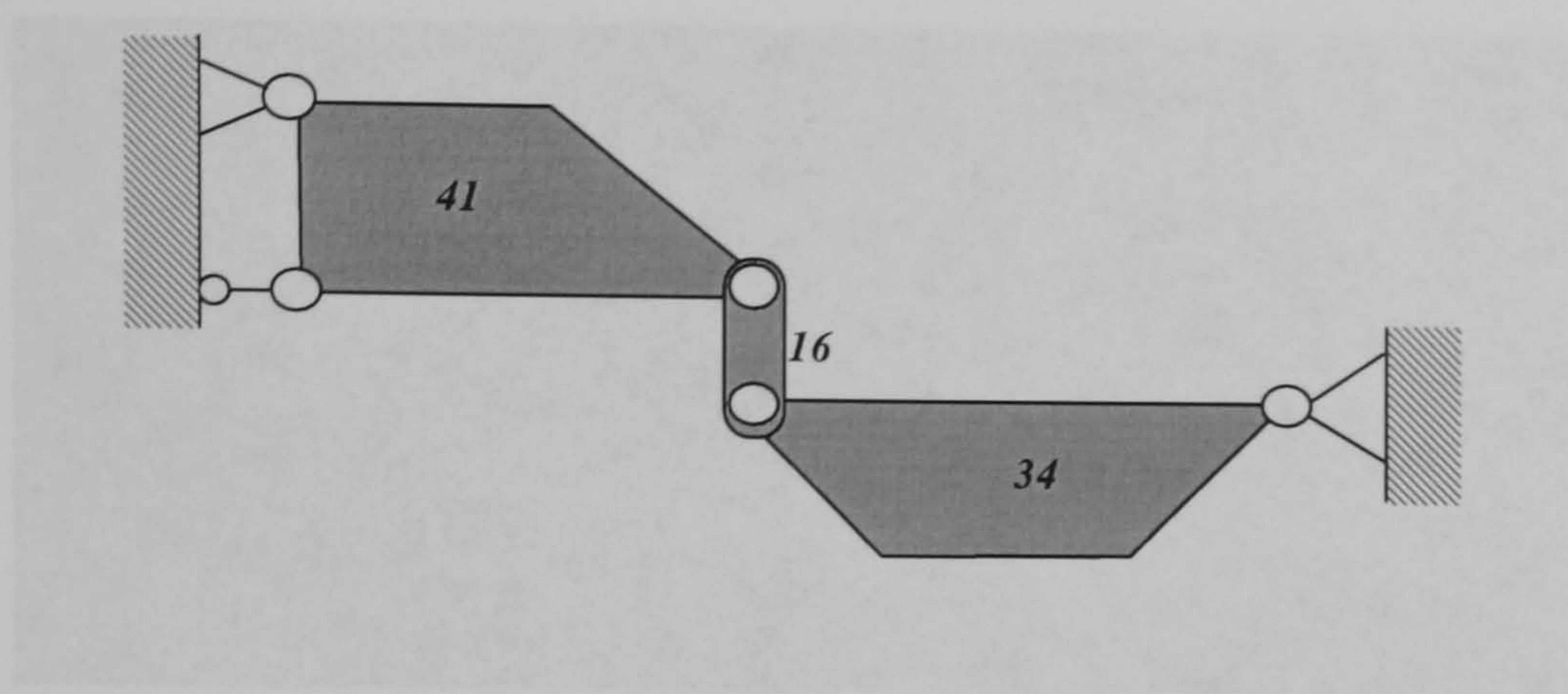
Steps	Components	Cluster Formed	Well-formedness Q	Damage demand	Nodal Degree	Distance
The structure: 						
----- Initial Clustering Stage -----						
	1+9+10		18.06×10^9	6.47	10	0.0
	2+12+13		18.06×10^9	6.47	13	12.0
	3+13+14		18.06×10^9	6.47	12	-
	4+5+15		18.06×10^9	6.47	10	-
	6+14+15		18.06×10^9	6.47	12	-
	7+11+12		18.06×10^9	6.47	13	-
Step 1	8+10+11		18.06×10^9	6.47	11	-
	17+23+24		24.77×10^9	14.42	10	26.4
	18+25+26		24.77×10^9	14.42	**12**	14.71

	19+20+27		24.77×10^9	14.42	9	-
	21+26+27		24.77×10^9	14.42	11	-
	22+24+25		24.77×10^9	14.42	11	-
	Forming Cluster 30	 as 	*Selection Criteria*: Higher nodal connectivity			
Step 2	21+27+30		48.2×10^9	14.42	15	11.2
	22+24+30		48.2×10^9	14.42	15	**18.7**
	Forming Cluster 31	 as 	*Selection Criteria*: Greater distance from Ref.			
Step 3	17+23+31		62.3×10^9	14.42	18	**22.4**
	21+27+31		62.3×10^9	14.42	18	11.2
	Forming Cluster 32	 as 	Selection Criteria: Greater distance from Ref..			
Step 4	21+27+32		71.6×10^9	14.42	21	11.2
	Forming Cluster 33	 as 	Selection Criteria: One choice. Increased well-formedness.			
	19+20+33		78.3×10^9	14.42	23	3.6

Step 5	Forming Cluster 34	 as 	Selection Criteria: One choice. Increased well-formedness.			
Step 6	1+9+10 : : 8+10+11 (Repeat part of Step 1)	 : : 	----	See Step 1	----	
	Forming Cluster 35	 as 	Selection Criteria: Higher nodal connectivity.			
Step 4	3+14+35		29.7×10^9	6.47	17	15
	7+11+35		**42.4×10^9**	6.47	16	9
	Forming Cluster 36	 as 	Selection Criteria: Higher well-formedness.			
Step 8	3+14+36		50.7×10^9	6.47	20	15
	8+10+36		50.7×10^9	6.47	19	-
	Forming Cluster 37	 as 	Selection Criteria: Higher nodal connectivity.			

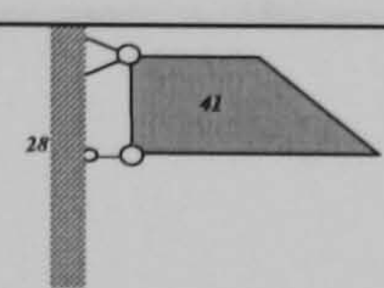
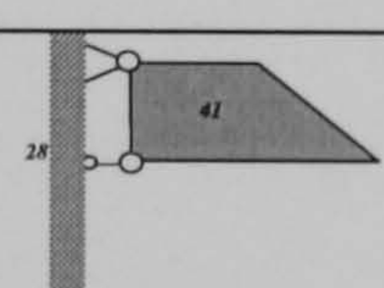
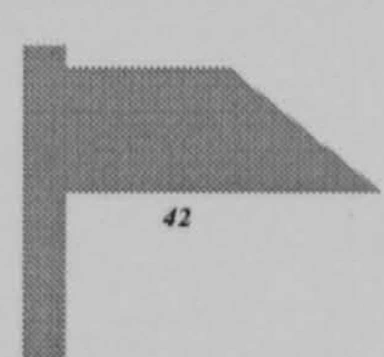
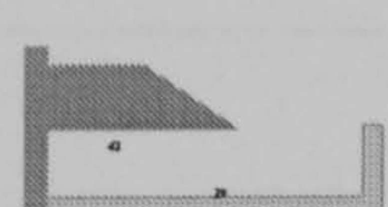
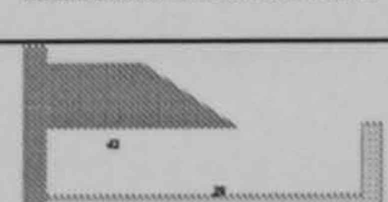
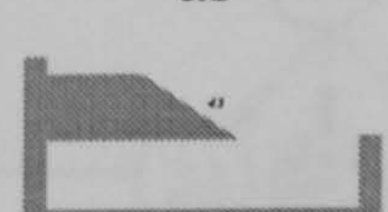
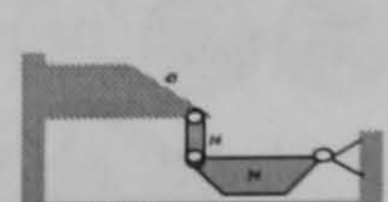
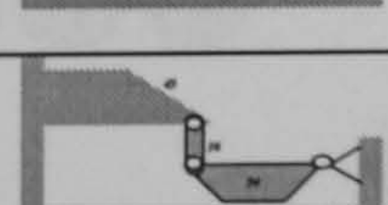
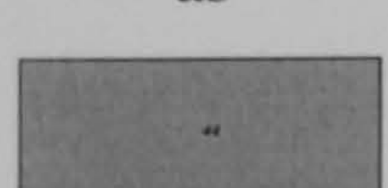
Step 9	6+15+37		**64.7×10⁹**	6.47	23	18
	8+10+37		56.3×10⁹	-	-	-
	Forming Cluster 38	 as 	Selection Criteria: Higher well-formedness.			
Step 10	4+5+38		67.5×10⁹	6.47	26	**21**
	8+10+38		67.5×10⁹	6.47	26	3
	Forming Cluster 39	 as 	Selection Criteria: Greater distance from Ref..			
Step 11	8+10+39		**69.6×10⁹**	6.47	29	3
	Forming Cluster 40	 as 	Selection Criteria: One choice. Increased well-formedness.			
Step 12	1+9+40		**76.8×10⁹**	6.47	31	0
	Forming Cluster 41	 as 	Selection Criteria: One choice. Increased well-formedness.			
End of Initial Clustering Stage						

The structure at the end of Initial Clustering Stage:



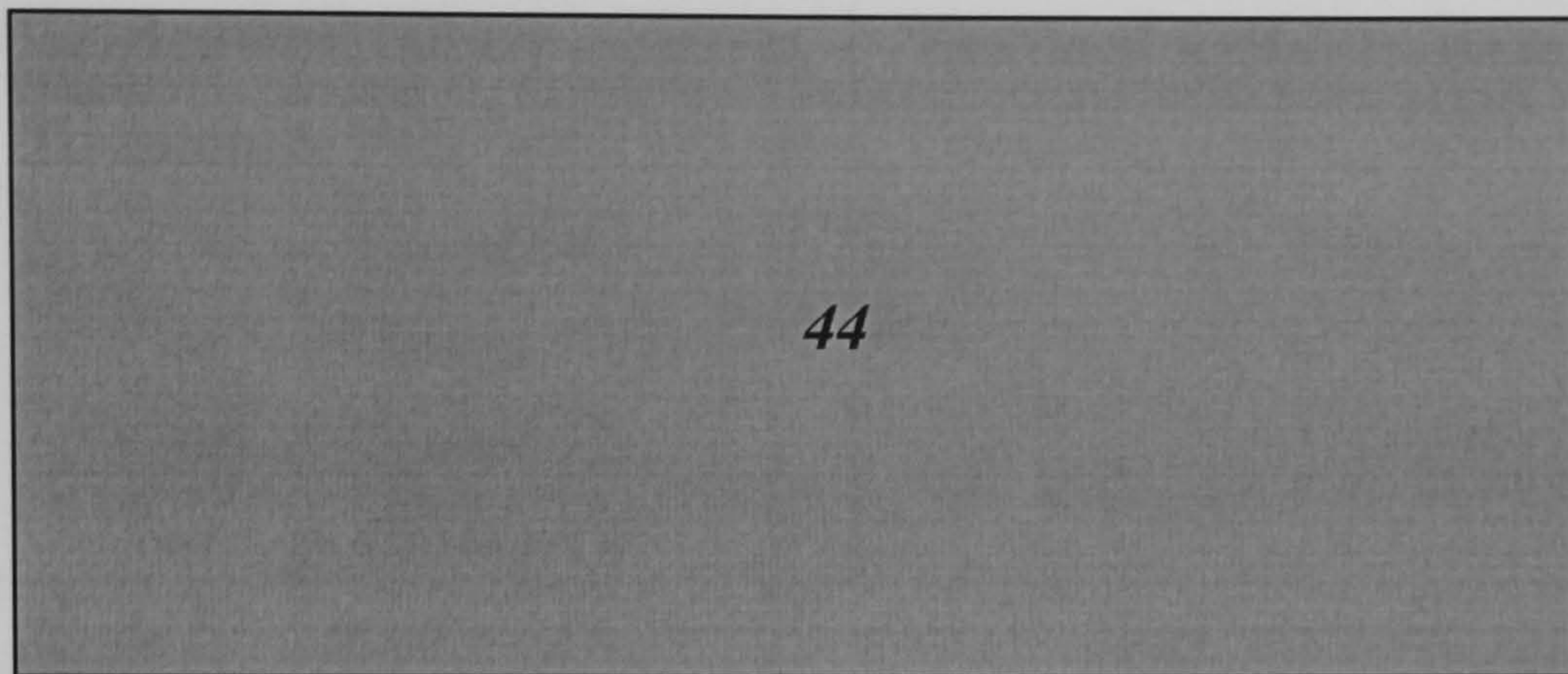
No secondary cluster identified.

----- Reference Clustering Stage -----

Step 13	28+41		**76.8×10^9**	6.47	31	0
	Forming Cluster 42	 as 	*Selection Criteria*: One choice.			
Step 14	29+42		**76.8×10^9**	6.47	31	0
	Forming Cluster 43	 as 	*Selection Criteria*: One choice.			
Step 15	16+34+43		**120×10^9**	6.47	54	12
	Forming Cluster 44	 as 	*Selection Criteria*: One choice.			

End of Reference Clustering Stage

The structure at the end of Reference Clustering Stage:



Cluster Formation Completed.

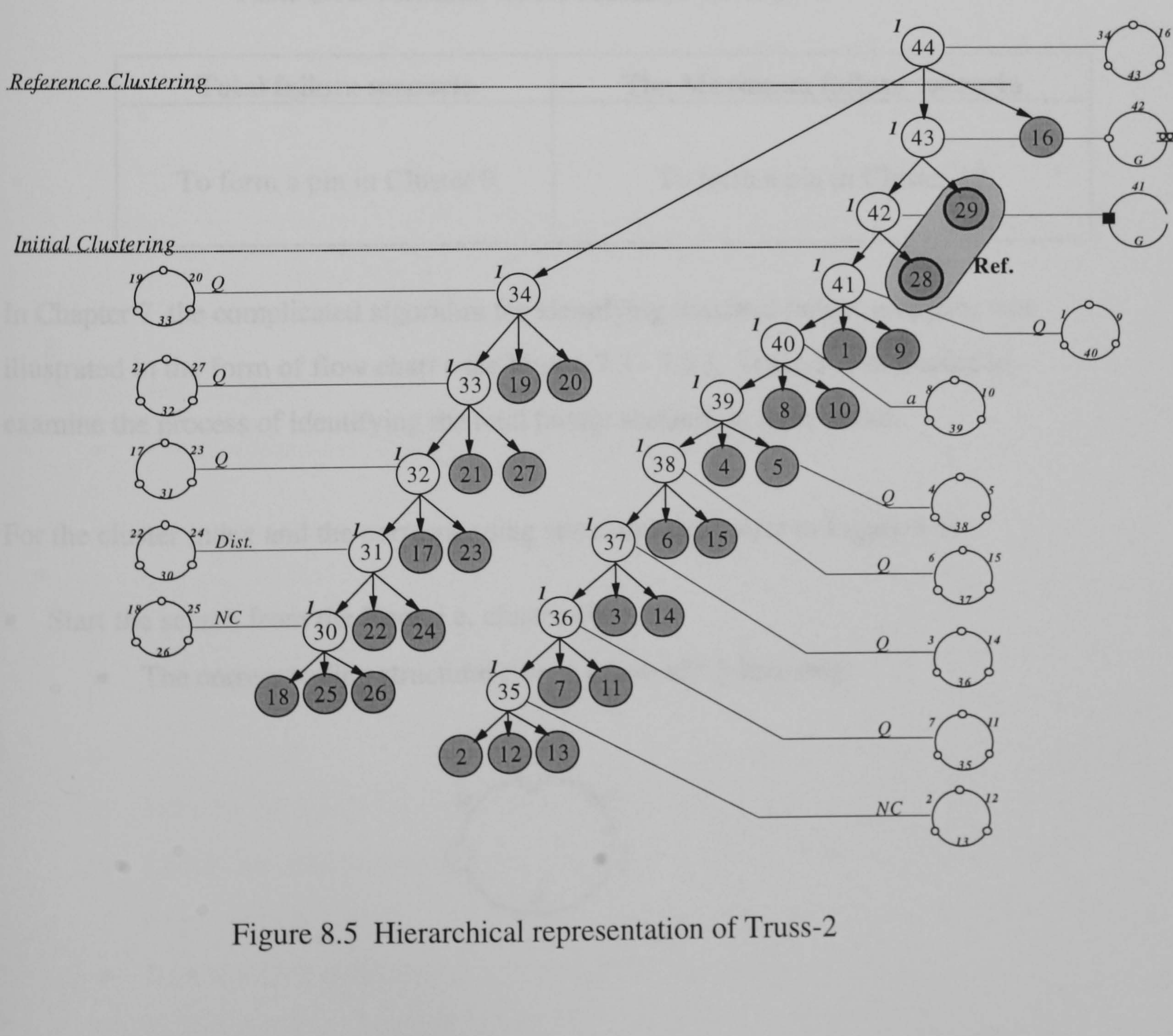


Figure 8.5 Hierarchical representation of Truss-2

Table 8.11 Minimal failure scenarios for Truss-2

Min. demand failure scenario	**The least well-formed cluster scenario**
To damage: Cluster 4 or Cluster 10 or Cluster 12 or Cluster 14	Cluster 16

In Truss-2, cluster 4, 10, 12 and 14 can be damaged most easily, however, because they connect and form clusters with others, they are not the least well-formed part of the structure. The latter is identified as cluster 16.

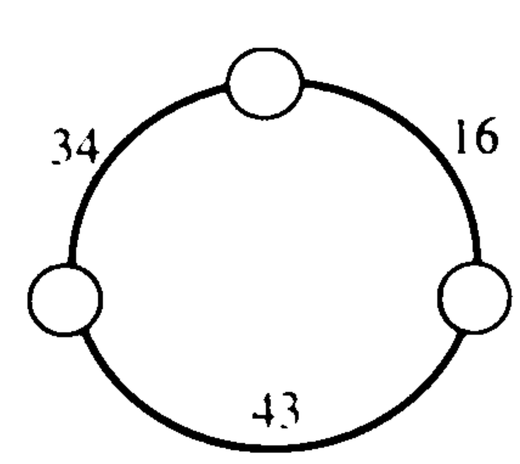
Table 8.12 Maximal failure scenarios for Truss-2

Total failure scenario	**The Maximum failure scenario**
To form a pin in Cluster 9.	To form a pin in Cluster 10.

In Chapter 7 the complicated algorithm for identifying maximal failure scenarios was illustrated in the form of flow chart (see Figure 7.3 - 7.5). Truss-2 is now used to examine the process of identifying the total failure scenario in more detail.

For the cluster index and the corresponding structural ring, refer to Figure 8.5.

- Start the search from the Root, i.e. cluster 44.
    - The corresponding structural ring is a just-stiff 3-link-ring:



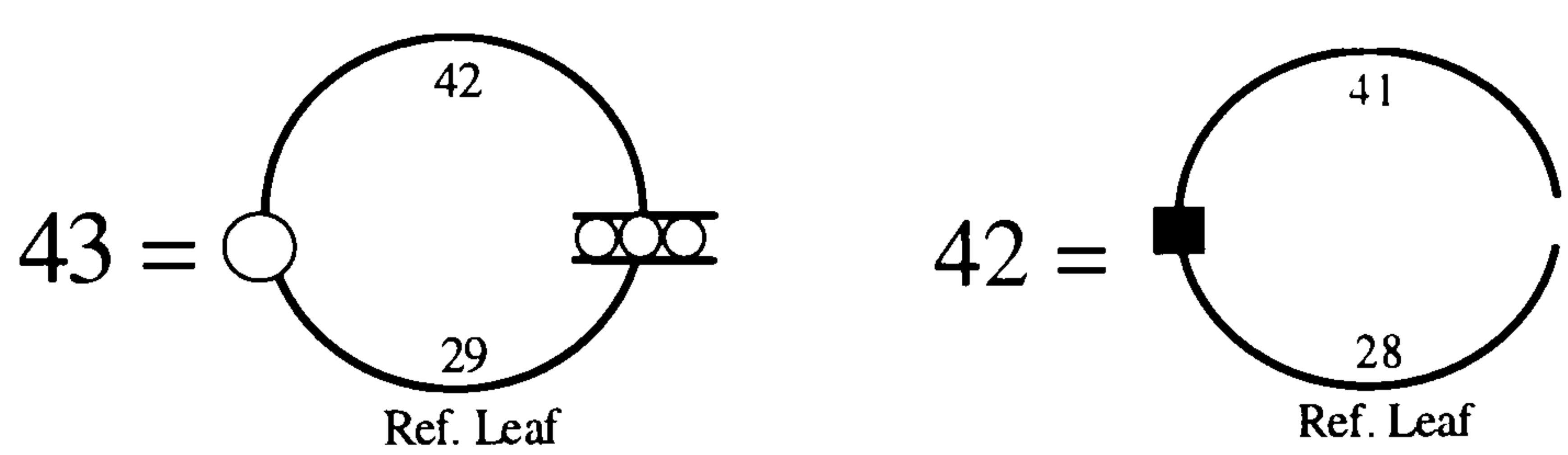
- Identify the child clusters of cluster 44, as cluster 16, 34 and 43.
- Check the deterioration event required to cause failure in this structural ring, (shown at the upper left corner of the structural ring in Figure 8.5), which is 1.
- Decide which child cluster is to break at this stage:

  Cluster 16 and 34 are identified as the dependent-clusters of cluster 43, therefore:

  - Ignore cluster 34 and 16.

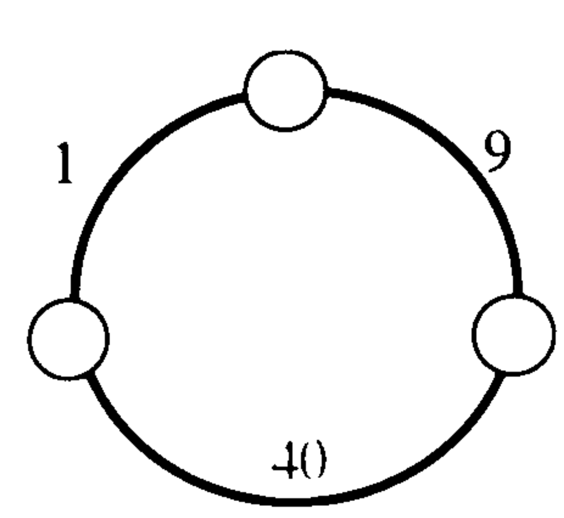
- Search down cluster 43.

  For next two steps, the two corresponding structural rings both contain a reference-leaf as one of the child clusters:



  Reference clusters are not considered as damagable, therefore, the process ignores the reference leaves in these two structural rings and goes on searching down the other child cluster.

- Search down cluster 41.
  - The corresponding structural ring is a just-stiff 3-link-ring:



  - Identify the child clusters of cluster 41, as cluster 1, 9 and 40.
  - Check the deteriorating event required to cause failure in this structural ring, which is 1.
  - Decide which child cluster is to break at this stage:

- There are no reference clusters among the child clusters of this cluster.
- Check the leaf-relationship with reference of all child clusters.

  Cluster 1 *can form a ring* with the reference, and cluster 9 *can form an overlap* with the reference.
- Choose cluster 9 to break.

- Decide on the appropriate action to take;
  - Check the number of deteriorating events required at this stage:

    There are no inherited events from upper levels of description, therefore, the number required is 1.
  - According to the number of deteriorating events required, *the formation of a pin* in cluster 9 can bring the right number of deteriorating events and has the minimum damage demand, therefore,
  - Take action: form a pin in cluster 9.
- Modify the total number of deteriorating events 1-1 = 0.

- Decide whether it is necessary to carry on search:
  - Check the number of events left.

    Events left = 0.
- Search completed.

Apart from the minimal and maximal failure scenarios, some of the interesting failure scenarios for Truss-2 are shown in Table 8.13. The relative damage demand, E_r, separateness, γ, and the vulnerability index, ξ, for each of the scenarios are listed. The vulnerability index is a relative measure and only valid at this stage when comparing various failure scenarios of the *same* structure.

Table 8.13 Some interesting failure scenarios for Truss-2

Failure scenarios	**E_r (Relative Damage Demand)**	**γ (Separateness)**	**ξ (Vulnerability Index)**
Pin Cluster 9	0.0152	1	65.7
Pin Cluster 10	0.0106	≈ 1	94.3
Pin Cluster 12	0.0106	0.73	68.9
Pin Cluster 4	0.0106	0.57	53.8
Pin Cluster 16	0.342	0.51	1.5

From Table 8.13, it is clear that for Truss-2 the maximum failure scenario is not the same as the total failure scenario. The maximum failure scenario is to form a pin in Cluster 10 because of the very low relative damage demand involved.

The results from these two truss structure examples seem to have confirmed a good engineer's common sense. The example in next section will prove that this intuition is not always obvious.

8.4 Frame Structure

For the frame structure used in this section, the shape of the structure is not uniform. The majority of the components have the same member properties, however, they are not all equal-spans (see Table 8.14 - 16). Member 11 has different member properties to create a situation where the internal form varies significantly.

Frame-1:

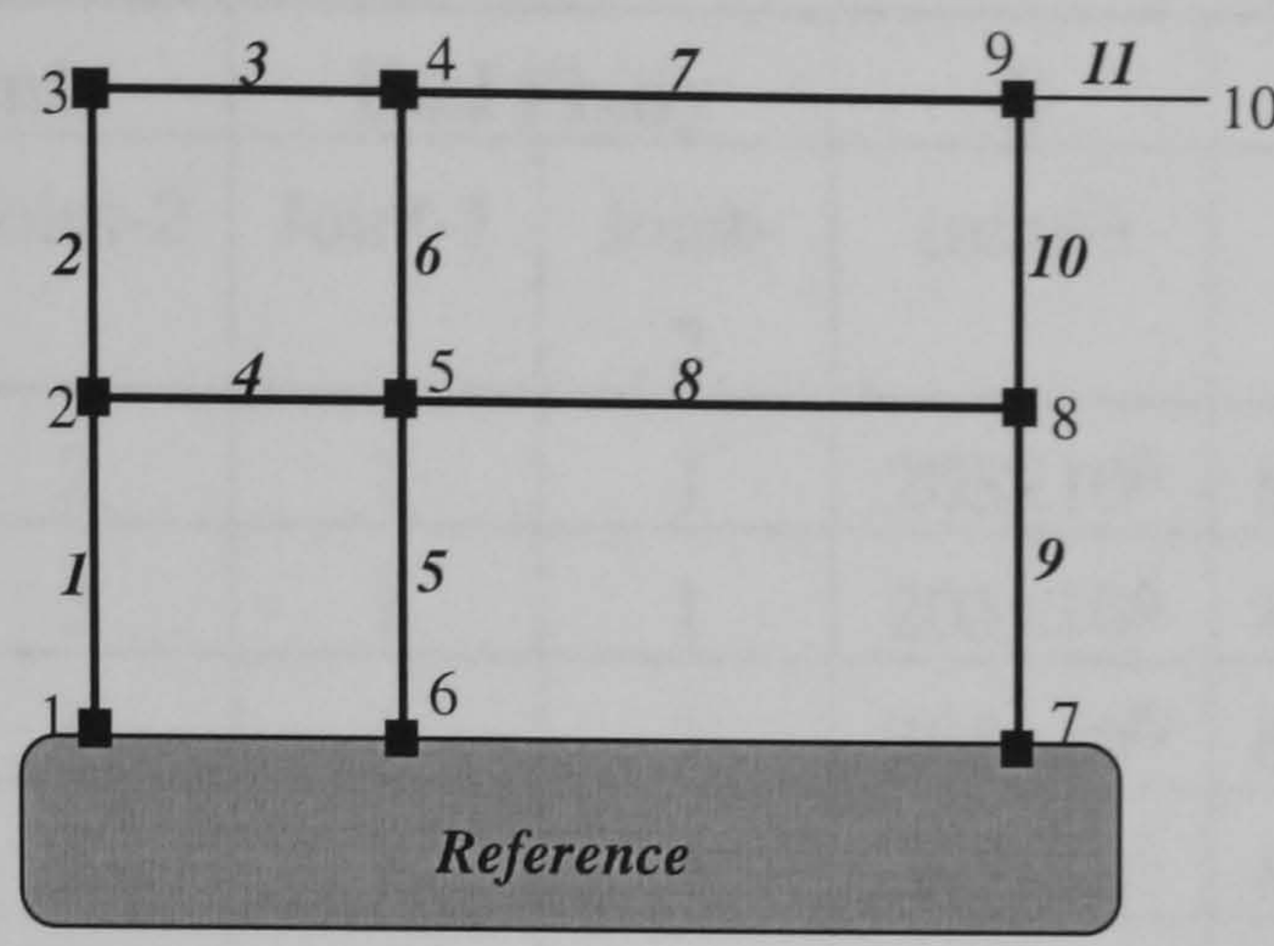


Figure 8.6 The structure --- Frame-1

Table 8.14 Joint co-ordinate table of Frame-1

Joint No.	**X Co-od.** (m)	**Y Co-od.** (m)
1	0.0	0.0
2	0.0	3.0
3	0.0	6.0
4	3.0	6.0
5	3.0	3.0
6	3.0	0.0
7	9.0	0.0
8	9.0	3.0
9	9.0	6.0
10	10.0	6.0

Table 8.16 Constraint condition of Frame-1

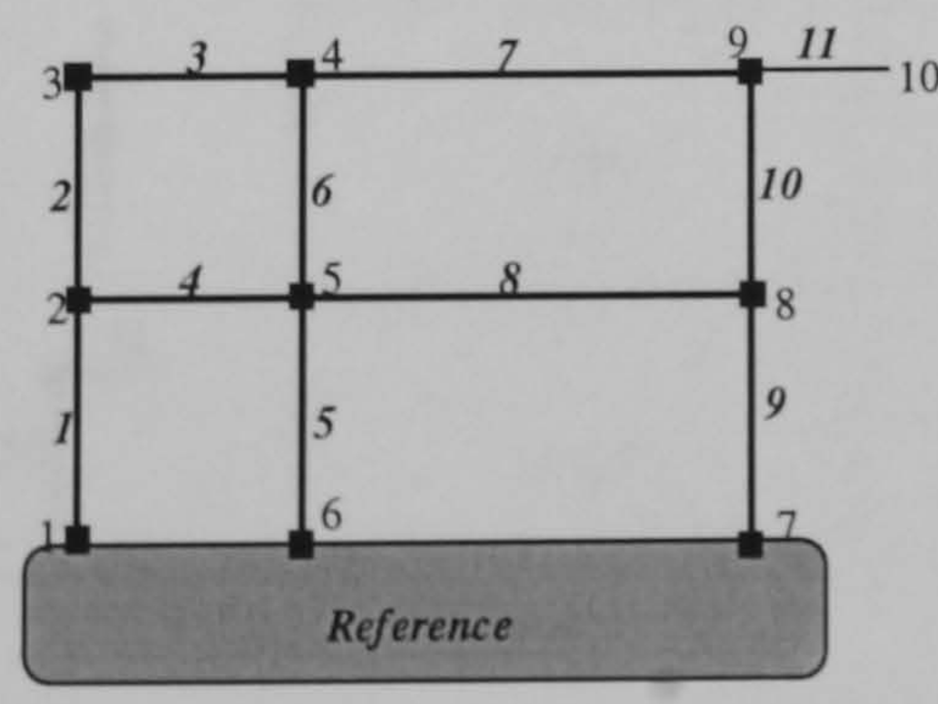
Constraint No.	**Joint No.**	x	y	θ
1	1	1	1	1
2	6	1	1	1
3	7	1	1	1

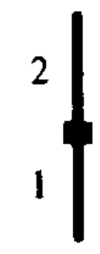
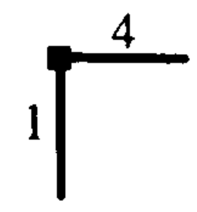
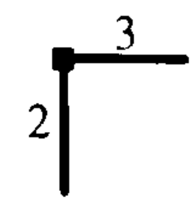
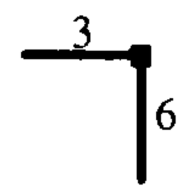
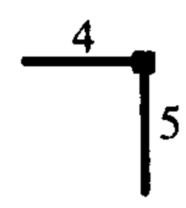
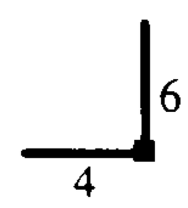
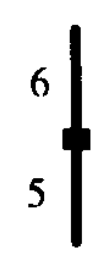
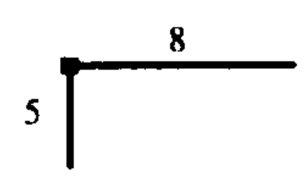
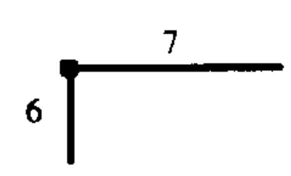
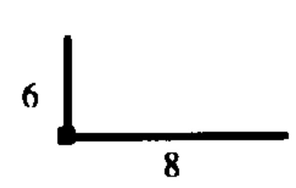
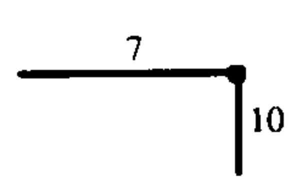
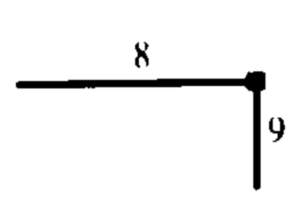
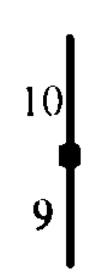
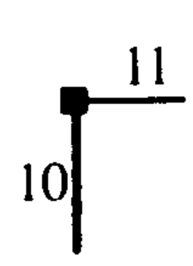
Table 8.15 Member properties table of Frame-1

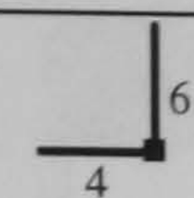
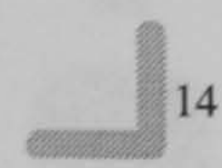
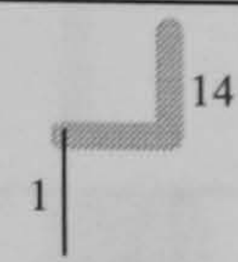
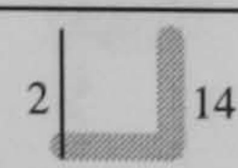
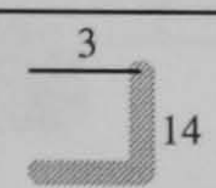
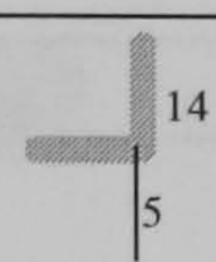
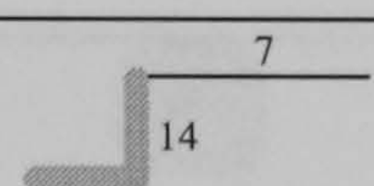
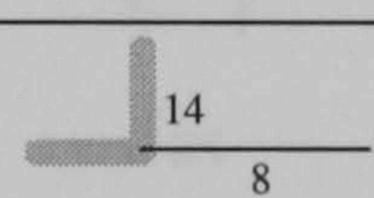
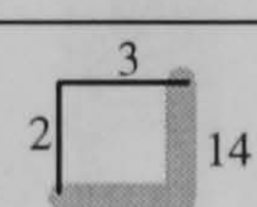
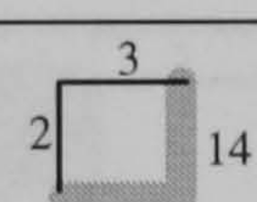
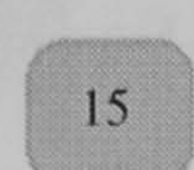
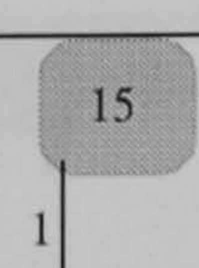
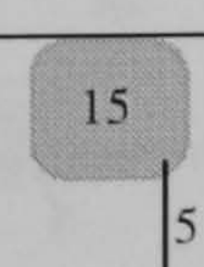
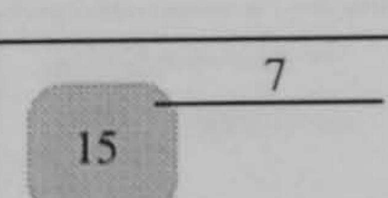
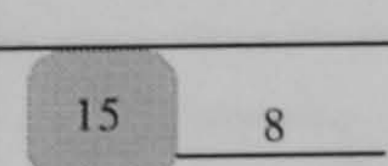
Member No.	End Joints		End Fixity		E	A	I
	Joint-1	Joint-2	Joint-1	Joint-2	(n/m^2)	(m^2)	(m^4)
1	1	2	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
2	2	3	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
3	3	4	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
4	2	5	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
5	5	6	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
6	4	5	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
7	4	9	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
8	5	8	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
9	7	8	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
10	8	9	1	1	205×10^6	8.08×10^{-2}	2.75×10^{-3}
11	9	10	1	1	205×10^6	0.32×10^{-2}	0.37×10^{-4}

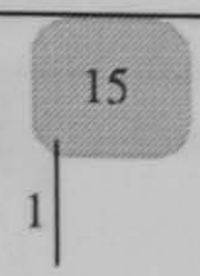
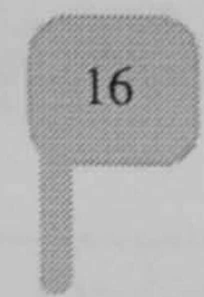
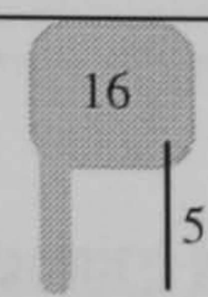
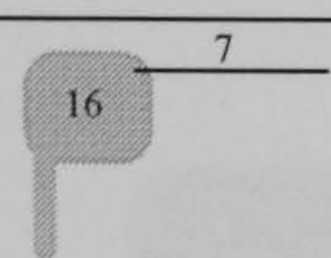
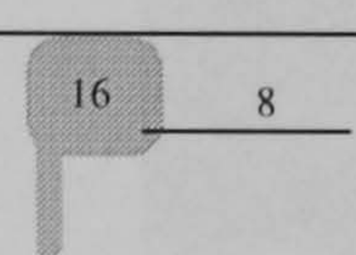
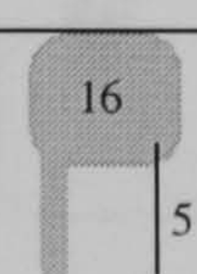
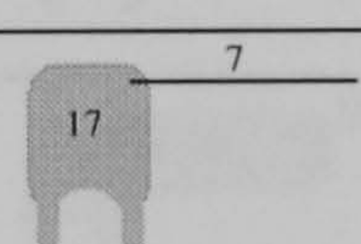
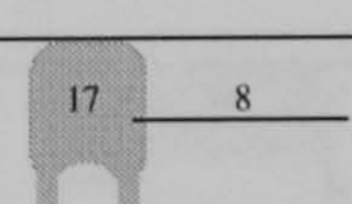
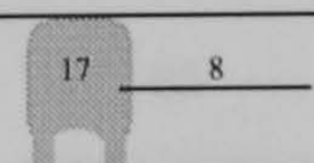
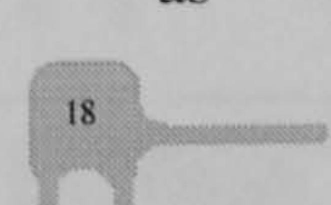
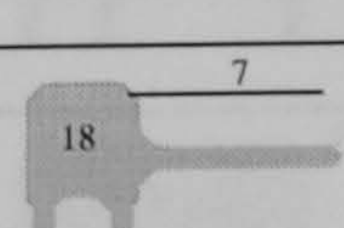
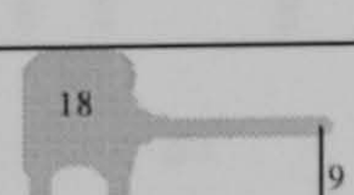
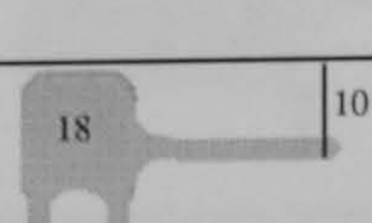
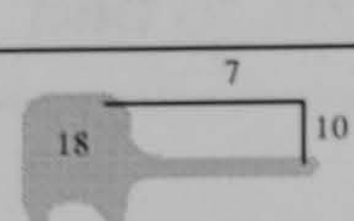
Table 8.17 Step-by-step cluster formation --- Frame-1

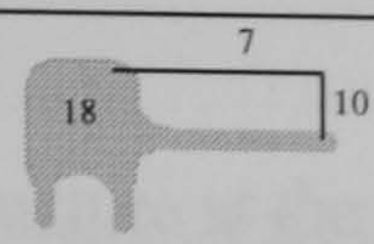
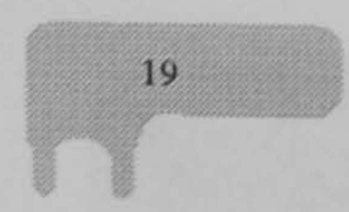
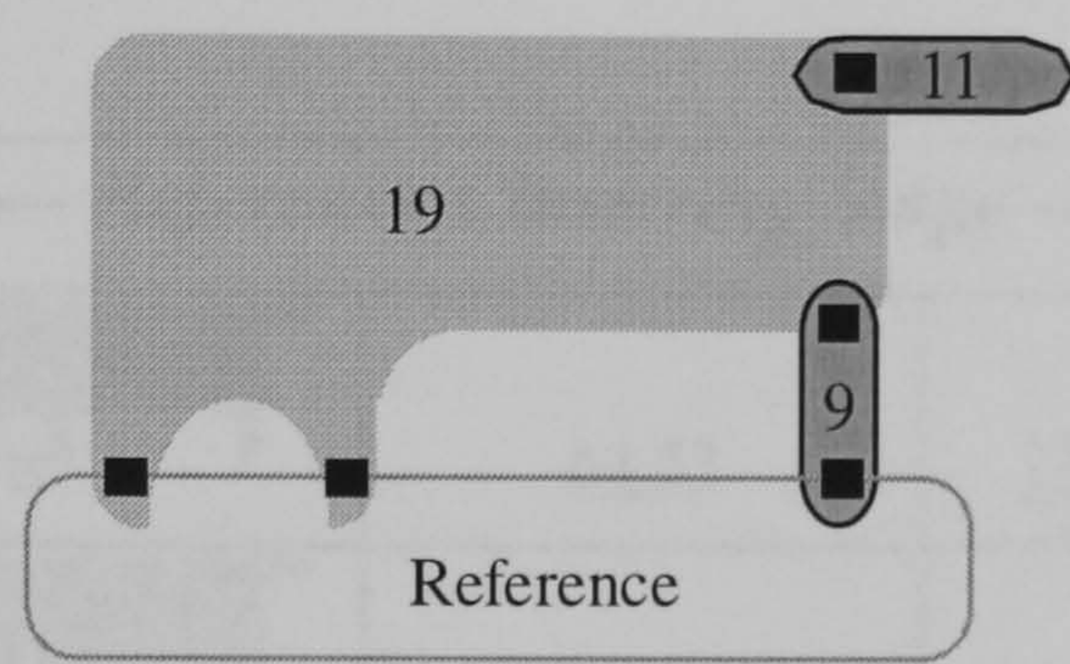
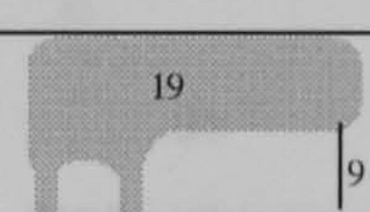
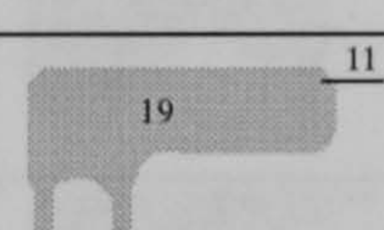
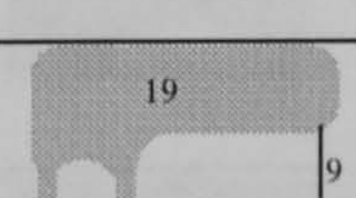
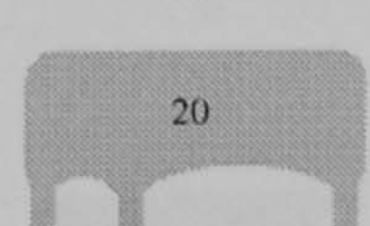
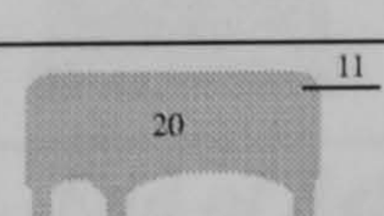
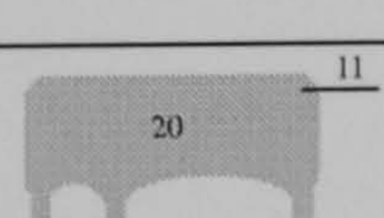
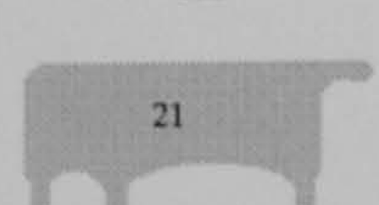
Steps	Components	Cluster Formed	Well-formedness (10^{18})	Damage demand	Nodal Degree	Distance
The structure:						
----- Initial Clustering Stage -----						



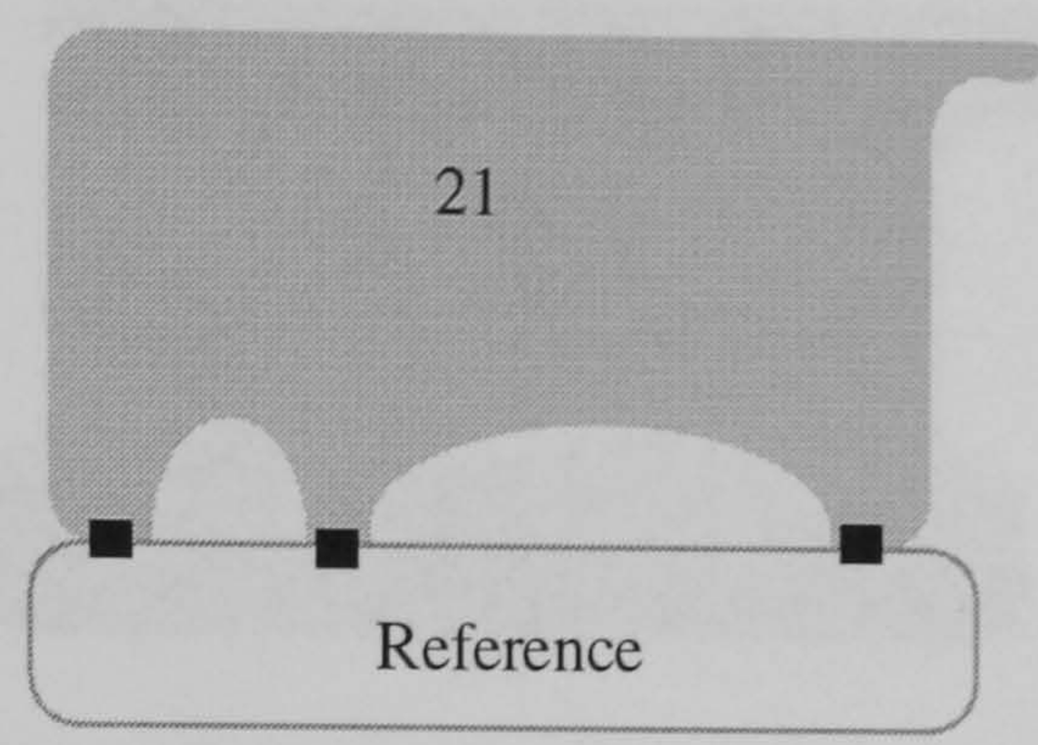
	1+2		2.95	960	6	3.0
	1+4		16.3	960	8	3.0
	2+3		16.3	960	8	9.0
	2+4		16.3	960	9	6.0
	3+6		16.3	960	9	9.0
	3+7		0.75	-	-	-
	4+5		16.3	960	8	-
	4+6		16.3	960	**10**	6.0
	4+8		0.75	-	-	-
Step 1	5+6		2.95	-	-	-
	5+8		6.1	-	-	-
	6+7		6.1	-	-	-
	6+8		6.1	-	-	-
	7+10		6.1	-	-	-
	7+11		0.06	-	-	-
	8+9		6.1	-	-	-
	8+10		6.1	-	-	-
	9+10		2.95	-	-	-
	10+11		1.16	-	-	-

	Forming Cluster 14	 as 	*Selection Criteria*: Higher nodal connectivity			
Step 2	1+14		24.36	960	11	3.0
	2+14		24.36	960	12	6.0
	3+14		24.36	960	12	9.0
	5+14		38.32	960	11	3.0
	7+14		16.7	-	-	-
	8+14		23.02	-	-	-
	2+3+14		**48.45**	960	12	12
	Forming Cluster 15	 as 	*Selection Criteria*: Higher well-formedness.			
Step 3	1+15		59.63	960	13	3.0
	5+15		59.63	960	13	3.0
	7+15		47.38	-	-	-
	8+15		47.38	-	-	-

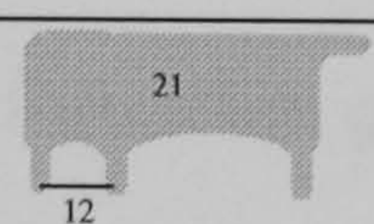
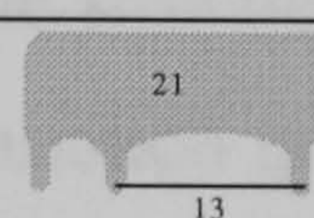
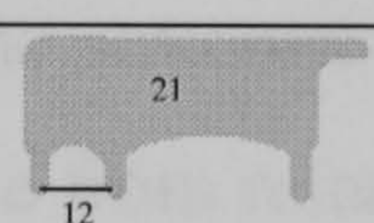
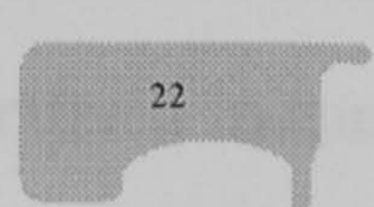
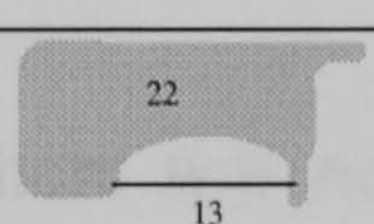
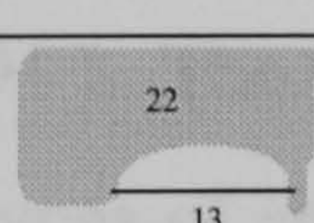
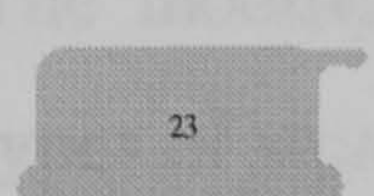
	Forming Cluster 16	 as 	*Selection Criteria*: Random choice from two clusters with highest well-formedness.			
Step 4	5+16		**67.08**	960	14	0.0
	7+16		56.87	-	-	-
	8+16		56.87	-	-	-
	Forming Cluster 17	 as 	*Selection Criteria*: Higher well-formedness.			
Step 5	7+17		63.65	425	17	6.0
	8+17		**72.99**	425	17	3.0
	Forming Cluster 18	 as 	*Selection Criteria*: Higher well-formedness.			
	7+18		69.25	425	20	6.0
	9+18		66.15	-	-	-
	10+18		66.15	-	-	-
Step 6	7+10+18		**73.76**	425	20	9.

	Forming Cluster 19	 as 	*Selection Criteria*: Higher well-formedness.			
End of Initial Clustering Stage						
The structure at the end of Initial Clustering Stage: 						
----- Secondary Clustering Stage -----						
Step 7	9+19		**71.13**	425	21	0.0
	11+19		66.14	-	-	-
	Forming Cluster 20	 as 	*Selection Criteria*: Higher well-formedness.			
Step 8	11+20		64.53	131	22	6.0
	Forming Cluster 21	 as 	*Selection Criteria*: One choice.			
End of Secondary Clustering Stage						

The structure at the end of Secondary Clustering Stage:

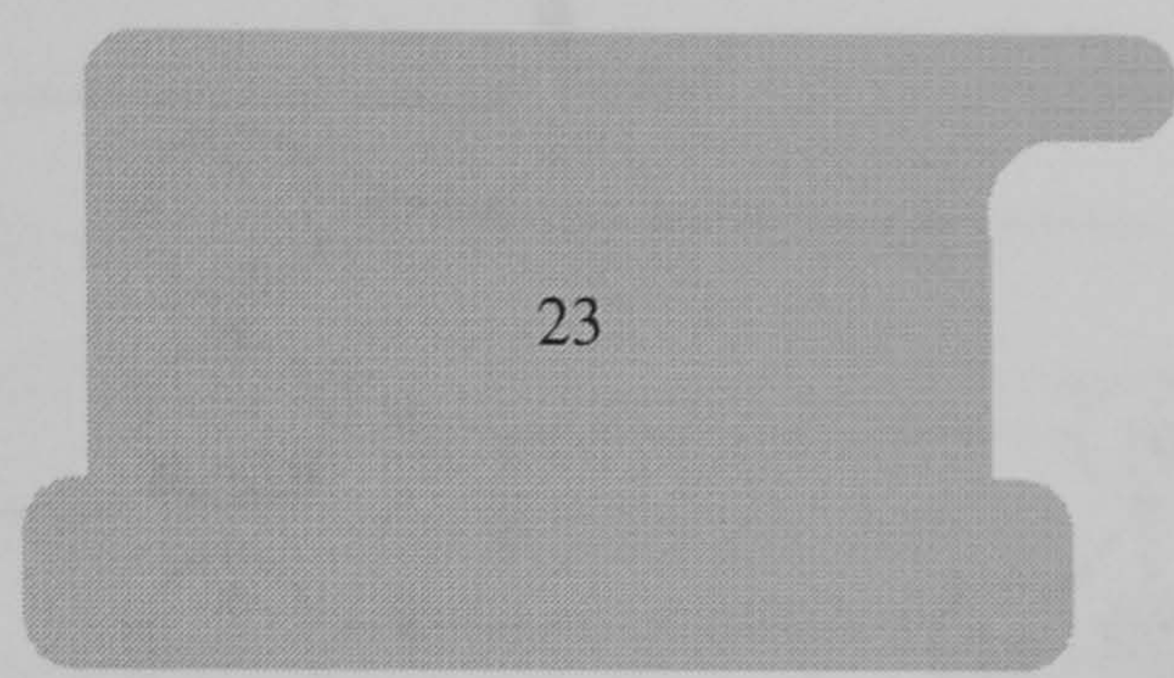


----- Reference Clustering Stage -----

	12+21		64.53	131	22	0.0
	13+21		64.53	131	22	0.0
	Forming Cluster 22	 as 	*Selection Criteria*: Random			
	13+22		64.53	131	22	0.0
	Forming Cluster 23	 as 	*Selection Criteria*: One choice.			

End of Reference Clustering Stage

The structure at the end of Reference Clustering Stage:



Cluster Formation Completed.

In Frame-1, the cluster formation process is mainly governed by the well-formedness criteria. It is because the structure has a variation in its internal form stemming from the unequal span and variation in member properties. In a case where the shape and member properties of the structure is highly uniform, the other criteria such as nodal connectivity and distance from reference will become vital. Frame-2 in Appendix-1 is such a structure. The cluster formation process shown in Appendix-1 illustrates the importance of the new criteria in modelling uniformed structures.

In the previous work, there was no theoretical limit for the number of the links in a structural ring (Wu, 1991). The size was artificially confined to four links. However, the newly defined two types of structural rings, i.e. 2-link-rings and 3-link-rings, have solved this problem. The modification allows all case of structural configurations presented with the two types of structural rings.

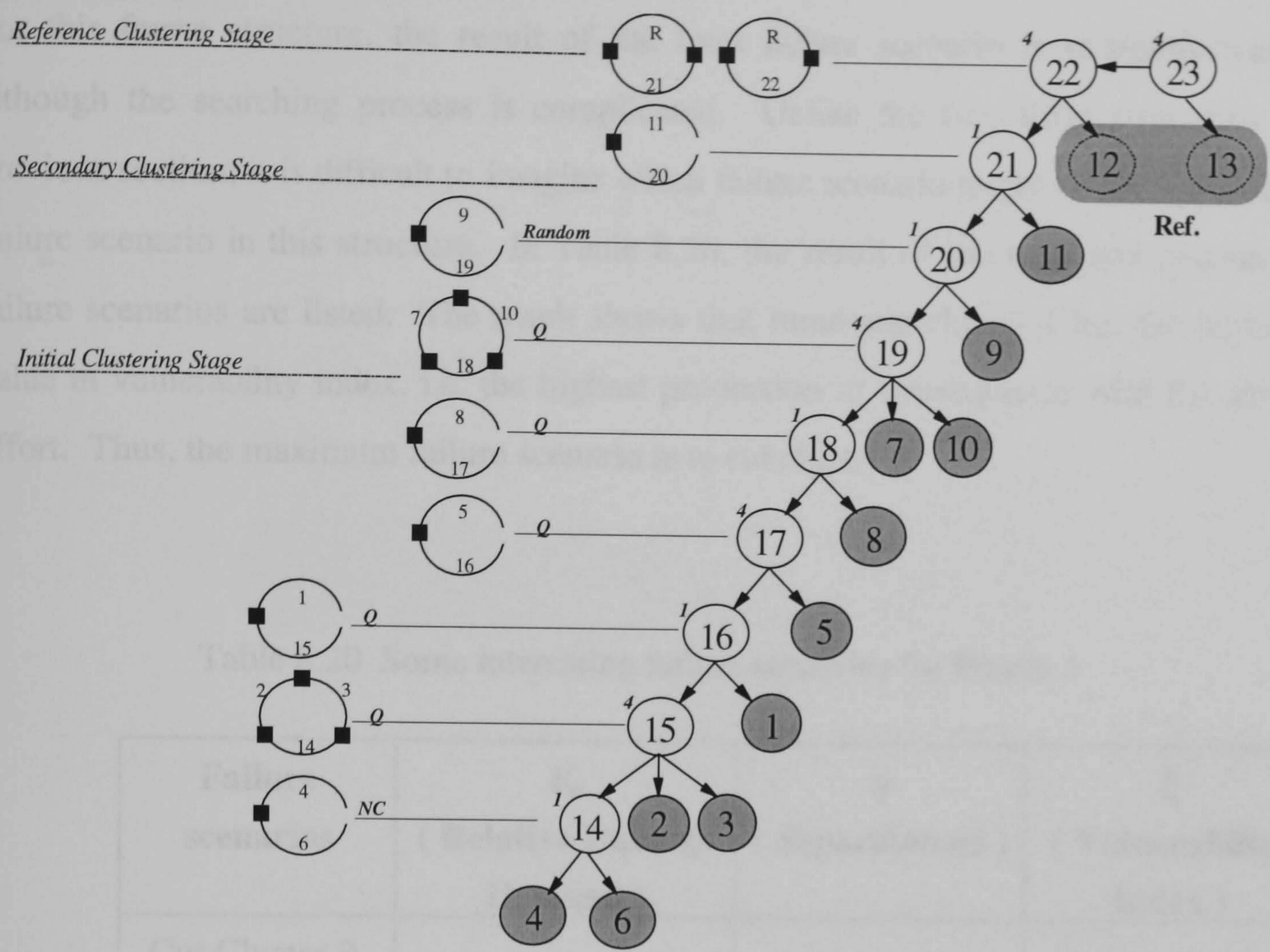


Figure 8.7 Hierarchical representation of Frame-1

Table 8.18 Minimal failure scenarios for Frame-1

Min. demand failure scenario	The least well-formed cluster scenario
To damage: Cluster 11	Cluster 11

In this structure, member 11 is identified, as expected, as a weak part of the structure in terms of the form of the structure. It is weak in the sense that it does not connect to the rest of the structure as tightly as the others.

Table 8.19 Maximal failure scenarios for Frame-1

Total failure scenario	The Maximum failure scenario
Cut in Cluster 1, Cut in Cluster 5, Cut in Cluster 9.	Cut in Cluster 4.

For this frame structure, the result of the total failure scenario is straightforward, although the searching process is complicated. Unlike the two truss structures in previous section, it is difficult to imagine which failure scenario might be the maximum failure scenario in this structure. In Table 8.20, the result of the total and maximum failure scenarios are listed. The result shows that removing cluster 4 has the highest value in vulnerability index, i.e. the highest proportion of consequence with the same effort. Thus, the maximum failure scenario is to cut cluster 4.

Table 8.20 Some interesting failure scenarios for Frame-1

Failure scenarios	E_r **(Relative Damage Demand)**	γ **(Separateness)**	ξ **(Vulnerability Index)**
Cut Cluster 9, Cut Cluster 5, Cut Cluster 1.	1.008	1	0.992
Cut Cluster 4	0.336	0.611	1.82
Cut Cluster 1	0.336	0.235	0.7

8.5 Combined Structures

This structure has been used in Chapter 4 and Chapter 5 to illustrate the process of cluster formation and the generation of the hierarchy. The same information will not be repeated here in this section. For details of cluster formation of this structure, refer to Section 4.6.

The combined structure contains a mixture of frame and truss structures. The algorithm is tested with the presence of both type of structures. The cluster formation process shows that the criteria are consistent for both type of structures (see Section 4.6).

Combined-1:

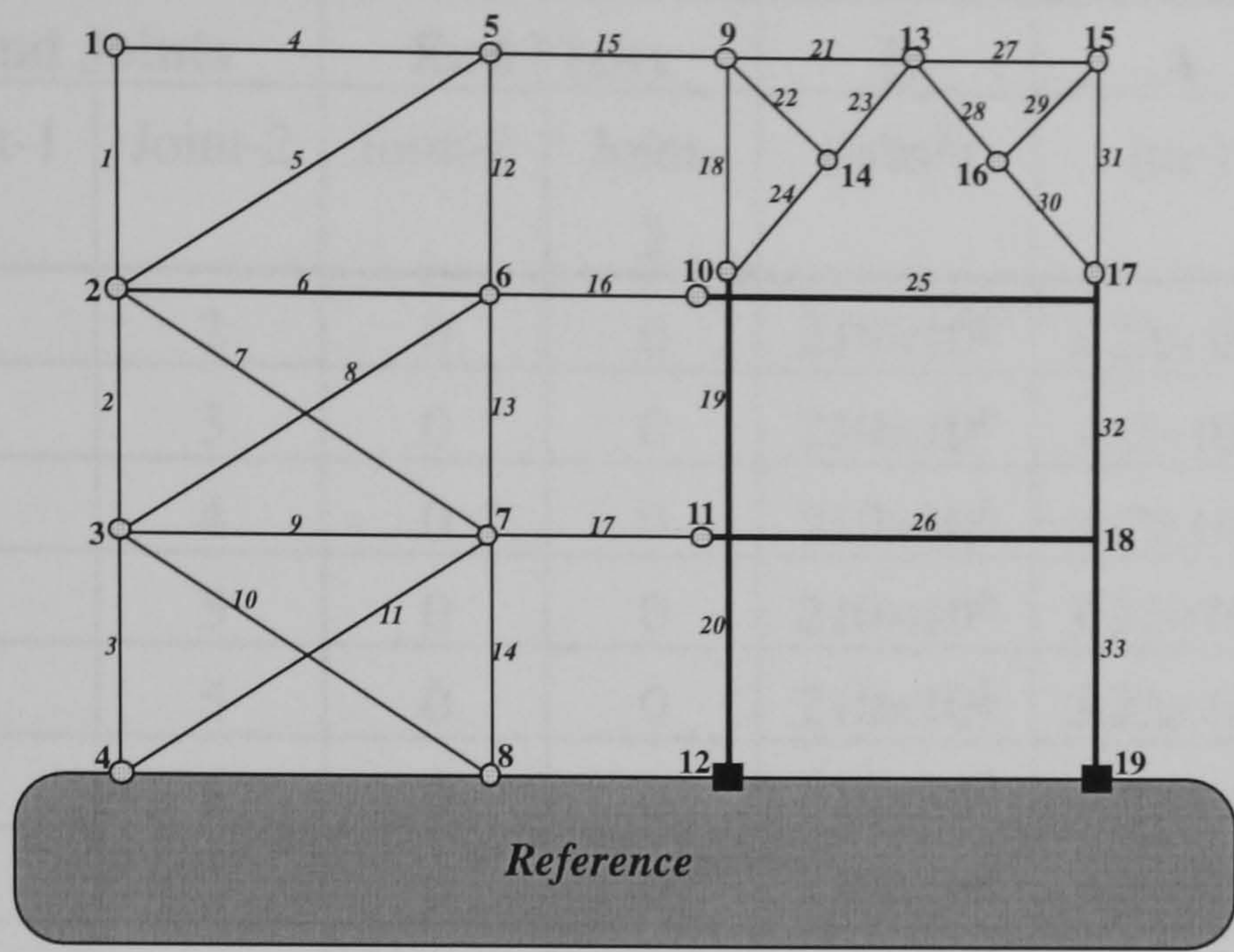


Figure 8.8 The structure --- Combined-1

Table 8.21 Joint co-ordinate table of Combined-1

Joint No.	X Co-od. (m)	Y Co-od. (m)
1	0.0	9.0
2	0.0	6.0
3	0.0	3.0
4	0.0	0.0
5	4.0	9.0
6	4.0	6.0
7	4.0	3.0
8	4.0	0.0
9	6.0	9.0
10	6.0	6.0
11	6.0	3.0
12	6.0	0.0
13	8.0	9.0
14	7.0	7.5
15	10.0	9.0
16	9.0	7.5
17	10.0	6.0
18	10.0	3.0
19	10.0	0.0

Table 8.22 Member properties table of Combined-1

Member No.	End Joints		End Fixity		E	A	I
	Joint-1	Joint-2	Joint-1	Joint-2	(n/m^2)	(m^2)	(m^4)
1	1	2	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
2	2	3	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
3	3	4	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
4	1	5	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
5	2	5	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
6	2	6	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
7	2	7	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
8	3	6	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
9	3	7	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
10	3	8	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
11	4	7	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
12	5	6	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
13	6	7	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
14	7	8	0	0	210×10^6	4.2×10^{-3}	4.9×10^{-5}
15	5	9	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
16	6	10	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
17	7	11	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
18	9	10	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
19	10	11	1	1	210×10^6	4.74×10^{-3}	5.5×10^{-5}
20	11	12	1	1	210×10^6	4.74×10^{-3}	5.5×10^{-5}
21	9	13	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
22	9	14	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
23	14	13	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
24	10	14	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
25	10	17	1	1	210×10^6	6.83×10^{-3}	11.7×10^{-5}
26	11	18	1	1	210×10^6	6.83×10^{-3}	11.7×10^{-5}
27	13	15	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
28	13	16	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
29	16	15	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
30	16	17	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
31	15	17	0	0	210×10^6	3.23×10^{-3}	3.7×10^{-5}
32	17	18	1	1	210×10^6	4.74×10^{-3}	5.5×10^{-5}
33	18	19	1	1	210×10^6	4.74×10^{-3}	5.5×10^{-5}

Table 8.23 Constraint condition of Combined-1

Constraint No.	Joint No.	x	y	θ
1	4	1	1	0
2	8	1	1	0
3	12	1	1	1
4	19	1	1	1

The detailed process of cluster formation has been shown in Chapter 4 (see 4.6). The hierarchical representation of the structure is:

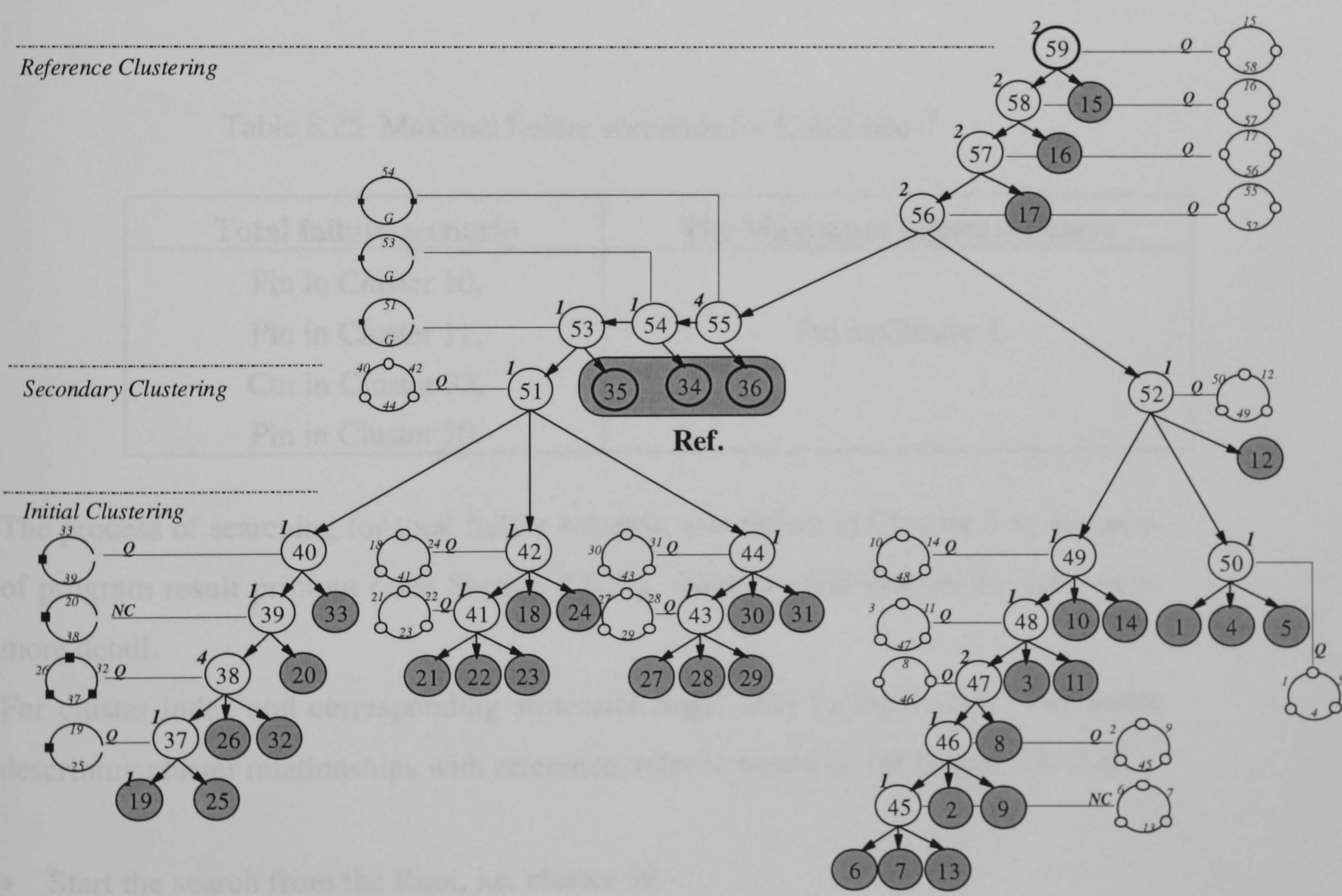


Figure 8.9 Hierarchical representation of Combined-1

The hierarchy shows the structure of the form of combined-1. From left to right is the sequence of cluster formation according to the form. In general, framed structures have higher well-formedness because of the type of structural connections. However. the type of joint is only one factor which influences the well-formedness. Depending on the member properties, truss structures can be more well-formed than frame structures. In Appendix-2, the member properties of the structure combined-1 are changed and the effect of the change on the internal structure of the form will be illustrated.

Table 8.24 Minimal failure scenarios for Combined-1

Min. demand failure scenario	The least well-formed cluster scenario
To damage: Cluster 5	Cluster 15

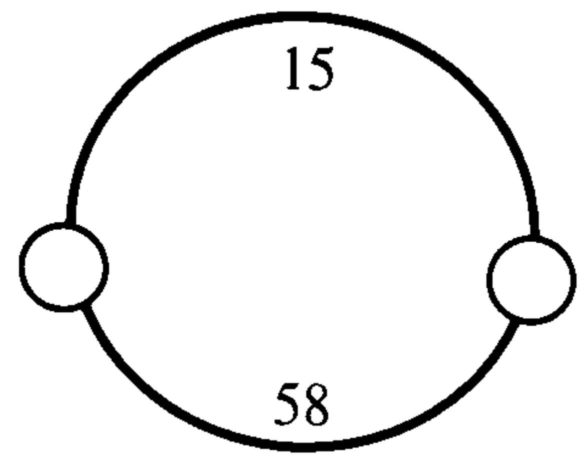
Table 8.25 Maximal failure scenarios for Combined-1

Total failure scenario	The Maximum failure scenario
Pin in Cluster 10, Pin in Cluster 11, Cut in Cluster 33, Pin in Cluster 20.	Pin in Cluster 4.

The process of searching for total failure scenario was shown in Chapter 7 in the form of program result printout (see Section 7.8.2). Here we will discuss the process in more detail.

For cluster index and corresponding structural rings, refer to Figure 8.9. For terms describing cluster relationships with reference, refer to notations for Figure 7.4 - 7.6.

- Start the search from the Root, i.e. cluster 59.
    - The corresponding structural ring is an over-stiff 2-link-ring:

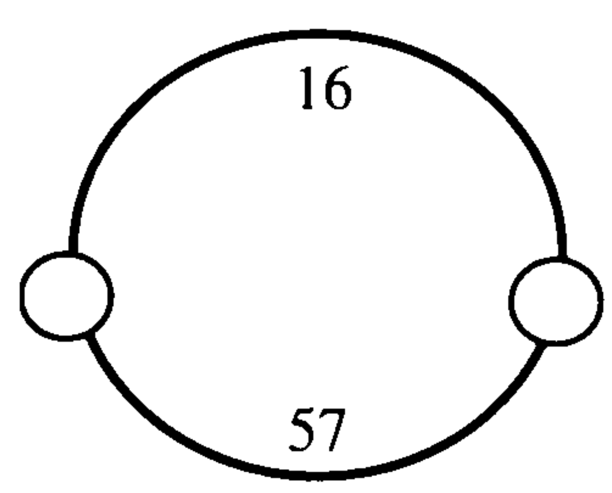


- Identify the child clusters of cluster 59, as cluster 15 and 58.
- Check the deterioration events required to cause failure in this structural ring, (shown at the upper left corner of the structural ring in Figure 8.9), which is 2.
- Decide which child cluster is to break at this stage:

  Cluster 15 is identified as the dependent-clusters of cluster 58, therefore:

  - Ignore cluster 15.

- Search down cluster 58.
  - The corresponding structural ring is an over-stiff 2-link-ring:

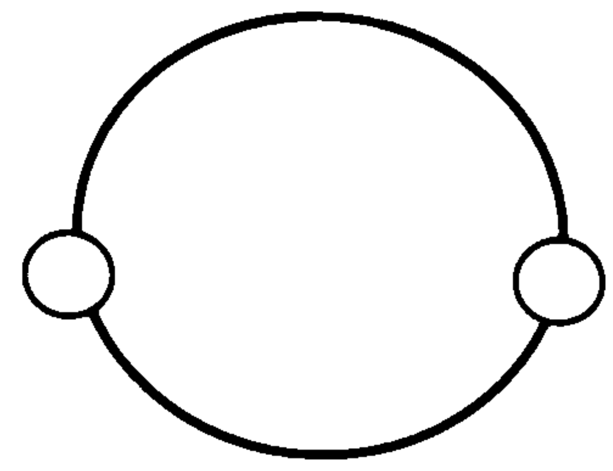


- Identify the child clusters of cluster 58, as cluster 16 and 57.
- Check the deterioration event required to cause failure in this structural ring, which is 2.
- Decide which child cluster is to break at this stage:

  Cluster 16 is identified as the dependent-clusters of cluster 57, therefore:

  - Ignore cluster 16.

- Search down cluster 57.
  - The corresponding structural ring is an over-stiff 2-link-ring:

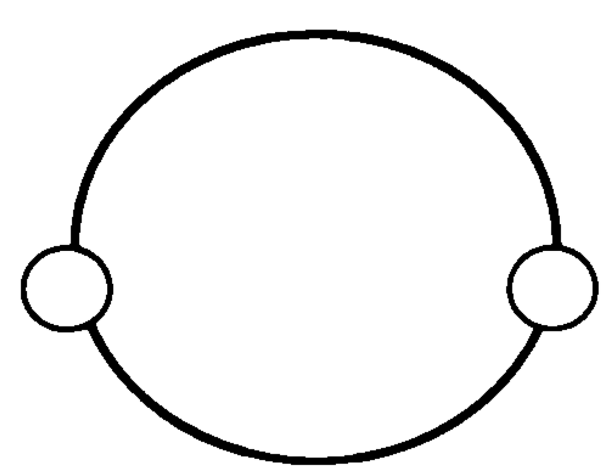


- Identify the child clusters of cluster 57, as cluster 17 and 56.
- Check the deterioration event required to cause failure in this structural ring, which is 2.
- Decide which child cluster is to break at this stage:

  Cluster 17 is identified as the dependent-clusters of cluster 56. therefore:

  - Ignore cluster 17.

- Search down cluster 56.
  - The corresponding structural ring is an over-stiff 2-link-ring:

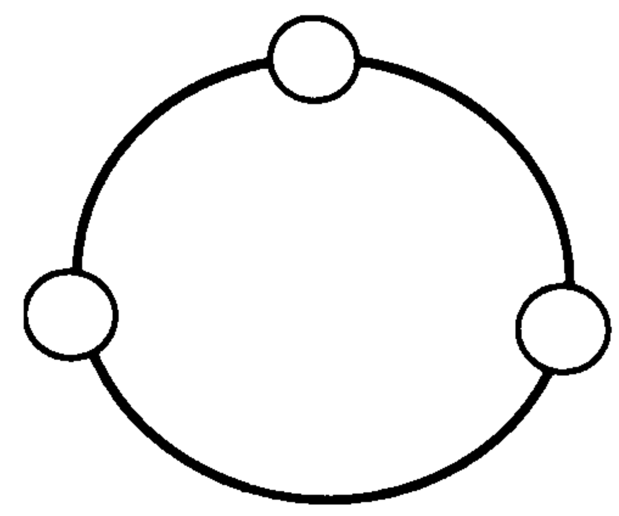


  - Identify the child clusters of cluster 56, as cluster 52 and 55.
  - Check the deterioration event required to cause failure in this structural ring, which is 2.
  - Decide which child cluster is to break at this stage:

    Cluster 52 and 55 are both independent branch clusters, therefore:

    - Search down both clusters. No inherited events for cluster 55 because the cluster *contains* reference. One inherit event is passed to cluster 52.

- Search down cluster 52.
  - The corresponding structural ring is a just-stiff 3-link-ring:

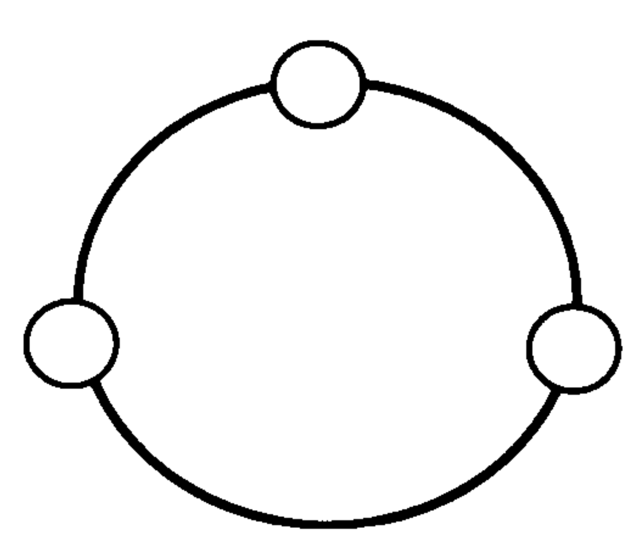


- Identify the child clusters of cluster 52, as cluster 12, 49 and 50.
- Check the deterioration event required to cause failure in this structural ring, which is 1. There is one inherited event. The total deteriorating event required is 2.
- Decide which child cluster is to break at this stage:

  Cluster 12 and 50 are identified as dependent-clusters of cluster 49, therefore:

  - Ignore cluster 12 and 50. One inherited event is passed to cluster 48.

- Search down cluster 49.
  - The corresponding structural ring is a just-stiff 3-link-ring:



  - Identify the child clusters of cluster 49, as cluster 10, 14 and 48.
  - Check the deteriorating event required to cause failure in this structural ring, which is 1. There is one inherited event. The total deteriorating event required is 2.
  - Decide which child cluster is to break at this stage:
    - Check leaf-relationship with reference of all child clusters:

      All three clusters *can form ring* with the reference, therefore:

    - Check minimum damage demand of all child clusters:

      Cluster 10 has the lowest minimum damage demand.

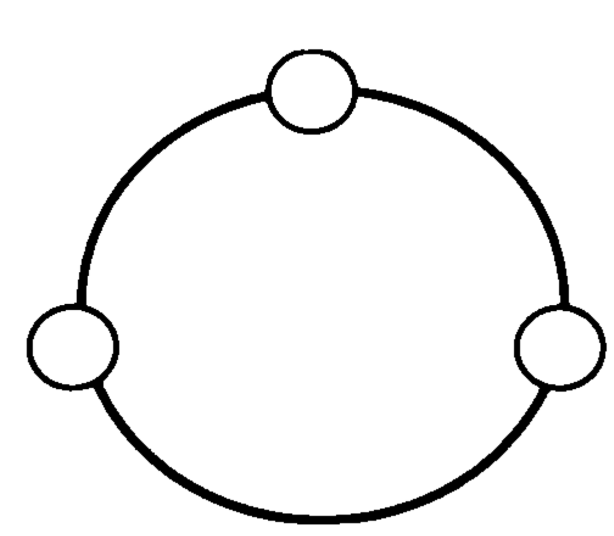
  - Decide on appropriate action:

        - Check the total deteriorating event required, which is 2. The appropriate action will be to form a pin, which causes one deteriorating event.
    - ***Take action***: to form a pin in cluster 10.
    - Modify the total number of deteriorating events 2-1 = 1.
- Decide whether it is necessary to carry on search:
    - Check the number of events left.

      Events left = 1. Therefore,
    - Carrying on search.
- Decide on which cluster to search next:
    - Check the remaining child clusters in cluster 49.
        - Check leaf-relationship with reference of the remaining child clusters:

          Both cluster 14 and 48 can form ring with the reference cluster.
        - Check minimum damage demand of the remaining child clusters:

          Cluster 48 has lower minimum damage demand, therefore,
- Search down cluster 48.
    - The corresponding structural ring is a just-stiff 3-link-ring:



    - Identify the child clusters of cluster 48, as cluster 3, 11 and 47.
    - Check the deteriorating event required to cause failure in this structural ring, which is 1. There is no inherited event
    - Decide which child cluster is to break at this stage:
        - Cluster 47 is identified as a dependent-cluster of cluster 3 and 11, therefore,
            - Ignore cluster 47.

- Check leaf-relationship with reference of the remaining child clusters:

  Both clusters *can form ring* with the reference. therefore:

- Check minimum damage demand of the remaining child clusters:

  Cluster 11 has the lowest minimum damage demand.

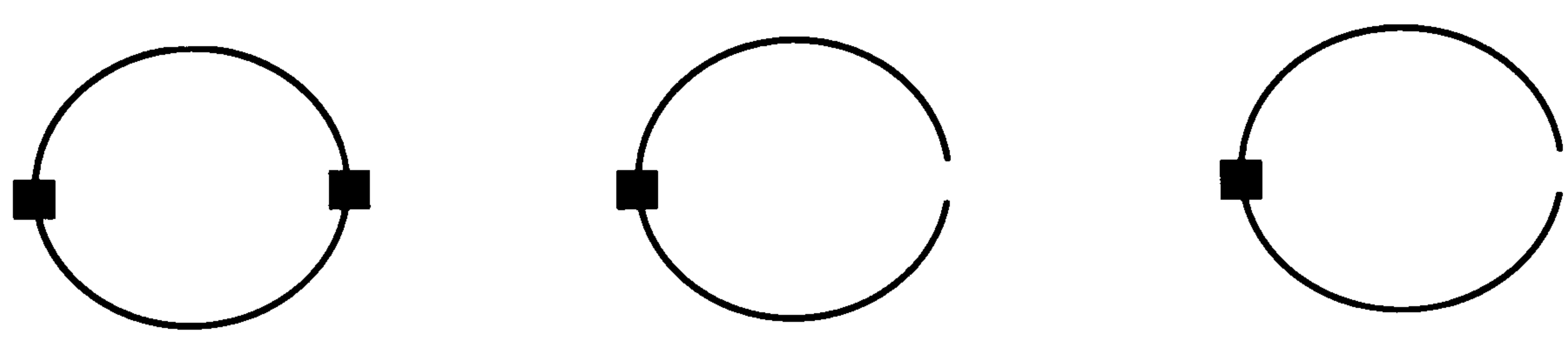
- Decide on appropriate action:
  - Check the total deteriorating event required, which is 1. The appropriate action will be to form a pin, which causes one deteriorating event.
- ***Take action***: to form a pin in cluster 11.
- Modify the total number of deteriorating events for cluster 52: 1-1 = 0.
- Decide whether it is necessary for further search in cluster 52:
  - Check the number of events left.

    Events left = 0. Therefore,

  - Search is completed for cluster 52.

- Search down cluster 55.

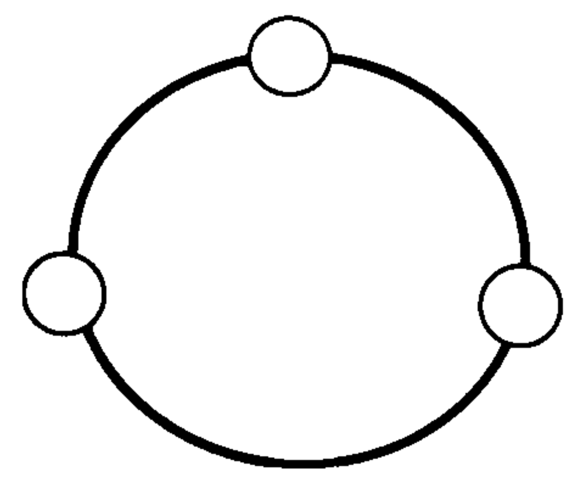
  For next three steps, the three corresponding structural rings all contain reference-leaf as one of the child clusters:



Reference clusters are not considered as damagable, therefore, the process ignore the reference leaves in these two structural rings and goes on search down the other child cluster.

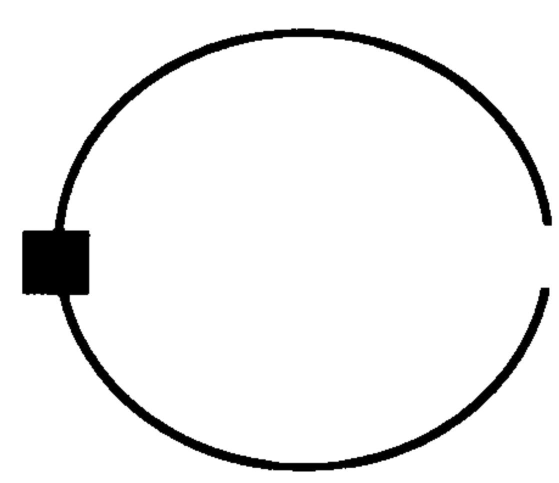
There are 4-1 = 3 inherited events being passed to cluster 51.

- Search down cluster 51.
  - The corresponding structural ring is a just-stiff 3-link-ring:



- Identify the child clusters of cluster 51, as cluster 40, 42 and 44.
- Cluster 42 and 44 are identified as dependent-clusters of cluster 40, therefore,
  - Ignore cluster 42 and 44. Three inherited event are passed to cluster 40.

- Search down cluster 40.
  - The corresponding structural ring is a just-stiff 2-link-ring:



- Identify the child clusters of cluster 40, as cluster 33 and 39.
- Check the deterioration event required to cause failure in this structural ring, which is 1. There are three inherited event. The total deteriorating event required is 1 + 3 = 4.
- Decide which child cluster is to break at this stage:
  - Check leaf-relationship with reference of both child clusters:

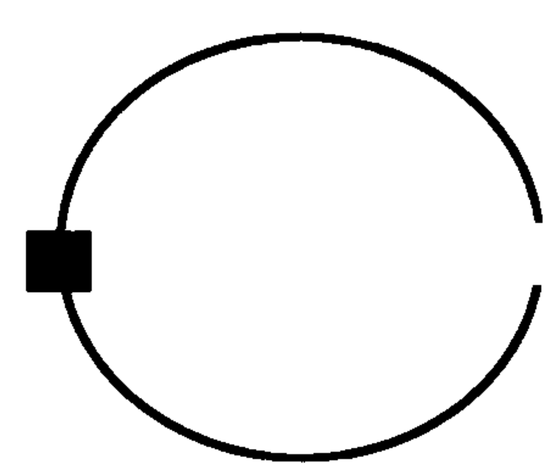
    Both cluster 33 and 39 can form overlap with reference.
  - Check minimum damage demand:

    Cluster 33 has lower minimum damage demand.
- Decide on appropriate action:
  - Check the total deteriorating event required, which is 4. The appropriate action will be to cut, which causes three deteriorating event.
- ***Take action***: to cut in cluster 33.

- Modify the total number of deteriorating events for cluster 40: 4-1 = 1.
- Decide whether it is necessary for further search in cluster 40:
    - Check the number of events left.

      Events left = 1. Therefore,
    - Carrying on search.

- Search down cluster 39.
    - The corresponding structural ring is a just-stiff 2-link-ring:



    - Identify the child clusters of cluster 39, as cluster 20 and 38.
    - Check the deterioration event required to cause failure in this structural ring, which is 1. The inherited event is 1 - 1 = 0. The total deteriorating event required is 1.
    - Decide which child cluster is to break at this stage:
        - Cluster 38 is identified as a dependent-cluster of cluster 20, therefore,
        - Ignore cluster 38, break cluster 20.
    - Decide on appropriate action:
        - Check the total deteriorating event required, which is 1. The appropriate action will be to form a pin, which causes one deteriorating event.
    - ***Take action***: to form a pin in cluster 20.
    - Modify the total number of deteriorating events for cluster 39: 1-1 = 0.
    - Decide whether it is necessary for further search in cluster 39:
        - Check the number of events left.

          Events left = 0. Therefore,
        - Search is completed for cluster 39.

- Search is completed.

The process of searching for the maximum failure scenario is similar to that for the total failure scenario, except the scale of failure is not complete separation from the reference. The result of the maximum failure scenario is, unexpectedly, to form a pin in cluster 4. Although the scale of damage does not seem to be large, the very small relative effort to cause the damage make the failure scenario highly vulnerable.

The details of the total, maximum and several other interesting failure scenarios are listed in Table 8.26. Apart from the maximum failure scenario, there are two other failure scenarios (to pin cluster 18 and to pin cluster 21) which are more vulnerable than the total failure scenario because of the relative ease of damaging the members.

Table 8.26 Some interesting failure scenarios for Combined-1

Failure scenarios	E_r **(Relative Damage Demand)**	γ **(Separateness)**	ξ **(Vulnerability Index)**
Pin Cluster 10, Pin Cluster 11, Cut Cluster 33, Pin Cluster 20.	0.1929	1	5.18
Pin Cluster 4	0.0128	0.2788	21.68
Pin Cluster 18	0.021	0.373	17.93
Pin Cluster 21	0.0388	0.3167	8.16
Cut Cluster 19	0.131	0.602	4.6
Cut Cluster 26	0.143	0.594	4.15

8.6 Conclusions

In this chapter, three different types of structure were used to test the operation of the program SAVE, which is a program developed to carry out vulnerability analysis. For each of the structures, the process of cluster formation, the hierarchical representation and result of various vulnerable failure scenarios were presented.

The step-by-step illustration of the process of cluster formation allows the examination of the candidate selection according to the new ordered set of clustering criteria. Compared to the old single measure clustering criteria used in previous research, the new criteria enable the algorithm to discriminate cases which previously are not possible. A second (damage demand), third (nodal connectivity) and fourth (distance from reference) measure is checked against before a final selection is made. The new criteria has improved the quality and resolution of candidate selection in the clustering algorithm.

The searching algorithm in Chapter 7 was illustrated in detail with two of the examples to allow close examination of the complicated algorithm.

The search process identifies the vulnerable failure scenarios including: minimum demand failure scenario, the least well-formed cluster scenario, the total failure scenario, the maximum failure scenario and other interesting failure scenarios.

Vulnerability analysis enables the identification of vulnerable failure scenarios which may not be intuitively obvious. This is done by using the vulnerability index which is a measure of the potential of disproportionate consequences in the event of damage or failure.

Part IV System vulnerability and structural monitoring

Chapter 9

Structural Monitoring with Vulnerability Analysis

9.1 Objectives

The objectives of this chapter are to:

- review briefly current practice and methodology of structural monitoring;
- discuss the systems approach in the general monitoring strategy and the potential application of structural vulnerability theory in planning a structural monitoring scheme.

9.2 Introduction

Currently, structural monitoring for building structures is mainly concerned with assessment and appraisal of structures. It tends to be only carried out when there is perceived to be a particular problem, such as when an under-designed member or joint is discovered whilst the structure is in service. However, it is important in a systems approach to safety management to investigate the potential of structural monitoring as an integral part of the design, construction and maintenance process.

9.3 Current practice of structural monitoring

Current structural monitoring practice covers a range of methods and technologies depending on the objectives of the monitoring. In building structures, it is mainly concerned with assessment and appraisal of structures relating to their safety and serviceability.

9.3.1 General purposes

The general purposes of structural monitoring are to:

- assess safety and serviceability of in-service structures;
- provide performance specifications of structures;
- improve understanding of behaviour of full-scale structures, therefore to provide feedback information to the design process;
- gain understanding of the environment of a structure, such as nature and change in loading, etc.;
- address specific problems identified with a particular structure, for instance movements during construction;
- detect abnormal response which is related to unexpected action effect, therefore to give early indication of serious structural deterioration and enable various courses of action to be considered.

Moss and Matthews (1995) summarised the following circumstances as appropriate for installing structural monitoring scheme:

- Modifications to existing structures.
- Monitoring of structures affected by external works.
- Monitoring building demolition.
- Structures subject to long term movements or degradation of materials.

- Fatigue assessment.
- Novel systems of construction.
- Feedback loop to design.

9.3.2 Methods and instrumentation

The most commonly used monitoring methods can be categorised into two types:

- manual methods, and
- automatic methods.

The manual methods are usually chosen for a monitoring scheme which involves a limited amount of instrumentation and requires either frequent measurement taken over a short term or infrequent measurement over a longer time scale. The manual methods include:

- visual inspection: such as observing cracks, damage, etc.,
- position and movement measurements (vertical and horizontal): using levelling surveys, theodolite and plumb-bobs, or more advanced, electronic and laser systems,
- corrosion measurement: such as resistivity measurements.

Automatic methods are those that employ an automated monitoring system which consists of a number of sensors and transducers combined with signal conditioning and connected to data-acquisition and recording equipment. The parameters of interest are measured as electrical signals and processed into an appropriate form to be recorded and stored. Many automatic systems can be controlled away from the site via a modem. Automatic methods enable systematic processing of data, hence reducing potential for certain human error.

A major consideration when choosing monitoring methods is cost. The balance of cost between manual and automatic methods changes constantly with the development of new technology.

Both methods may involve tasks ranging from semi-skilled (recording temperature, pressure, etc.), skilled (frequency analysis, data logging, etc.) to highly skilled (pattern recognition and interpretation, acoustic emission). Instrumentation used in structural monitoring can be categorised into two headings: those for measuring action effects on structures and those for measuring responses of structures. Details of monitoring instrumentation has been reviewed by various researchers at BRE (Moss, Brooke and Sagar, 1993; Moss and Matthews, 1995).

With automatic methods, one issue is efficient data collecting and storage. Data reduction can be achieved by intelligent control and adaptive logging (Davis and Vann, 1992). The strategy involves setting up a threshold value to distinguish "normal" and "significant" data and adapting the sampling rate during logging. The principle of active control in monitoring is illustrated in Figure 9.1 with an example of load effect monitoring of a bridge.

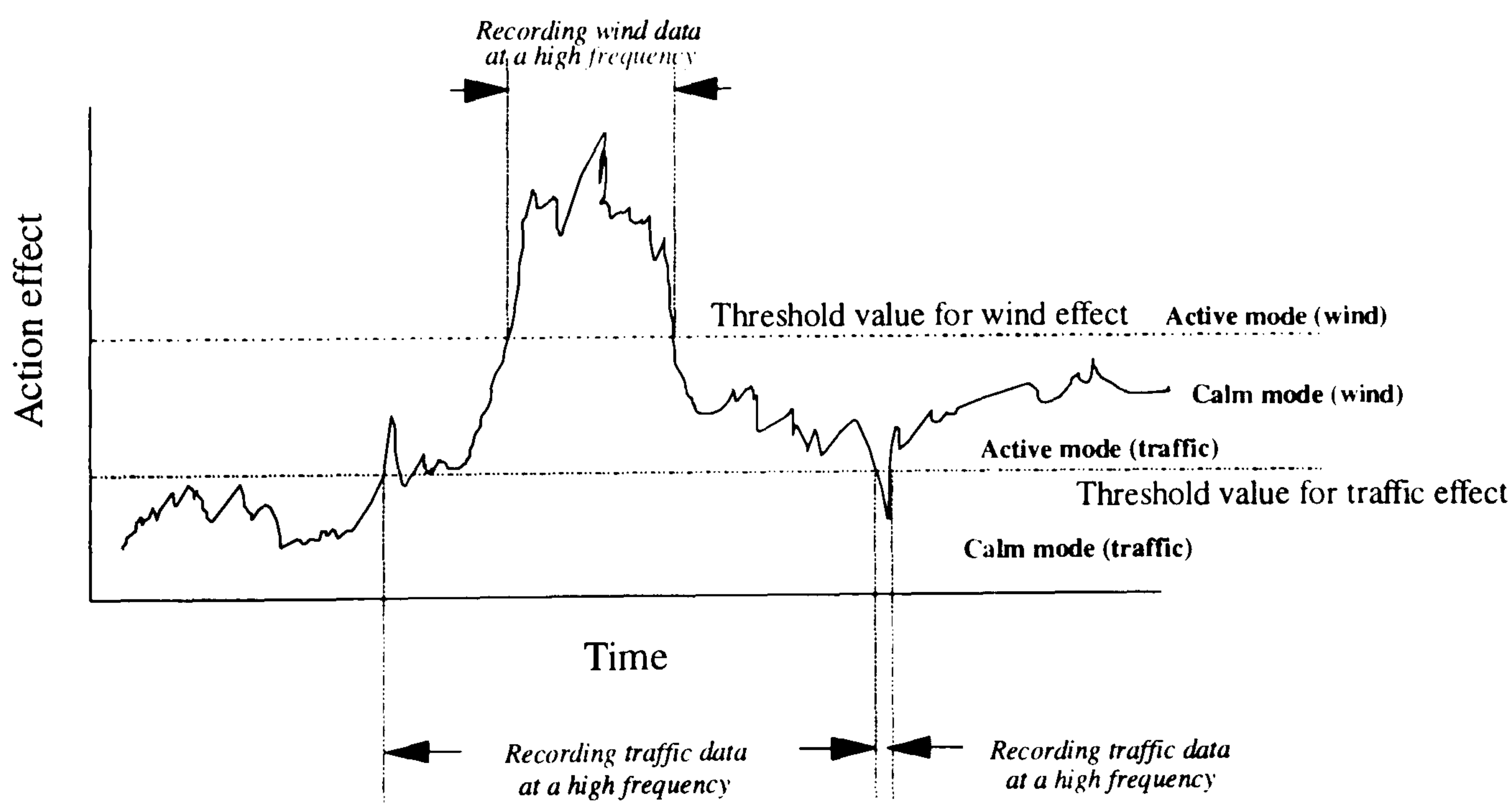


Figure 9.1 Demonstration of active control principle of automatic monitoring system

Another important practice is to make a clear and rigorous procedure as guideline for monitoring. A general procedure made by Dynamic Testing Agency (DTA) provides clear guideline on planning, viability analysis, detailed design, implementation, operation and review of condition monitoring (DTA, 1995).

9.3.3 Problems

There are several general problems associated with defining a monitoring scheme. In some cases, the objectives and precise benefits of monitoring may be unclear and careful consideration needs to be given to the choice of adequate parameters and appropriate instrumentation and methods. To reduce costs, the position and frequencies of measurement have to be considered carefully. For long-term monitoring, especially in harsh conditions, there may be problems related to the reliability and performance of instrumentation, therefore consideration must be given to fitness for purpose of the instrumentation and suitable level of redundancy in the monitoring system. Uncertainty may also come from modelling and interpretation of acquired data.

9.4 Vulnerability analysis and structural monitoring

Currently, structural monitoring in building structures is only carried out when there is perceived to be a particular problem. Such general approach is reactive. For a more proactive approach, investigation should be made to integrate structural monitoring as part of the whole process of design, construction and maintenance to achieve maximum quality and financial benefits. Such practice has already been implemented in some industry where safety concern is paramount and resource to monitoring is justified. In specific case such as nuclear-safety-related structures, structural monitoring is an essential part of maintenance scheme throughout the operational life time of the structures (Smith, 1996).

Monitoring for safety management purpose requires that special consideration is given to the allocation of resources. Hence one of the first issues is to determine appropriate position for deploying monitoring instrumentation.

Good engineering judgement and intuition is always important but not sufficient. A theoretical approach is required to address the problem. One powerful tool is reliability theory, with which the probability of failure of a structure can be studied and estimated, therefore giving guidance as to where in the structure needs to be monitored.

However, as discussed in Chapter 2, reliability theory addresses only one side of the problem, that is, what is the most likely failure scenario. Vulnerability lies in a structure where unlikely events occur and cause disproportionate consequences. The vulnerable parts in a large and complex structural system may easily be overlooked because they are not the most likely failure scenario identified by reliability analysis or perceived by engineers.

The vulnerability theory presented in Chapter 2 ~ 6 is a theory of structural form and able to identify the vulnerable part or parts of a structure in terms of its form and failure consequences. These vulnerable parts are very important in a monitoring scheme aimed at safety management.

Using the structure shown in Figure 8.8 (Chapter 8):

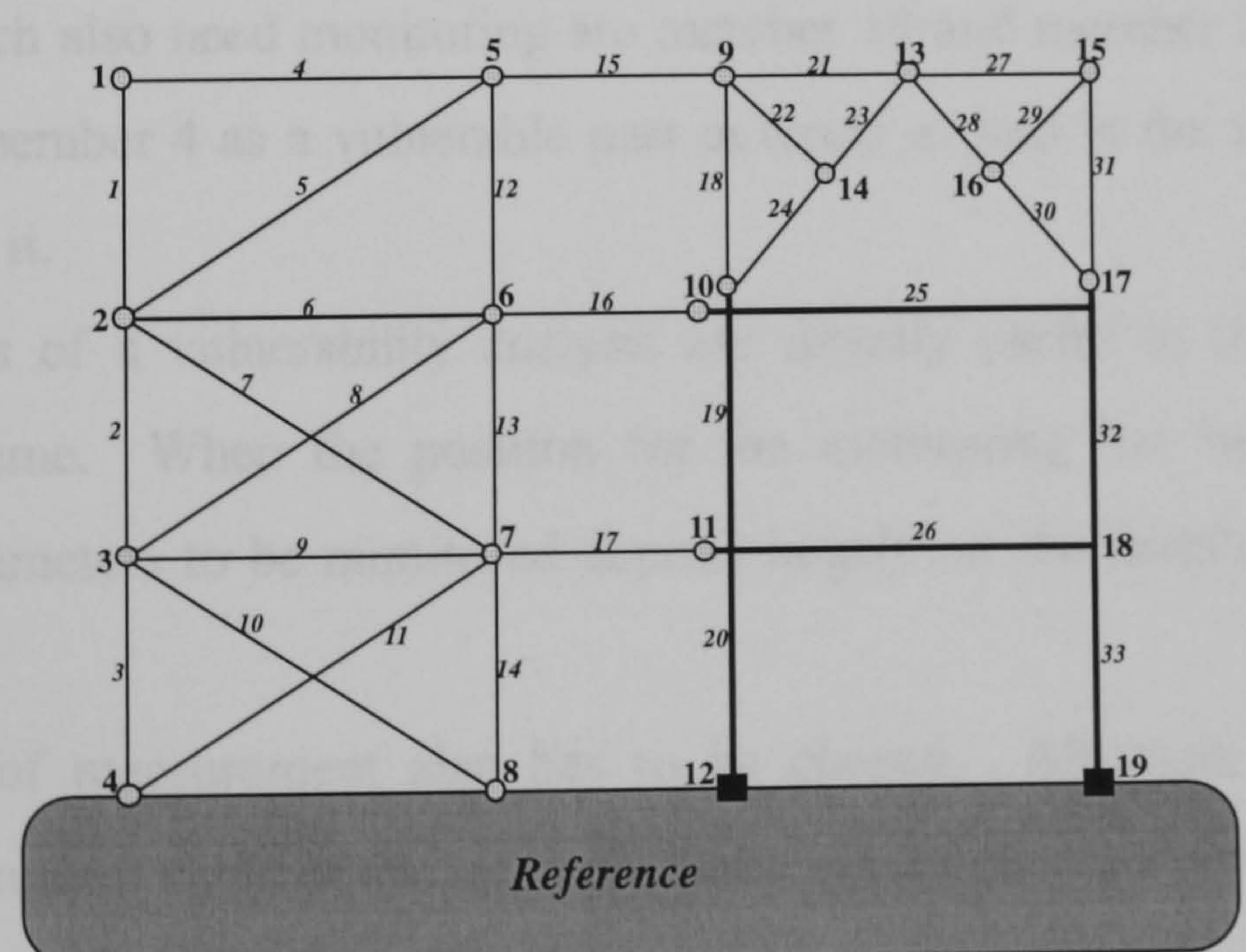


Figure 8.8 The structure --- Combined-1

the vulnerable failure scenarios have been listed in Table 8.26 (Chapter 8):

Table 8.26 Some interesting failure scenarios for Truss-2

Failure scenarios	E_r **(Relative Damage Demand)**	γ **(Separateness)**	ξ **(Vulnerability Index)**
Pin Cluster 10, Pin Cluster 11, Cut Cluster 33, Pin Cluster 20.	0.1929	1	5.18
Pin Cluster 4	0.0128	0.2788	21.68
Pin Cluster 18	0.021	0.373	17.93
Pin Cluster 21	0.0388	0.3167	8.16
Cut Cluster 19	0.131	0.602	4.6
Cut Cluster 26	0.143	0.594	4.15

The total failure scenario identified member 10, 11, 20 and 33 as the parts which obviously need to be protected or monitored. Judged by the vulnerability index, the vulnerable parts, according to the maximal and interesting failure scenarios given in Table 8.26, which also need monitoring are member 18 and member 21. The analysis also identified member 4 as a vulnerable part in terms of loss in the structural form if failure occurs in it.

Thus the results of a vulnerability analysis are directly useful in the planning of a monitoring scheme. When the position for the monitoring has been selected, the appropriate parameters to be monitored depend largely on the details of each specific project.

The frequency of measurement also has to be chosen. Although the issue is not covered by the current stage of the research, future development in vulnerability theory as discussed in Chapter 2, (see Section 2.) may contribute to tackle the problem since vulnerability of a structural system will be considered as a process through time.

9.5 Conclusions

As far as building structures are concerned, current structural monitoring practice is mainly reactive to circumstances. It is mainly concerned with assessment and appraisal of structures and tends to be only carried out when there is perceived to be a particular problem. Development in technology provides more methods and improves accuracy and efficiency of monitoring measurement. However, a more proactive approach in which structural monitoring is considered as an integral part of the whole design, construction and maintenance process should be investigated for a systems approach to safety management.

In determining appropriate positions in a structure for monitoring for the purpose of safety management, the part or parts which has high probability of failure can be analysed with reliability theory. However, it is only part of the problem. A structure is vulnerable if unlikely events could occur and cause disproportionate consequences. Vulnerability theory is a theory of structural form and is able to identify the vulnerable part or parts in a structure according to the form and failure consequences. It will help to reduce the risk of overlooking vulnerable but otherwise unlikely failure scenarios.

Part V Conclusion

Chapter 10

Conclusions & Recommendations

10.1 Conclusions

1. Current safety and reliability procedures for structural design are the limit state design method and reliability theory. Problems arise from the difficulty in reliability theory of dealing with complex dependencies between random variables describing the system. This research adopted an approach that looks at the whole problem from a different perspective which identifies the weakest links in the system.

2. The vulnerability of a structural system is the susceptibility of it to disproportionate consequences in the event of some damage or failure. Internal vulnerability lies within the system and stems from its internal configuration and form.

3. A structural system is vulnerable if certain damage processes in the system are capable of causing severe and disproportionate consequences at the system level. A structural system is robust if it can withstand damage in the system without loss of its required functions.

4. A graph model of a structure consists of a set of joint objects and a set of member joints. Each member object is defined by a pair of joint objects and is a link, or a communication channel between joint objects.

5. A structural ring is a concept of the form of a structure. It is a pattern in the graph model which represents a set of structural objects that is capable of transmitting force along arbitrary direction in space. Unlike previous definitions of it as an object, a structural ring is a pattern. Just like a circle is not an object but a concept or a description of an object.

6. Two types of structural ring, i.e. 3-link-ring and 2-link-ring, can represent most cases of structural configuration. The previous theory has no theoretical limit on the number of objects in a structural ring. This is important for potential implementation.

7. Well-formedness is a measure of the quality of the form of a structure. The measure is independent of the co-ordinate system. It is related to the principal stiffness coefficients of the joints, the type of joints, the stiffness of the members and the configuration of the members in the structure.

8. In the graph model, a structural cluster is a subset of a structure, the objects of which must be (a) able to form a structural ring, and (b) more tightly connected to each other than to those not inside the cluster.

9. The clustering criteria has been improved from the single measure criterion in the previous theory to an ordered set of measures which are related to the form of the structure and the damage potential of the cluster. The problem of the previous criteria was that when used for uniform structures, it did not distinguish clusters which have identical well-formedness. Decisions in such cases was made by random choice and the best candidates, in terms of future steps, may have been missed out. Apart from the measure of well-formedness of a cluster, the minimum damage demand, the nodal connectivity of a cluster and the distance from the reference for a cluster have been added to the new clustering criteria. The new criteria can distinguish clusters which are identical according to the old criteria, therefore enabling significant improvement in the resolution of the candidate selection in the process of cluster formation.

10. The form of a structural system can be represented with a hierarchy of structural clusters. In a hierarchical model of a structural system, each structural cluster is a holon. The structural clusters at lower levels have more detailed information of the structure than those at higher levels of description.

11. In a structural system, the damage process can be described in terms of failure scenarios of structural rings. For a structural ring, a failure scenario is defined as a path in the DHSR (Deterioration Hierarchy of Structural Rings) such that the final element is a mechanism. A failure scenario is a pattern in which the structural ring deteriorates to a failure state.

12. As a measure of failure consequence of a failure scenario, the separateness has been redefined as the ratio of the loss in structural well-formedness which is caused by the failure scenario to the well-formedness of the intact structure. In the previous work, the failure consequence was measured by the ratio of the structural well-formedness of the clusters which are structurally disconnected from the reference to that of the clusters which are still connected to the reference. One major problem with the old measure is that the divisor may be zero.

13. The vulnerability index has been defined as a measure of the vulnerability. It is defined as the ratio of the separateness of the damaged structure from the reference cluster to the relative damage demand of a failure scenario. For a failure scenario, the vulnerability index indicates the level of damage upon the structural system for a given level of effort.

14. Structural vulnerability analysis is mainly concerned with the identification of various vulnerable failure scenarios. There are five important vulnerable scenarios. They are: the minimum demand failure scenario; the least well-formed cluster scenario; the total failure scenario; the maximum failure scenario, and interesting failure scenarios.

15. An algorithm for implementing the theory has been derived and a computer program SAVE (Structural Analysis for Vulnerability Estimation) has been developed. The program is written in C. Its main purpose is to demonstrate and test structural vulnerability theory.

16. A set of three different types of structure have been used to test the operation of the program SAVE and to demonstrate vulnerability analysis using the improved theory. Examples have been given where the new measures of the clustering criteria were shown to be critical.

17. Current structural monitoring practice is mainly reactive and tends to be only carried out when there is perceived to be a particular problem. However, a more proactive approach in which structural monitoring is considered as an integral part of the whole design, construction and maintenance process should be investigated for a systems approach to safety management.

18. Vulnerability theory is a theory of structural form and is able to identify the vulnerable part or parts in a structure according to the form and failure consequences. It can help to determine appropriate positions in a structure for structural monitoring for the purpose of safety management and reduce the risk of overlooking vulnerable but otherwise unlikely failure scenarios.

10.2 Recommendations for future work

1. This research is concerned with the system internal vulnerability of a structural system, which is determined by its internal form and configuration. Future work is needed to study the system vulnerability which is related to specific actions, such as certain types of loading. Analysis of the internal vulnerability

may be carried out alongside structural analysis to investigate action-related vulnerability in the structural system.

2. At current stage, the theory only deals with two dimensional structures. Further development is required to extend the theory so that three dimensional structures can be analysed.

3. Future work is needed to generalise the measure of well-formedness to any system. The theory is potentially applicable to any system that can be described by a graph. The major issue is to find a suitable measure of well-formedness.

4. The clustering criteria can similarly be refined and improved further.

5. Although the structural vulnerability theory was developed for civil engineering structures, the principles and techniques can be generalised so that internal vulnerability of general engineering systems can be studied.

REFERENCES

Adams, J. G. U. (1987)
The channel tunnel - a risk not worth taking.
New Civil engineer, 1987, Vol. 761, pp16 - 17.

Alexander, C. J. (1964)
Notes on the synthesis of form.
Harvard University Press, Cambridge, Mass., USA.

Anton, H. (1984)
Elementary linear algebra.
John Wiley & Sons., London.

ASCE (1973)
Structural failures: modes, causes, responsibilities.
ASCE, New York.

Banerjee, S. and Rosenfeld, A. (1993)
Model-based cluster analysis.
Pattern Recognition, Vol. 26, No. 6, pp963 - 974.

Benedetti, D., Benzoni, G. and Parisi, M. A. (1988)
Seismic vulnerability and risk evaluation for old urban nuclei.
Earthquake Engineering and Structural Dynamics, Vol. 16, pp183 - 201.

Benjamin, J. R. and Cornell, C. A. (1970)
Probablity, statistics and decision for civil engineers.
McGraw-Hill., London.

Berge, C. (1962)
The theory of graphs and its applications.
Methven, London.

Bignell, V. Peters, G. and Pym, C (1977)
Catastrophic failures.
The OU Press, Milton Keynes.

Biggs, N. L., Lloyds, E. K. and Wilson, R. J. (1936)
Graph theory, 1736 - 1936.
Oxford University Press, Oxford.

Blaikie, P., Cannon, T., Davis, I. and Wisner, B. (1994)
At risk: natural hazards, people's vulnerability, and disasters.
Routledge, London.

Blockley, D. I. (1980)
The nature of structural design and safety.
Ellis Horwood Limited, Chichester.

Blockley, D. I. (1990)
Open world problems in structural reliability.
Proc. ICOSSAR '89, **3**, 1989, pp1659 - 1665.

Blockley, D. I. (ed.) (1992a)
Engineering safety.
McGraw-Hill, London.

Blockley, D. I. (1992b)
Engineering from reflective practice.
Res. Eng. Des., 1992, **4**, pp13 - 22.

Blockley, D. I. (1993)
Structural failure and hazard engineering.
Structural Engineering International, 4, 1993, pp253 - 257.

Blockley, D. I. (1994)
Uncertain ground: on risk and reliability in geotechnical engineering.
Dept. of Civil Engineering, University of Bristol.

Bondy, J. A. and Murty, U. S. R. (1976)
Graph theory with applications.
Macmillan Press Ltd, London.

Bonner, R. E. (1964)
On some clustering techniques.
IBM Journal, Jan. 1964, pp22 - 32.

Capra, F. (1996)
The web of life.
HarperCollins Publishers, London.

Chandra, S. (1992)
Knowledge-based physical process modelling and explanations.
PhD Thesis, University of Bristol.

Chandra, S., Blockley, D. I. and Woodman, N. J. (1992)
An interacting objects physical process model.
Computing Systems in Engineering, Vol. 3, No. 6, pp661 - 670.

Checkland, P. (1984)
Systems thinking, systems practice.
John Wiley & Sons, Chichester.

Chistofides, N. (1975)
Graph theory: an algorithmic approach.
Academic Press Inc., London.

Coates, R. C. and Kong, F. K. (3rd Edition)
Structural analysis.
Van Nostremd Reinhold (UK).

Comerford, J. B. (1989)
The use of knowledge-based systems in interpretation of measurements.
PhD Thesis, University of Bristol.

Comerford, J. B. and Blockley, D. I. (1993)
Managing safety and hazard through dependability.
Structural safety, **12**, (1993), pp21 - 33.

Cornell, C. A. (1967)
Bounds on the reliability of structural systems.
Journal of Structural Division, Vol. 93, ST1, February, 1967, ASCE.

Cook, J. (1989)
An accident waiting to happen.
Unwin Hyman Limited, London.

Cui, W. and Blockley, D. I. (1991)
On the bounds for structural system reliability.
Structural safety, Vol. 9, (1991), pp247 - 259.

Davis, J. P. and Vann, A. M. (1992)
Intelligent monitoring of civil engineering systems.
Applications of Artificial Intelligence in Engineering VII - Proc. of *AIENG92*.
Coomputational Mechanics Publications.

Dawkins, R. (1986)
The blind watchmaker.
Harlow, Longman, London.

De Bono, E. (1971)
The mechanism of mind.
Harmondsworth, Pelican.

De Bono, E. (1976)
Practical thinking.
Harmondsworth, Pelican.

Department of Environment (1993)
Earthquake hazard and risk in the U.K.
HMSO, London.

Dias, W. P. S. and Blockley, D. I. (1995)
Reflective practice in engineering design.
Proc. Instn Civ. Engrs, Civ. Engineering, 1995, **108**, Nov., pp160 - 168.

Ditlevsen, O. and Bjerager, P. (1986)
Methods for structural systems reliability.
Structural safety, Vol. 3, pp195 - 229.

Dynamic Testing Agency, (1995)
DTA condition monitoring primer.
Interim Publication, DTA.

EERI Committee on Seismic Risk (1984)
Glossary of terms for probabilistic seismic risk and hazard analysis.
Earthquake Spectra, Vol. 1, No. 1, pp35 - 40.

Elms, D. G. (1983)
From a structure to a tree.
Civil Engineering System, **1**, (1983), pp95 - 106.

Emery, F. E. (1969)
Systems thinking.
Penguin Books, London.

Everitt, B. (1974)
Cluster analysis.
Heinemann Educational Books, London.

Fenves, S. J. (1966)
Structural analysis by networks, matrices, and computers.
ASCE. Vol. 92, No. ST1, February, pp199 - 221.

Fischer, J. (1995)
Hierarchical cluster analysis.
Computational Statistics, Vol. 10, (1995), pp21 - 27.

Frost, R. A. (1986)
Introduction to knowledge base systems.
Collins, London.

Gonnet, G. H. and Baeza-Yates, R. (1991)
Handbook of algorithms and data structures in Pascal and C.
Addison-Wesley Publishing Company, London.

Green, A. E. (1983)
Safety systems reliability.
John Wiley & Sons, Chichester.

Hashimoto, M. (1994)
Vulnerability and reliability of structural systems.
PhD Thesis, University of Bristol.

Kessler, A. Ormsbee, L. and Shamir, U. (1990)
A methodology for least-cost design of invulnerable water distribution networks.
Civil Engineering Systems, Vol. 7, No. 1, pp20 - 28.

Klaassen, K. B. and Van Peppen, J. C. L. (1989)
System reliability - concepts and applications.
Edward Arnold, New York.

Koestler, A (1967)
The ghost in the machine.
Hutchinson & Co, London.

Krauthammer, T., Muralidharan, R. and Schmidt, W. (1992)
Combined symbolic-numeric structural damage assessment for post-attack conditions.
Structures Under Shock and Impact 2. Proc. 2nd international conference, Portsmouth, UK, June, 1992, pp271 - 282.

Kuhn, T. S. (1962)
The structure of scientific revolutions.
University of Chicago Press.

Lacayo, R. (1993)
Tower terror. (World Trade Centre)
Time, March, 8, (1993), pp23 - 29.

Laman, G. (1970)
On graphs and rigidity of plan skeletal structures.
Journal of Engineering Mathematics, Vol. 4, No. 4, pp331 - 340.

Lance, G. N. and Williams, W. T. (1966)
A general theory of classificatory sorting strategies. (II. Clustering systems)
Computer Journal, Vol. 9, pp271 - 277.

Mansell, G. (1993)
The failure method and soft systems methodology.
Systemist, Vol. 15, No. 4, Nov. 1993, pp190 - 204.

Marshall, W. T. and Nelson, H. M. (1977)
Structures (2nd Edition)
Longman Scientific & Technical, Harlow.

Mathieu, R. G. and Gibson, J. E. (1993)
A methodology for large-scale R & D planning based on cluster analysis.
IEEE Transactions on Engineering Management, Vol. 40, No. 3, Aug. 1993.

Melchers, R. E. and Tang, L. K. (1984)
Dominant failure modes in stochastic structural systems.
Structural safety, Vol. 2, pp127 - 143.

Melchers, R. E. (1987)
Structural Reliability, analysis and prediction.
Ellis Horwood Limited, Chichester.

Mesarovic, M. D. and Macho, D. (1969)
Foundations for a scientific theory of hierarchical systems.
Hierarchical Structures, Elsevier, New York. pp29 - 50.

Meyer, B., (1988)
Object-oriented software construction.
Prentice Hall International Ltd., London.

Moses, F. (1982)
System reliability developments in structural engineering.
Structural safety, Vol. 1, (1982), pp3 - 13.

Moses, F. (1990)
New directions and research needs in system reliability research.
Structural safety, Vol. 7, (1990), pp93 - 100.

Moss, R. M. and Matthews, S. L. (1995)
In-service structural monitoring: a state of the art review.
The Structural Engineer, Vol. 73, No. 2, Jan. 1995, pp23 - 31.

Doyle, N. (1993)
Design aggravated NY bomb impact. (World Trade Centre)
New Civil Engineer, 4, March, (1993), pp3 - 4.

Pellegrino, S. and Calladine, C. R. (1986)
Matrix analysis of statically and kinematically indeterminate frameworks.
J. Solids Structures, Vol. 22, No. 4, pp409 - 428.

Sanchez-Silva, Taylor, C. A. and Blockley, D. I. (1994)
Evaluation of proneness to failure of a project in an earthquake.
Fifth U.S. National Conference on Earthquake Engineering, Chicago, July 1994.

Sangiovanni-Vincentelli, A., Chen, L. and Chua, L. O. (1977)
An efficient heuristic cluster algorithm for tearing large-scale networks.
IEEE Transactions on Circuits and Systems, Vol. CAS-24, No. 12, pp709 -717.

Sedgewick, R. (1983)
Algorithms.
Addison-Wesley publishing company, London.

Shekar, B., Murty, N. and Krishna, G. (1989)
Structural aspects of semantic-directed clusters.
Pattern Recognition, Vol.22, No.1, pp65 - 74.

Simon, H. A. (1965)
The architecture of complexity.
General Systems Yearbook, 1965, Vol. 10, pp63 - 76.

Smith, L. M. (1996)
In-service monitoring of nuclear-safety-related structures.
The Structural Engineer, Vol. 74, No. 12, pp210 - 211.

Sowa, J. F. (1984)
Conceptual Structures.
Addison-Wesley, London.

Steinberg, D. I. (1974)
Computational matrix algebra.
McGraw-Hill, Inc. New York.

Stone, J. (1989)
Expert system learning from structural failures.
PhD Thesis, University of Bristol.

Swamy, M. N. S. and Thulasiraman, K. (1981)
Graphs, Networks, and Algorithms.
John Wiley & Sons, New York.

Templeman, A. B. (1982)
Civil engineering systems.
The MacMillan Press Ltd, London.

Terry, G. J. (1991)
Engineering system safety.
Mechanical Engineering Publication Ltd, London.

Thoft-Christensen, P. and Baker, M. J. (1982)
Structural reliability and its application.
Springer-Verlag, Berlin.

Thomson, J. R. (1987)
Engineering safety assessment - an introduction.
Longman Scientific & Technical, New, York.

Vemuri, V. (1978)
Modelling of complex systems - an introduction.
Academic Press, New York.

Waddington, C. H. (1977)
Tools for thought.
Basic Books, Inc., New York.

Whyte, L. L., Wilson, A. G. and Wilson, D. (1969)
Hierarchical structures.
American Elsevier Publishing Company, Inc. New York.

Williams, G. (1976)
Computational linear algebra with models.
Allyn and Becon, Inc.

Wilson, B. (1984)
Systems: Concepts, Methodologies and Applications.
John Wiley & Sons, Chichester.

Wirth, N. (1986)
Algorithms and data structures.
Prentice/Hall international, Inc. London.

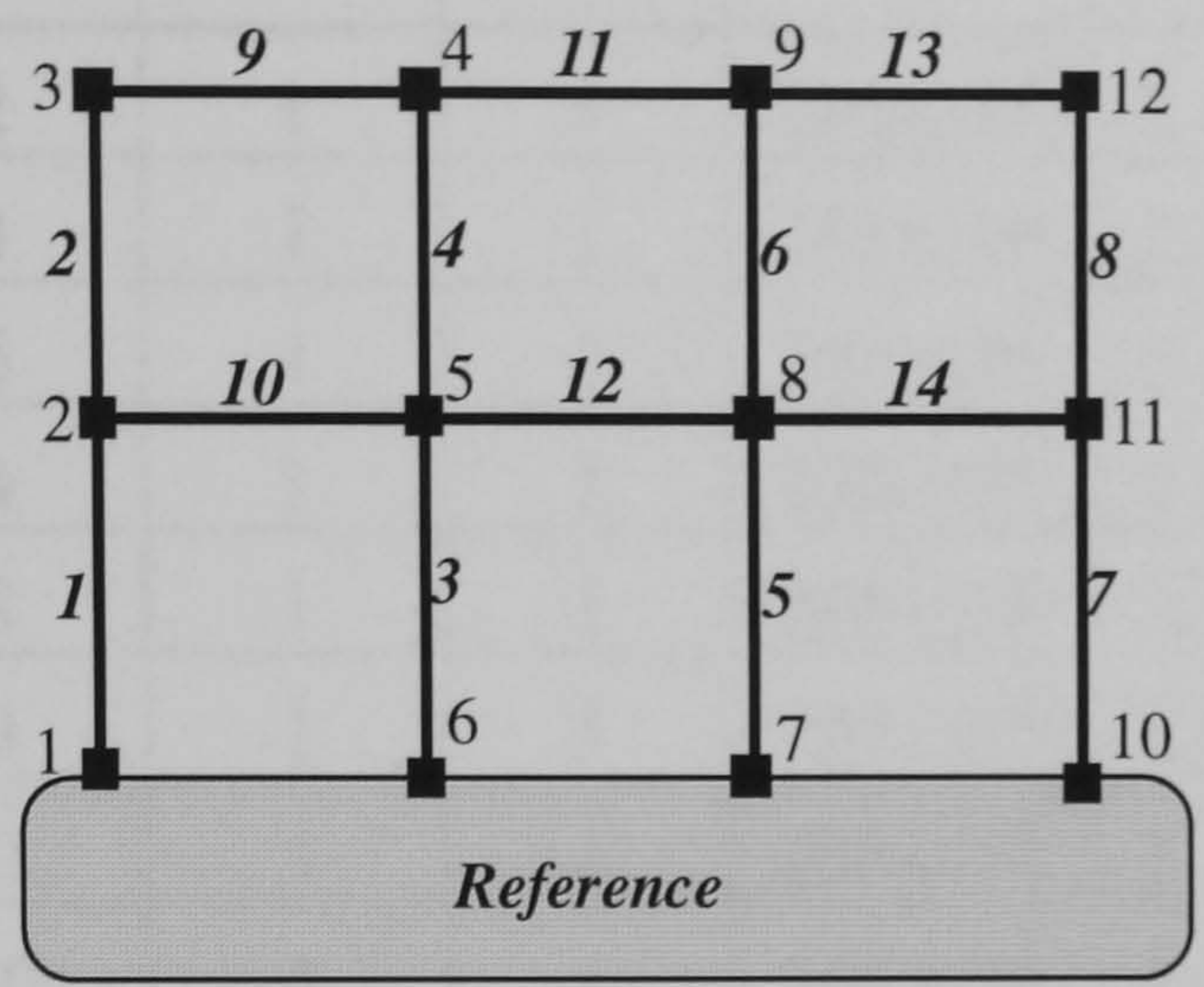
Wu, X. (1991)
Vulnerability analysis of structural systems.
PhD Thesis, University of Bristol.

Yao, J. T. P. (1985)
Safety and reliability of existing structures.
Pitman, Boston.

Zalka, K. A. and Armer, G. S. T. (1992)
Stability of large structure.
Building Research Establishment.

Zimmerman, J. J., Corotis, R. B. and Ellis, J. H. (1992)
Collapse mechanism identification using a system-based objective.
Structural safety, **11** (1992), pp157 - 171.

APPENDIX-1



The structure --- Frame-2

Joint co-ordinate table of Frame-2

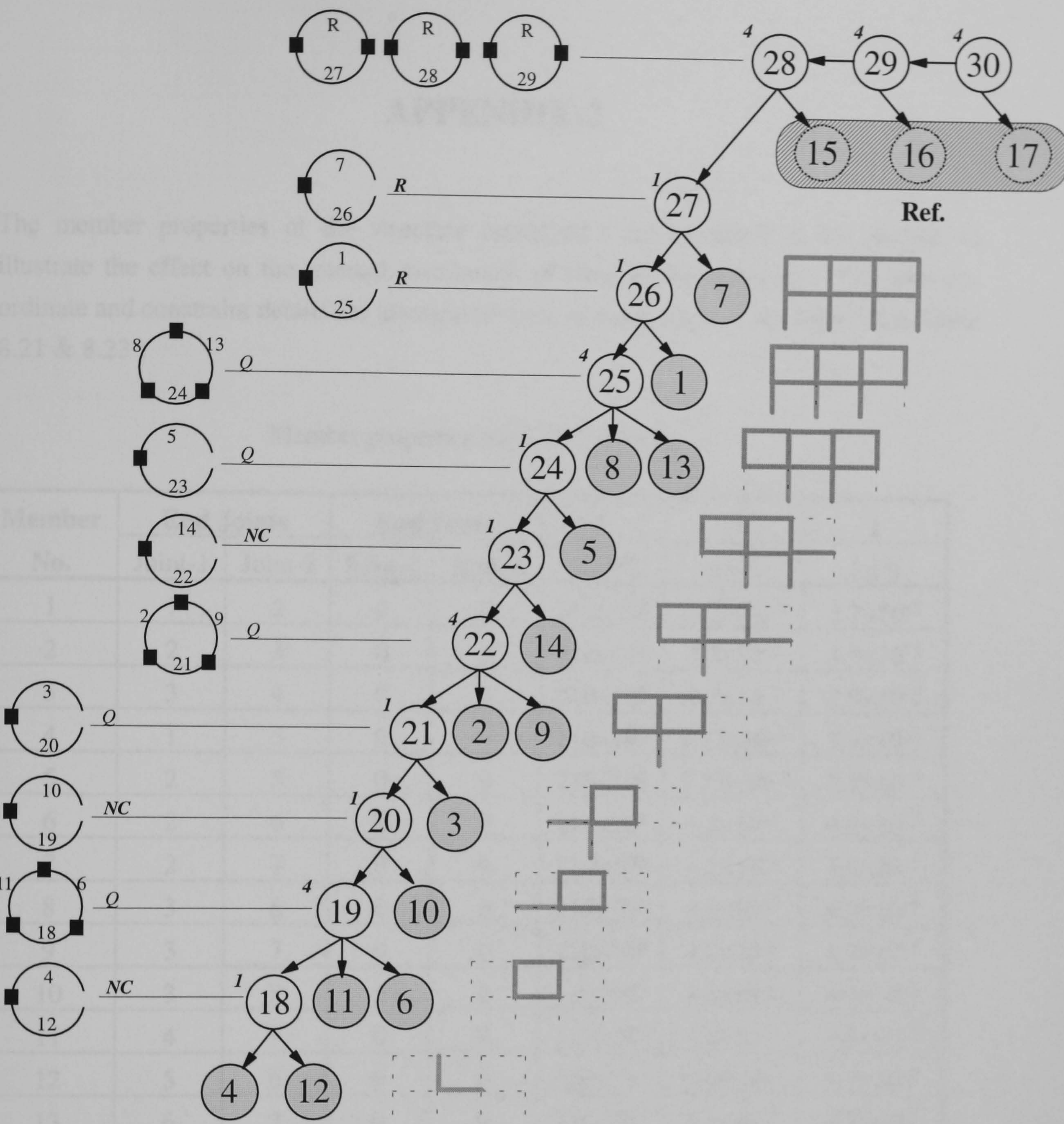
Joint No.	X Co-od. (m)	Y Co-od. (m)
1	0.0	0.0
2	0.0	3.0
3	0.0	6.0
4	3.0	6.0
5	3.0	3.0
6	3.0	0.0
7	6.0	0.0
8	6.0	3.0
9	6.0	6.0
10	9.0	6.0
11	9.0	3.0
12	9.0	0.0

Member properties table of Frame-2

Member No.	**End Joints**		**End Fixity**		**E**	**A**	**I**
	Joint-1	Joint-2	Joint-1	Joint-2	(n/m^2)	(m^2)	(m^4)
1	1	2	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
2	2	3	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
3	5	6	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
4	5	4	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
5	7	8	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
6	8	9	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
7	12	11	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
8	11	10	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
9	3	4	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
10	2	5	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
11	4	9	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
12	5	8	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
13	9	10	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}
14	8	11	1	1	205×10^6	2.16×10^{-2}	1.7×10^{-3}

Constraint condition of Frame-2

Constraint No.	**Joint No.**	**x**	**y**	**θ**
1	1	1	1	1
2	6	1	1	1
3	7	1	1	1
4	12	1	1	1



Hierarchy of Frame-2

The step-by-step cluster formation of Frame-2 is shown without the details of the measures. However, the governing criteria for each step is shown with the hierarchical representation, structural rings and illustration of the position in the structure.

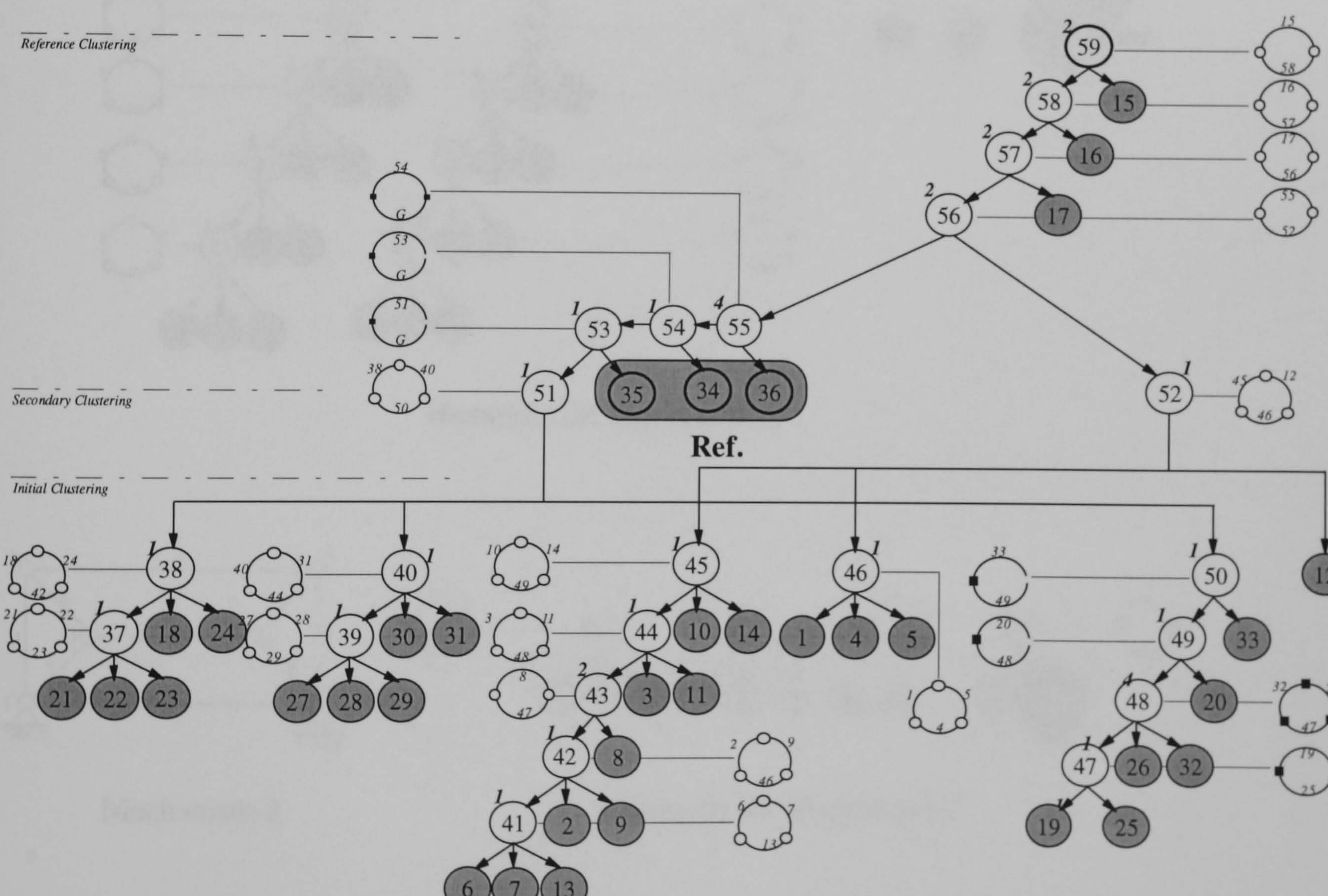
APPENDIX-2

The member properties of the structure combined-1 are modified in this section to illustrate the effect on the internal distribution of form of the structure. The joint co-ordinate and constraint details are identical to those of combined-1. (see Figure 8.8. Table 8.21 & 8.23)

Member properties table of Combined-1a

Member	End Joints		End Fixity		E	A	I
No.	Joint-1	Joint-2	Joint-1	Joint-2	(n/m^2)	(m^2)	(m^4)
1	1	2	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
2	2	3	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
3	3	4	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
4	1	5	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
5	2	5	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
6	2	6	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
7	2	7	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
8	3	6	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
9	3	7	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
10	3	8	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
11	4	7	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
12	5	6	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
13	6	7	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
14	7	8	0	0	210×10^{6}	4.2×10^{-3}	4.9×10^{-5}
15	5	9	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
16	6	10	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
17	7	11	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
18	9	10	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
19	10	11	1	1	210×10^{6}	***0.14×10^{-3}***	***0.5×10^{-5}***
20	11	12	1	1	210×10^{6}	***0.14×10^{-3}***	***0.5×10^{-5}***
21	9	13	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}

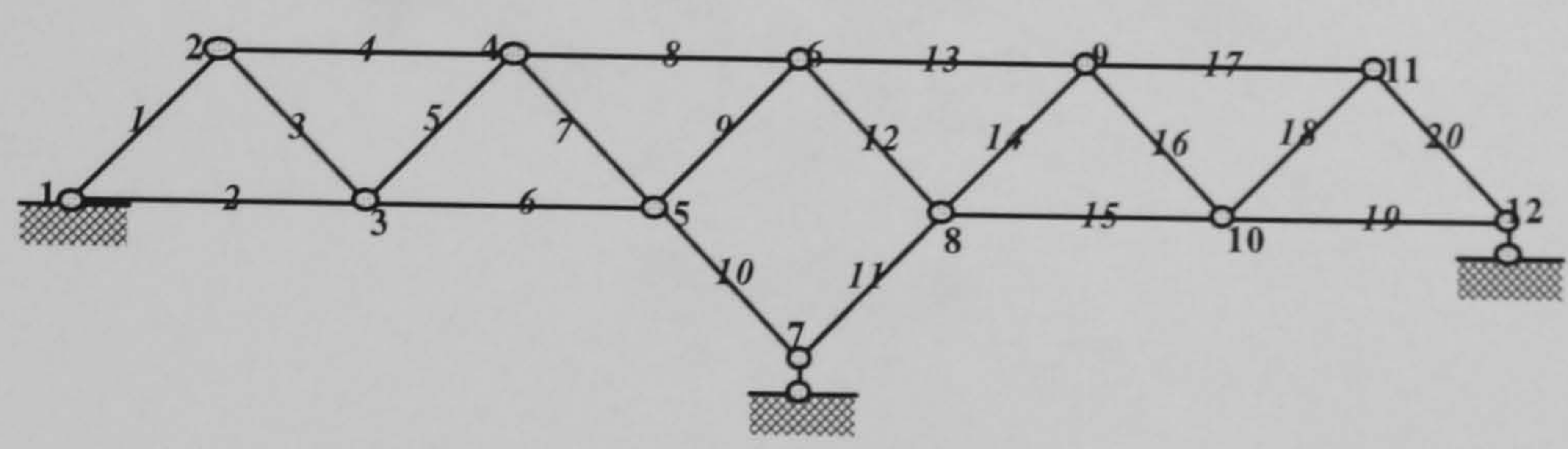
22	9	14	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
23	14	13	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
24	10	14	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
25	10	17	1	1	210×10^{6}	***0.14×10^{-3}***	***0.5×10^{-5}***
26	11	18	1	1	210×10^{6}	***0.14×10^{-3}***	***0.5×10^{-5}***
27	13	15	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
28	13	16	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
29	16	15	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
30	16	17	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
31	15	17	0	0	210×10^{6}	3.23×10^{-3}	3.7×10^{-5}
32	17	18	1	1	210×10^{6}	***0.14×10^{-3}***	***0.5×10^{-5}***
33	18	19	1	1	210×10^{6}	***0.14×10^{-3}***	***0.5×10^{-5}***



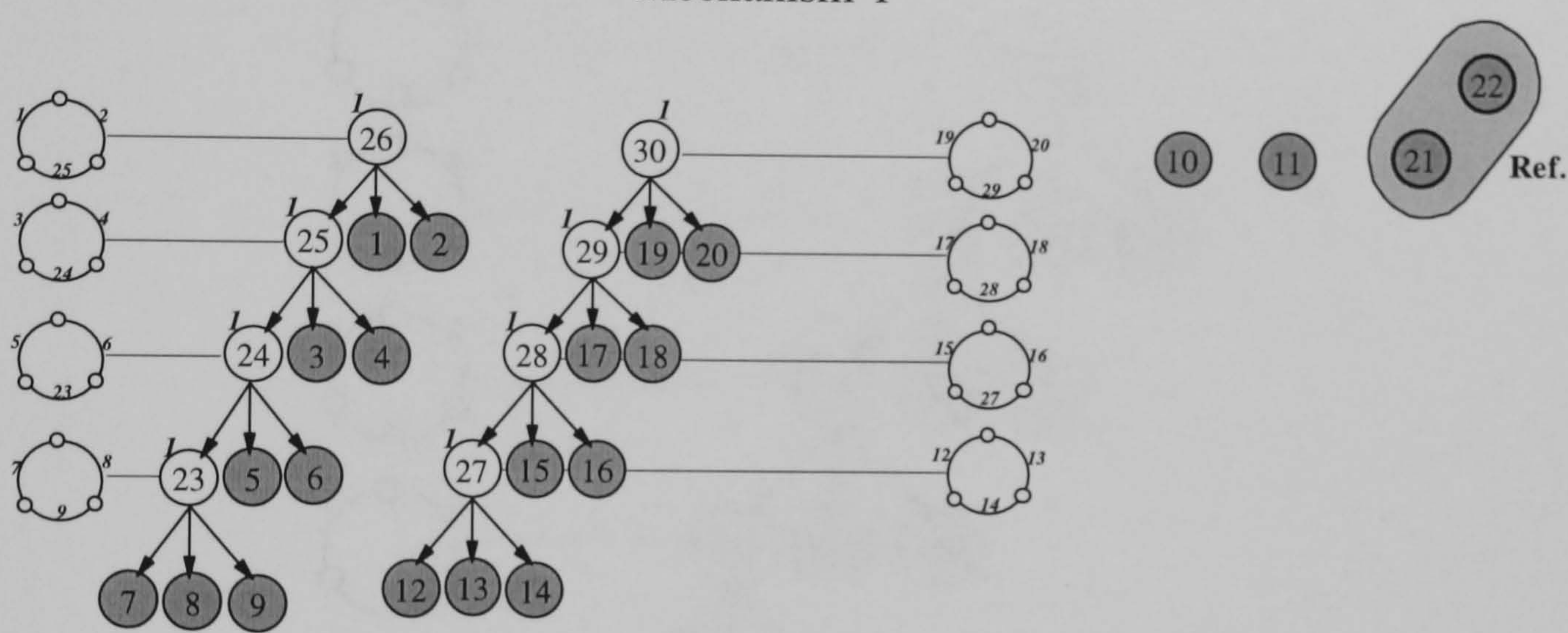
The hierarchy for combined-1a

APPENDIX-3

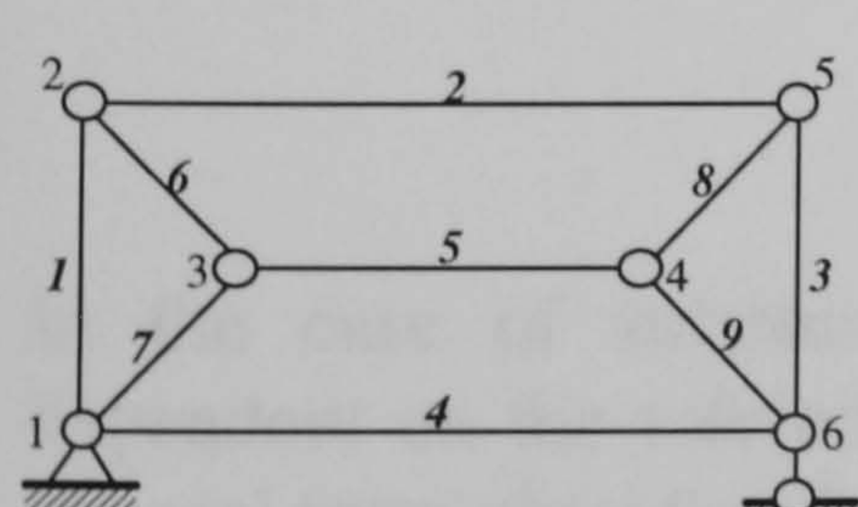
1. Mechanisms:



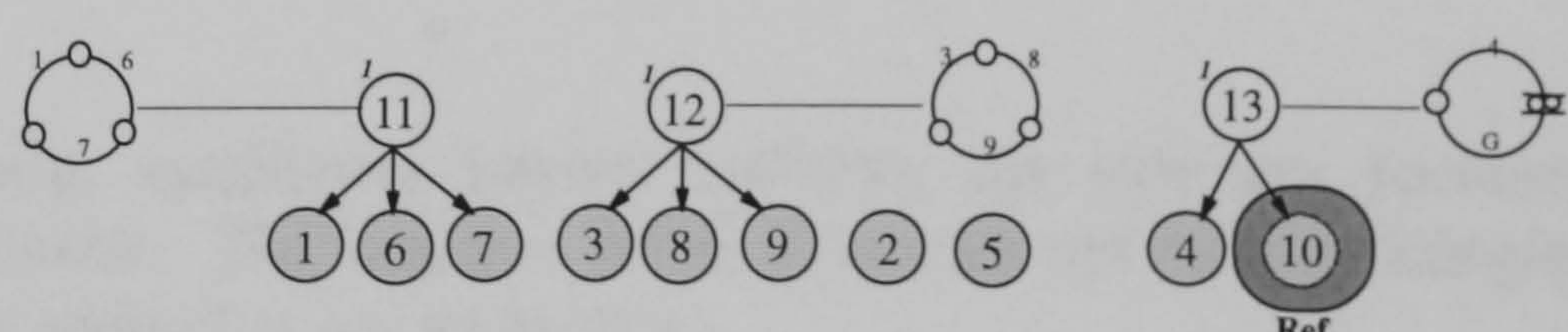
Mechanism-1



Hierarchy for Mechanism-1



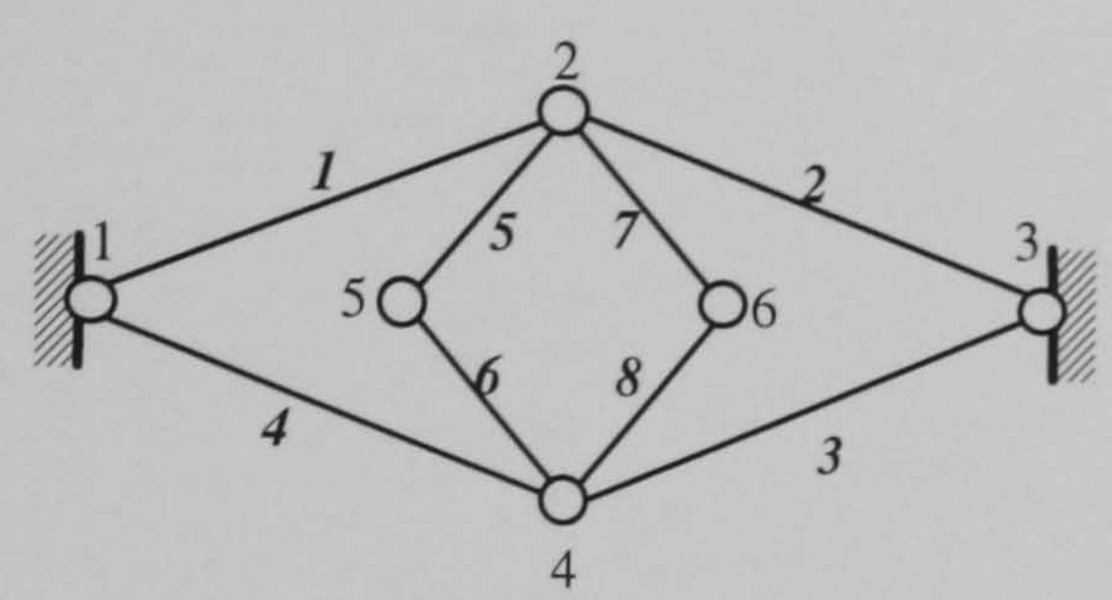
Mechanism-2



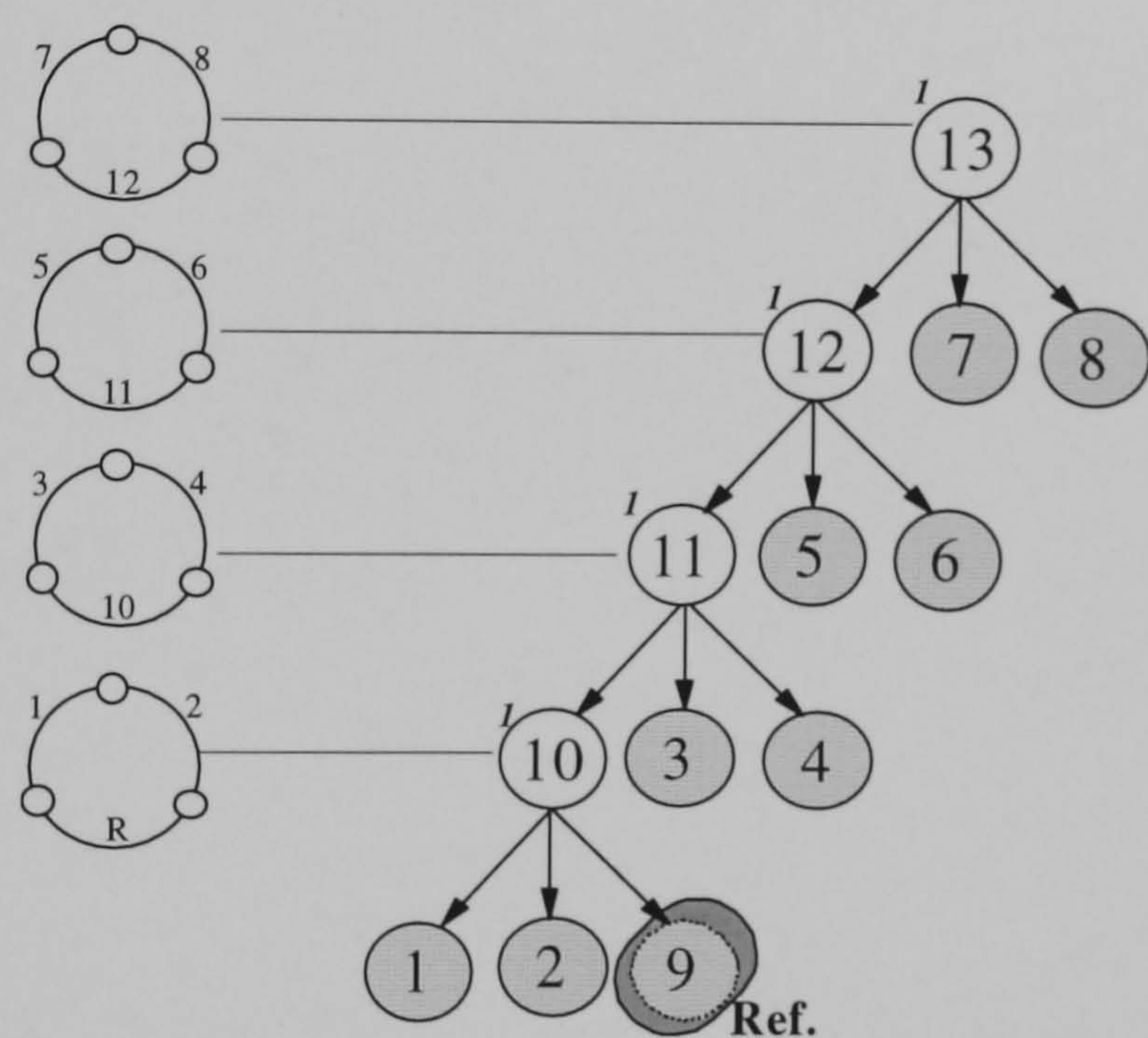
Hierarchy for Mechanism-2

Note that in mechanism-1, the cluster formation process produced five clusters in the at the last step, and three in mechanism-2. If more than one cluster exists at the end of cluster formation process, a mechanism is identified and further analysis aborted.

2. Structure with Insufficient Internal Stiffness:



Structure-A



Hierarchy for Structure-A

In the case of structures with insufficient internal stiffness, the structure becomes dependent on the reference cluster. The structure on its own does not have meaningful internal form, therefore further analysis is not applicable.